KULDIP S. RATTAN + NATHAN W. KLINGBEIL

Introductory
Mathematics for Engineering Applications

REVISED
PRELIMINARY EDITION

Introductory Mathematics for Engineering Applications

Revised Preliminary Edition

Kuldip S. Rattan
Wright State University

Nathan W. Klingbeil
Wright State University

WILEY

VP EXECUTIVE PUBLISHER	Dave Fowley
ASSOCIATE PUBLISHER	Daniel Sayre
ASSISTANT EDITOR	Samantha Mandel
MARKETING MANAGER	Christopher Ruel
PRODUCTION MANAGER	Lucile Buonocore
SENIOR PRODUCTION EDITOR	Sujin Hong
COVER PHOTO:	©Mark Watson 2010/Flickr/Getty Images, Inc.

This book is typeset by the authors

Founded in 1807, John Wiley & Sons, Inc. has been a valued source of knowledge and understanding for more han 200 years, helping people around the world meet their need and fulfill their aspirations. Our company is built on a foundation of principles that include responsibilty to the communities we serve and where we live and work. In 2008, we lunched a Corporate Citizenship Initiative, a global effort to address the envionmental, social, economic, and ethical challenges we face in our business. Among the issues we are addressing are carbon impact, paper specifications and procurement, ethical conduct within our business and among our vendors, and community and cheritable support. For more information, please visit our website: www.wiley.com/citizenship.

ISBN-13 978-1-118-46616-2

10 9 8 7 6 5 4 3 2 1

The authors would like to thank all those who have contributed to the development of this text. This includes their outstanding staff of TA's, who have not only provided numerous suggestions and revisions, but also played a critical role in the success of the first-year engineering math program at Wright State University. The authors would also like to thank their many colleagues and collaborators who have joined in their nationwide quest to change the way math is taught to engineers. Special thanks goes to Jennifer Serres, Werner Klingbeil and Scott Molitor, who have contributed a variety of worked examples and homework problems from their own engineering disciplines. Finally, the authors would like to thank their wives and families, whose unending patience and support have made this effort possible.

This material is based upon work supported by the National Science Foundation under Grant Numbers EEC-0343214, DUE-0618571, DUE-0622466 and DUE-0817332. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

PREFACE

This book is intended to provide first-year engineering students with a comprehensive introduction to the application of mathematics in engineering. This includes math topics ranging from pre-calculus and trigonometry through calculus and differential equations, with all topics set in the context of an engineering application. Specific math topics include linear and quadratic equations, trigonometry, 2-D vectors, complex numbers, sinusoids and harmonic signals, systems of equations and matrices, derivatives, integrals and differential equations. However, these topics are covered only to the extent that they are *actually used* in core first- and second-year engineering courses, including Physics, Statics, Dynamics, Strength of Materials and Electric Circuits, with occasional applications from upper-division courses. Additional motivation is provided by a wide range of worked examples and homework problems representing a variety of popular engineering disciplines.

While this book provides a *comprehensive introduction* to both the math topics and their engineering applications, it provides *comprehensive coverage of neither*. As such, it is not intended to be a replacement for any traditional math or engineering textbook. It is more like an advertisement or movie trailer. Indeed, everything covered in this book will be covered again in either an engineering or mathematics classroom. This gives the instructor an enormous amount of freedom – the freedom to integrate math and physics by *immersion*. The freedom to leverage student intuition, and to introduce new physical contexts for math without the constraint of prerequisite knowledge. The freedom to let the physics help explain the math and the math help explain the physics. The freedom to teach math to engineers the way it really ought to be taught – within a context, and for a *reason*.

Ideally, this book would serve as the primary text for a first-year engineering mathematics course, which would replace traditional math prerequisite requirements for core sophomore-level engineering courses. This would allow students to advance through the first two years of their chosen degree programs without first completing the required calculus sequence. Such is the approach adopted by Wright State University and a growing number of institutions across the country, which are now enjoying significant increases not only in engineering student retention, but also in engineering student performance in their first required calculus course.

Alternatively, this book would make an ideal reference for any freshman engineering program. Its organization is highly compartmentalized, which allows instructors to pick and choose which math topics and engineering applications to cover. Thus, any institution wishing to increase engineering student preparation and motivation for the required calculus sequence could easily integrate selected topics into an existing freshman engineering course, without having to find room in the curriculum for additional credit hours. Finally, this book would provide an outstanding resource for non-traditional students returning to school from the workplace, for students who are undecided or are considering a switch to engineering from another major, for math and science teachers or education majors seeking physical contexts for their students, or for upper-level high school students who are thinking about studying engineering in college. For all of these students, this book represents a one-stop shop for how math is really used in engineering.

