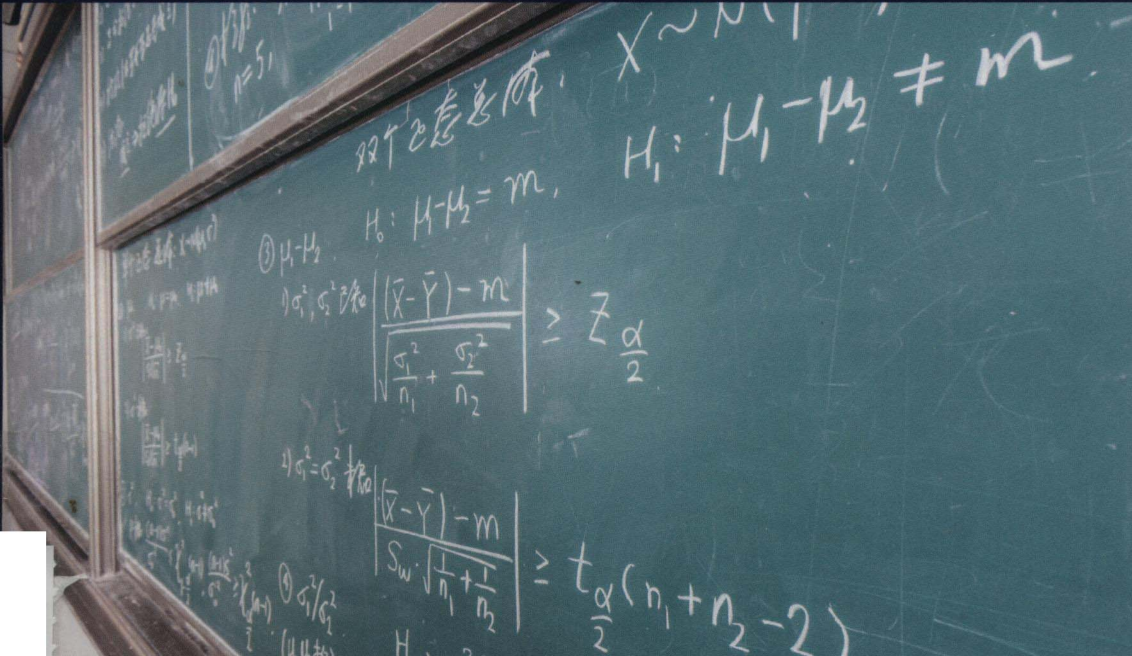
Semi-Markov Models

Control of Restorable Systems with Latent Failures



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Preface

The improvement of industrial systems' reliability and production quality is a real problem of the modern industry. The automatic systems of technological processes management enable solving this problem. The local technical control constitutes an important part of the system.

In spite of the diversity and high level of checkout and measurable instruments, the problem of latent failures detection and elimination is still significant.

The latent failure is a failure which cannot be detected by standard methods or visually, but by maintenance or special methods of diagnostics only.

In this monograph, by the latent failure we refer to one which can be detected during control execution only.

In complex industrial systems, the periodical control is applied. The reason is the difficulty of checking individual operation of all units and details (components). It means the control is carried out at fixed (in general case random) time periods, which should be optimal for the whole system by ensuring its maximum reliability and efficiency. The problem can be solved by constructing mathematical models of control of restorable systems with latent failures.

The present monograph is dedicated to building such models on the basis of the theory of semi-Markov processes with arbitrary phase state space, as well as to the definition of optimal periodicity of latent failures control. The problems of application of the results obtained are considered.

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Introduction

Technical control is an important part of the production quality control department of any enterprise. The rapid development of technologies, increase in quality, and reliability requirements result in considerable growth of technical control expenses. As noted in [6], metal-processing industry spends 8–15% of expenses on the quality control. It takes from 5 hours to several weeks to make the project of a single detail control, and from 40 minutes to several hours to execute its control. That is why the tasks of reduction of control expenses and efficiency increase are significant.

Mathematical models of technical control execution can serve to solve the problem. These models allow one to analyze the efficiency of different control strategies and to define optimal periods of their execution. The present monograph is dedicated to the control modeling with regard to latent failures of the technical system.

According to the possibility of detection, there are two types of failures [20]:

- evident failure, which can be detected visually or by standard methods of control and diagnostics in the process of object preparation and exploitation;
- latent failure, which can be detected by maintenance or special methods of diagnostics only.

A great number of parametric failures are referred to as latent.

As stated in the Preface, by the latent failure we mean the one which can be detected in the control process.

In the present monograph, to build control models, the approach introduced by V.S. Korolyuk, A.F. Turbin, and their disciples [13–17] is used. It is based on the application of the theory of semi-Markov processes with arbitrary phase space. This approach allows us to omit some restrictions, in particular the assumption of exponential distribution laws of random variables, describing the system. It enables obtaining applicable system operation characteristics. In cases of high model dimensions, algorithms of phase merging serve as an efficient approximation method [14–17].

In this present monograph, the concept of a system component is involved. A component is a constituent part or element of a system. If a system functionally consists of one element (component), not divisible from the point of view of failures, it is called a one-component. The system consisting of $n \geq 2$ indivisible components is named multicomponent [4, 20].

In the present work one- and two-component restorable systems with latent failures control are investigated. However, the approach can be applied to multicomponent systems [21, 22].

In Chapter 1 of the monograph, preliminaries are given.

Chapter 2 covers semi-Markov models for different control strategies in one-component systems. Their stationary characteristics of reliability and efficiency are defined. For the characteristics approximation, we apply the method offered in [14]. It has common background with algorithms of asymptotic phase merging.

Chapter 3 is dedicated to semi-Markov models of latent failures control in two-component systems.

In Chapter 4, on the basis of the results obtained in Chapters 2 and 3, the problems of optimal periodicity of control execution are solved.

Chapter 5 contains comparative analysis of analytical and imitational modeling of some one- and two-component systems. The possibility of practical application of the results represented in the present monograph is considered.

In Chapter 6 semi-Markov models of systems of different function are considered:

- model of queuing system with losses;
- model of system with a cumulative reserve of time;
- model of two-phase system with a intermediate buffer;
- model of technological cell with nondepreciatory failures.

Appendices include data, to support the reader's understanding of the basic text.

Contents

Chapter 1

Preliminaries

Chapter Outline

1.1 STRATEGIES AND CHARACTERISTICS OF TECHNICAL CONTROL

Automatic checkout systems consist of the object, engineering devices, programs, and operator, which enable to carry out automatic control. Control strategy usually means the rule defining the choice of checkout means with regard to the system controlled. There exist efficiency control and preventive control [6]. Efficiency control is checkout of the production capability to fulfil functions under parameters, determined by manuals. Preventive control is a technical checkout for detection and prevention of defects or flaws.

In the monograph, efficiency control is investigated. Efficiency control is devided into ideal and nonideal [4]. Under ideal efficiency control, all the failures are detected immediately and reliably [4]. Under nonideal efficiency control, latent failures and automatic checkout system failures take place [4].

In Figure 1.1, a general scheme of efficiency control execution with the help of automatic checkout systems is presented. One of the control characteristics is its periodicity. The control periodicity is time period between two successive checkout processes, executed by certain control instrument [6].

According to the object, continuous, periodical, and casual kinds of control are singled out. Under continuous control, the information on parameters is received constantly, while under periodical control it happens at certain time intervals. Casual control is carried out at random time intervals [6]. Casual control includes single control. The latter is executed, for instance, before the use of stored system, in case the system reliability is ensured by the storage measures.

In the monograph, periodic control with full efficiency restoration is investigated.

In Sections 2.1, 2.4, 3.3–3.5 and Sections 2.2, 2.3, 3.1, 3.2, system efficiency control with component deactivation and without deactivation while control execution are considered, respectively.