CONTENTS

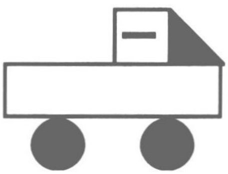
CHAPTER 1

Straight Lines in Engineering

In this chapter, the applications of straight lines in engineering are introduced. It is assumed that the students are already familiar with this topic from their high school algebra course. This chapter will show, with examples, why this topic is so important for engineers. For example, the velocity of a vehicle while braking, the voltage-current relationship in a resistive circuit, and the relationship between force and displacement in a preloaded spring can all be represented by straight lines. In this chapter, the equations of these lines will be obtained using both the slope-intercept and the point-slope forms.

1.1 Vehicle during Braking

The velocity of a vehicle during braking is measured at two distinct points in time, as indicated in Fig. 1.1.



t, s	$v(t)$, m/s
1.5	9.75
2.5	5.85

Figure 1.1: A vehicle while braking.

The velocity satisfies the equation

$$v(t) = at + v_o \tag{1.1}$$

where v_o is the initial velocity in m/s and a is the acceleration in m/s^2.

(a) Find the equation of the line $v(t)$ and determine both the initial velocity v_o and the acceleration a.

(b) Sketch the graph of the line $v(t)$ and clearly label the initial velocity, the acceleration, and the total stopping time on the graph.

The equation of the velocity given by equation (1.1) is in the slope-intercept form $y = mx + b$, where $y = v(t)$, $m = a$, $x = t$, and $b = v_o$. The slope m is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Therefore, the slope $m = a$ can be calculated using the data in Fig. 1.1 as

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5.85 - 9.75}{2.5 - 1.5} = -3.9 \text{ m/s}^2.$$

The velocity of the vehicle can now be written in the slope-intercept form as

$$v(t) = -3.9\,t + v_o.$$

The y-intercept $b = v_o$ can be determined using either one of the data points. Using the data point $(t, v) = (1.5, 9.75)$ gives

$$9.75 = -3.9\,(1.5) + v_o.$$

Solving for v_o gives

$$v_o = 15.6 \text{ m/s}.$$

The y-intercept $b = v_o$ can also be determined using the other data point $(t, v) = (2.5, 5.85)$, yielding

$$5.85 = -3.9\,(2.5) + v_o.$$

Solving for v_o gives

$$v_0 = 15.6 \text{ m/s}.$$

The velocity of the vehicle can now be written as

$$v(t) = -3.9\,t + 15.6 \text{ m/s}.$$

The total stopping time (time required to reach $v(t) = 0$) can be found by equating $v(t) = 0$, which gives

$$0 = -3.9\,t + 15.6.$$

Solving for t, the stopping time is found to be $t = 4.0$ s.

Fig. 1.2 shows the velocity of the vehicle after braking. Note that the stopping time $t = 4.0$ s and the initial velocity $v_o = 15.6$ m/s are the x- and y-intercepts of the line, respectively. Also, note that the slope of the line $m = -3.90$ m/s^2 is the acceleration of the vehicle during braking.

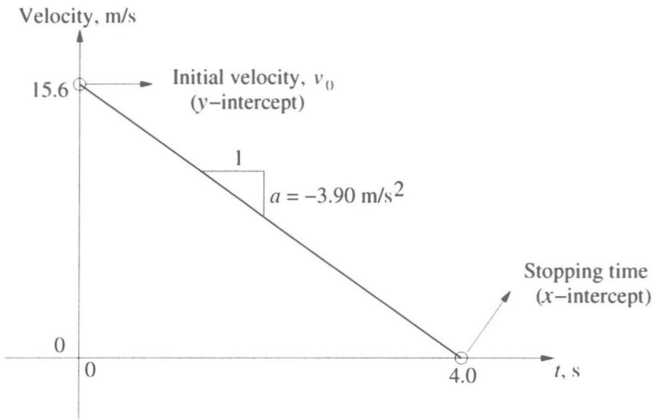
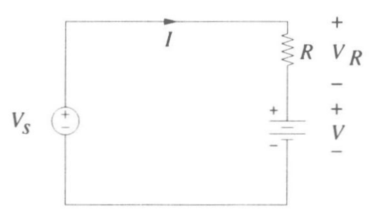


Figure 1.2: Velocity of the vehicle after braking.