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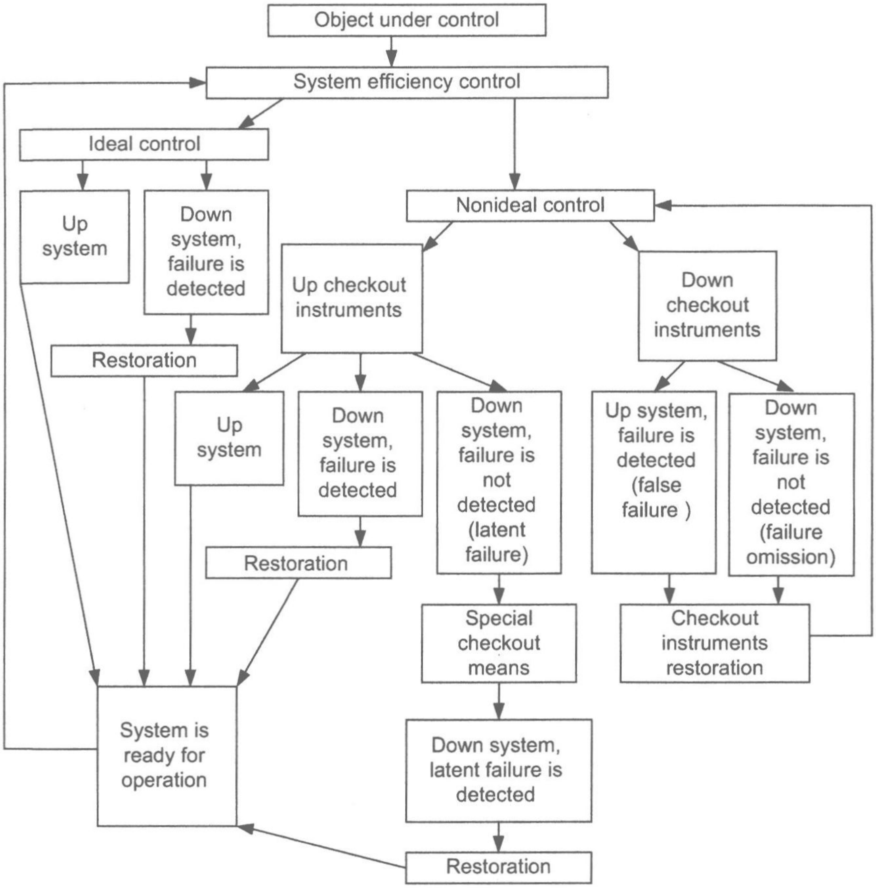

FIGURE 1.1 General scheme of efficiency control execution

1.2 PRELIMINARIES ON RENEWAL THEORY

In the present section, renewal theory review is made [1–3, 7]. The information is mainly given according to monograph [3].

Renewal theory origins from the simplest restoration model: after each failure the system is restored immediately.

Definition [3]. Renewal process is a sequence $\{\alpha_n; n \geq 1\}$ of non-negative independent random variables (RVs) with the same distribution function (DF) $F(t)$.

RVs $T_k = \sum_{n=1}^{k} \alpha_n, k \geq 1, T_0 = 0$ are called renewal moments. Renewal process is often defined as a sequence of RV $\{T_k\}$ as well. The counting renewal process, defined with the help of renewal moments, is of particular interest

$$N(t) = \max\{k : T_k \leq t\}.$$

For each moment t, the value of $N(t)$ determines the random number or renewal moments in $[0,t]$. Renewal function $H(t)$, defining the mean of renewal moments in $(0,t]$, plays fundamental role in renewal theory:

$$H(t)=E[N(t)]=\sum_{k=1}^{\infty}kP\{N(t)=k\}=\sum_{k=1}^{\infty}F^{*(k)}(t), \tag{1.1}$$

where $F^{*(k)}(t)$ is a k-fold convolution of DF $F(t)$, $F^{*(1)}(t)=F(t)$.