1.2 Voltage-Current Relationship in a Resistive Circuit

For the resistive circuit shown in Fig. 1.3, the relationship between the applied voltage V_s and the current I flowing through the circuit can be obtained using **Kirchhoff's voltage law (KVL)** and **Ohm's law**. For a closed-loop in an electric circuit, KVL states that the sum of the voltage rises is equal to the sum of the voltage drops, for example

$$\text{Kirchhoff's voltage law: } \Rightarrow \quad \sum Voltage\ rise = \sum Voltage\ drop.$$



V_s, V	I, A
10.0	0.1
20.0	1.1

Figure 1.3: Voltage and current in a resistive circuit.

Applying KVL to the circuit of Fig. 1.3 gives

$$V_s = V_R + V. \tag{1.2}$$

Ohm's law states that the voltage drop across a resistor V_R in volts (V) is equal to the current I in amperes (A) flowing through the resistor multiplied by the resistance R in ohms (Ω), for example

$$V_R = I\,R. \tag{1.3}$$

Substituting equation (1.3) into equation (1.2) gives a linear relationship between the applied voltage V_s and the current I as

$$V_s = I\,R + V. \tag{1.4}$$

The objective is to find the value of R and V when the current flowing through the circuit is known for two different voltage values given in Fig. 1.3.

The voltage-current relationship given by equation (1.4) is the equation of a straight line in the slope-intercept form $y = mx + b$, where $y = V_s$, $x = I$, $m = R$, and $b = V$. The slope m is given by

$$m = R = \frac{\Delta y}{\Delta x} = \frac{\Delta V_s}{\Delta I}.$$

Using the data in Fig. 1.3, the slope R can be found as

$$R = \frac{20 - 10}{1.1 - 0.1} = 10\,\Omega.$$

Therefore, the source voltage can be written in slope-intercept form as

$$V_s = 10\,I + b.$$

The y-intercept $b = V$ can be determined using either one of the data points. Using the data point $(V_s, I) = (10, 0.1)$ gives

$$10 = 10\,(0.1) + V.$$

Solving for V gives

$$V = 9\text{ V}.$$

The y-intercept V can also be found by finding the equation of the straight line using the point-slope form of the straight line $(y - y_1) = m(x - x_1)$ as

$$V_s - 10 = 10(I - 0.1) \Rightarrow V_s = 10I - 1.0 + 10.$$

Therefore, the voltage-current relationship is given by

$$V_s = 10I + 9. \tag{1.5}$$

Comparing equations (1.4) and (1.5), the values of R and V are given by

$$R = 10\,\Omega, \quad V = 9\text{ V}.$$

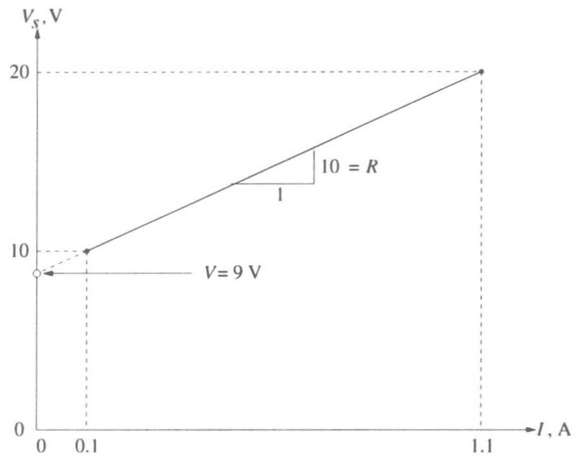


Figure 1.4: Voltage-current relationship for the data given in Fig. 1.3.

Fig. 1.4 shows the graph of the source voltage V_s versus the current I. Note that the slope of the line $m = 10$ is the resisance R in Ω and the y-intercept $b = 9$ is the voltage V in volts.

The values of R and V can also be determined by switching the interpretation of x and y (the independent and dependent variables). From the voltage-current relationship $V_s = I\,R + V$, the current I can be written as a function of V_s as

$$I = \frac{1}{R} V_s - \frac{V}{R}. \tag{1.6}$$

This is an equation of a straight line $y = m\,x + b$, where x is the applied voltage V_s, y is the current I, $m = \frac{1}{R}$ is the slope, and $b = -\frac{V}{R}$ is the y-intercept. The slope and y-intercept can be found from the data given in Fig. 1.3 using the slope-intercept method as

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V_s}.$$

Using the data in Fig. 1.3, the slope m can be found as

$$m = \frac{1.1 - 0.1}{20 - 10} = 0.1.$$

Therefore, the current I can be written in slope-intercept form as

$$I = 0.1\,V_s + b.$$

The y-intercept b can be determined using either one of the data points. Using the data point (V_s, I) = (10, 0.1) gives

$$0.1 = 0.1\,(10) + b.$$

Solving for b gives

$$b = -0.9.$$

Therefore, the equation of the straight line can be written in the slope-intercept form as

$$I = 0.1\,V_s - 0.9. \tag{1.7}$$

Comparing equations (1.6) and (1.7) gives

$$\frac{1}{R} = 0.1 \quad \Rightarrow \quad R = 10\,\Omega$$

and

$$-\frac{V}{R} = -0.9 \quad \Rightarrow \quad V = 0.9(10) = 9\text{ V}.$$

Fig. 1.5 is the graph of the straight line $I = 0.1V_s - 0.9$. Note that the y-intercept is $-\dfrac{V}{R} = -0.9$ A and the slope is $\dfrac{1}{R} = 0.1$.

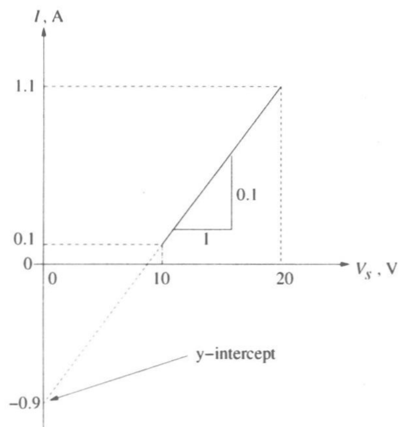


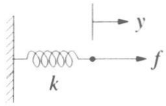
Figure 1.5: Straight line with I as independent variable for the data given in Fig. 1.3.

1.3 Force-Displacement in a Preloaded Tension Spring

The force-displacement relationship for a spring with a preload f_o is given by

$$f = ky + f_o, \tag{1.8}$$

where f is the force in *Newtons* (N), y is the displacement in *meters* (m), and k is the spring constant in N/m.



f, N	y, m
1	0.1
5	0.9

Figure 1.6: Force-displacement in a preloaded spring.

The objective is to find the spring constant k and the preload f_o, if the values of the force and displacement are as given in Fig. 1.6.

Method 1: Treating the displacement y as an independent variable, the force-displacement relationship $f = ky + f_o$ is the equation of a straight line $y = mx + b$, where the independent variable x is the displacement y, the dependent variable y is the force f, the slope m is the spring constant k, and the y-intercept is the preload f_o. The slope m can be calculated using the data given in Fig. 1.6 as

$$m = \frac{5-1}{0.9-0.1} = \frac{4}{0.8} = 5.$$

The equation of the force-displacement equation in the slope-intercept form can therefore be written as

$$f = 5y + b.$$

The y-intercept b can be found using one of the data points. Using the data point $(f,\, y) = (5,\, 0.9)$ gives

$$5 = 5(0.9) + b.$$

Solving for b gives

$$b = 0.5\ \mathrm{N}.$$

Therefore, the equation of the straight line can be written in slope-intercept form as

$$f = 5y + 0.5. \tag{1.9}$$

Comparing equations (1.8) and (1.9) gives

$$k = 5\ \mathrm{N/m}, \quad f_o = 0.5\ \mathrm{N}.$$

Method 2: Now treating the force f as an independent variable, the force-displacement relationship $f = ky + f_o$ can be written as $y = \frac{1}{k} f - \frac{f_o}{k}$. This relationship is the equation of a straight line $y = mx + b$, where the independent variable x is the force f, the dependent variable y is the displacement y, the slope m is the reciprocal of the spring constant $\frac{1}{k}$, and the y-intercept is the negated preload divided by the spring constant $-\frac{f_o}{k}$. The slope m can be calculated using the data given in Fig. 1.6 as

$$m = \frac{0.9-0.1}{5-1} = \frac{0.8}{4} = 0.2.$$