The function is a solution of the integral renewal equation:

$$H(t)=F(t)+\int_0^t H(t-x)dF(x).$$

Renewal function derivative $h(t)=H'(t)=\sum_{k=1}^{\infty}f^{*(k)}(t)$ is called renewal density. It satisfies the following integral equation:

$$h(t)=f(t)+\int_0^t h(t-x)f(x)dx, \tag{1.2}$$

where $f(t)$ is the density of DF $F(t)$.

The following formulas are true:

$$\int_0^t \overline{F}(s)h(t-s)ds+\overline{F}(t)=1, \int_0^t H(t-s)\overline{F}(s)ds+\int_0^t \overline{F}(s)ds=t. \tag{1.3}$$

Under sufficiently small Δt, the value $h(t)\Delta t$ approximately equals the probability of renewal moment appearance in $(t,t+\Delta t]$.

Explicit form of functions $H(t)$ for some renewal processes can be found in [3, 7].

The functions:

$$\tilde{H}(t)=1+H(t), \hat{H}(t)=\begin{cases}1+H(t), t>0,\\ 0, t=0.\end{cases} \tag{1.4}$$

will be applied as well.

The function $\hat{H}(t)$, having a unit jump at $t=0$, is used to write down expressions in form of Stieltjes integral.

The following theorems, describing asymptotic behavior (under $t\to+\infty$) of the function $H(t)$, take place.

Renewal theorem (elementary) [3]. For any distribution law of $F(t)$

$$\lim_{t\to+\infty}\frac{H(t)}{t}=\frac{1}{\mu}, \tag{1.5}$$

where $\mu=E\alpha$ is expectation of RV α.

Renewal theorem (key) [3]. If $F(t)$ is not an arithmetic distribution, and $g(t)$ is integrable nonincreasing in $(0,+\infty)$ function, then

$$\lim_{t\to+\infty}\int_0^t g(t-x)\,dH(x)=\frac{1}{\mu}\int_0^\infty g(x)\,dx. \tag{1.6}$$

The process of direct residual time $V_t=\tau_{N(t)+1}-t, t\geq 0$, V_t being residual time to failure by t, is connected with the renewal process $\{\alpha_n; n\geq 1\}$. V_t is a homogeneous Markov process with phase state $(0,+\infty)$.

DF $V(t,x)=P\{V_t\leq x\}$ of the direct residual time is defined by the formula [3]:

$$V(t,x)=F(t+x)-\int_0^t \overline{F}(t+x-s)\,dH(s), \tag{1.7}$$

and corresponding distribution density is:

$$v(t,x)=f(t+x)+\int_0^t f(t+x-u)h(u)\,du. \tag{1.8}$$

The expectation of the direct residual time equals:

$$E(V_t)=E\alpha(1+H(t))-t. \tag{1.9}$$

In the renewal process, the restoration of failed system is considered to be negligible in comparison with operating time. This assumption does not take place in practice. That is why the following system renewal process is considered [3].

For the first time, the system fails in a random time period α_1 and is restored in random time β_1. The restored system operates for α_2 time, then it fails, and is restored in β_2, and so on. Time moments $T_1=\alpha_1, T_2=\alpha_1+\beta_1+\alpha_2,\ldots$, of the system failure are called failure moments or moments of 0-renewation, and the moments $S_1=\alpha_1+\beta_1$, $S_2=\alpha_1+\beta_1+\alpha_2+\beta_2,\ldots$, of the restoration end are restoration moments (or 1-renewations).

Definition [3]. If $\{\alpha_n; n\geq 1\}$ and $\{\beta_n; n\geq 1\}$ are two sequences of independent similarly distributed RVs, the sequence $\{(\alpha_n,\beta_n); n\geq 1\}$, as well as $\{(T_n,S_n); n\geq 1\}$, is called an alternating renewal process.

Alternating renewal process can be equivalently given by $\{Z(t), t\geq 0\}$ with the help of

$$Z(t)=\begin{cases}0, \text{if } t\in(T_k,S_k),\\ 1, \text{otherwise.}\end{cases}$$