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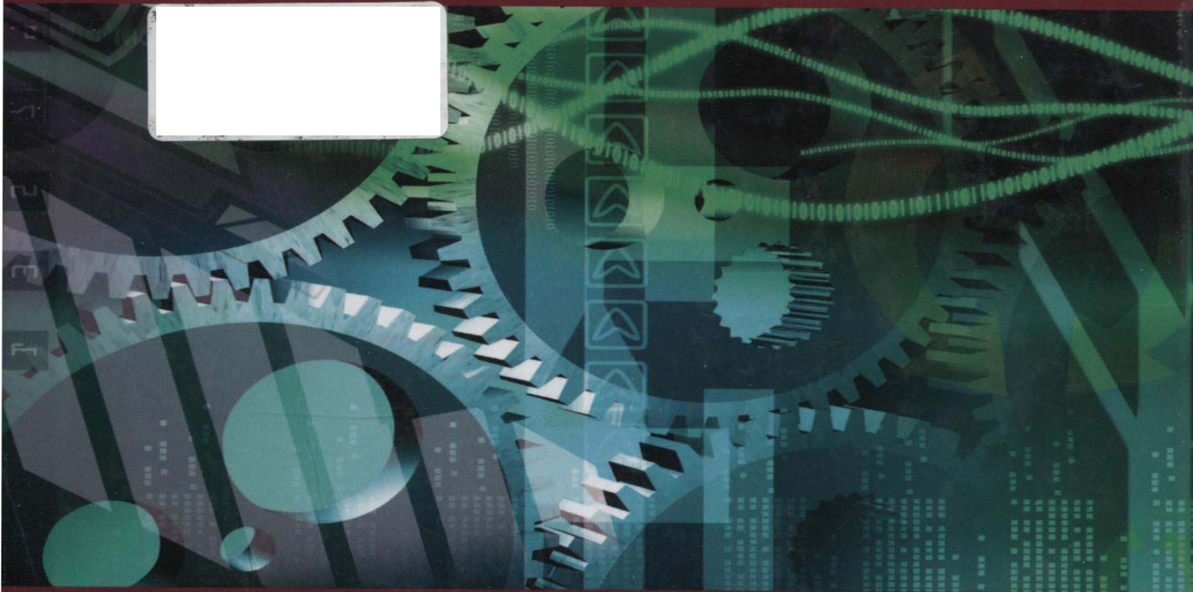

Optimization in Engineering Sciences

Exact Methods

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Florin Gh. Filip and Dan Stefanoiu

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Foreword

The optimization theory field is already well defined, strong and mature, with plenty of theoretical results and remarkable applications. Nowadays, it takes courage to publish a new book on classical optimization issues. Although it is said that anyone can conceive a new optimization technique, outperforming the existing algorithms in terms of convergence speed and efficient implementation is rather difficult. However, improvements should be possible.

What makes this book interesting, and original at the same time, is something that is often missing from publications of quality scientific literature: the engineering point of view. As Albert Einstein said so well, it is quite sad to see how a beautiful theory is destroyed by an ugly reality. In this spirit, optimization theory has plenty of pure theoretical results that are quite impossible to transform into efficient numerical procedures to be employed later in real applications. However, the milestone of this book is, seemingly, the optimization algorithm for the benefit of application.

The authors succeed in describing quite a large panoply of optimization techniques, from simple ones like linear or dynamic programming, to complex ones including nonlinear programming, large-scale systems, system identification or automatic control strategies. Of course, no-one can encompass in a single volume all the optimization methods that the authors refer here to as "exact", i.e. non-heuristic, or stochastic. For example, the recent group of Linear Matrix Inequality (LMI) optimization techniques, based on the interior point methods, is not presented here. However, I assume that the final goal to fulfill here was not to cover all possible topics in optimization, as in a treatise. The authors rather intended to meet the engineering need for clear and efficient optimization procedures ready to be implemented and, moreover, easy to adapt to specific applications. In spite of optimization toolboxes or dynamic link libraries that can be found on various software platforms, the user is faced with two major problems when approaching applications that require optimization of some criteria. First of all, he/she does not know very well the meaning of input arguments to be set for a function implementing some optimization technique. This book enlightens the user in this aim, by revealing how to configure the numerical procedure associated with each optimization technique. Second, he/she could not modify the optimization function if some application requirements are to be met. On the contrary, very often, problems within specific applications are reformulated, in order to adapt to some available optimization procedure, which, of course, could change the initial nature of those applications. This book describes the steps of each algorithm in a clear and concise manner, so that anyone can implement it in some particular way, if necessary.

The methods described in the book include: linear programming with various implementations, nonlinear programming, dynamic programming with various application examples, Hopfield networks, optimization in systems identification, optimization of dynamic system with particular application to process control, optimization of large-scale and complex systems using decomposition techniques, optimization and information systems.

As described above, the reader may understand that the book is just an optimization algorithms compendium, which is not true at all. It is much more than that. For each algorithm, where possible, a sound analysis concerning its foundation, convergence, complexity and efficiency is presented. Easy to follow examples also exist, where possible. Most of the numerical procedures introduced here are improved compared to the original or other improved procedures found in the scientific literature.

As a final word, I am pleased to see that exact optimization methods could be improved and, moreover, help the engineer, regardless of the fields of activity, to better understand them and how to apply them, and what their limitations are, etc. The authors were clearly inspired to write such a book, which, I hope, will be welcomed both by the scientific community and practitioners.

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November 2012

Preface

The purpose of this book is to introduce the most important methods of static and dynamic optimization, from an engineering point of view.

The methods are *exact*, in the sense that optimum solutions are searched by means of accurate, deterministic numerical algorithms, the convergence being soundly proven for most of them.

In order to focus on the optimization algorithms and to make the presentation friendly, the proofs of various results are often not developed. However, some remarks or short rationales regarding the principles of various proposed algorithms, sometimes with additional references allowing the interested reader to explore the optimization topics in depth, are given.

When the optimization algorithms are not too complex, some easy to follow and reproducible implementation examples are presented.

The methods described within the book include:

– linear programming with various implementations;

– nonlinear programming, which is a particularly important topic, given the wide variety of existing algorithms;

– dynamic programming with various application examples;

– Hopfield networks;

– optimization in systems identification;

– optimization of dynamic systems with particular application to process control;

– optimization of large-scale and complex systems;

– optimization and information systems.

Optimization techniques for difficult problems implementing metaheuristic, stochastic and suboptimal approaches will be addressed in a different book.

This book was produced within the framework of the European FP7 project ERRIC (*Empowering Romanian Research on Intelligent Information Technology*), contract FP7-REGPOT-2010-1/264207 and developed in cooperation between French and Romanian scientists.

Pierre BORNE, Dumitru POPESCU, Florin Gh. FILIP and Dan STEFANOIU
Lille and Bucharest
November 2012

Acronyms

AIVM	adaptive instrumental variables method
AIVMλ	adaptive instrumental variables method with exponential window
AIVM□	adaptive instrumental variables method with rectangular window
ALSM	adaptive least squares methods
ALSMλ	adaptive least squares methods with exponential window
ALSM□	adaptive least squares methods with rectangular window
ARE	algebraic Riccati equation
ARMAX	class of autoregressive moving average with exogenous control identification models
BFGS	Broyden-Fletcher-Goldfarb-Shanno algorithm
DAS	decision assistance systems
DFP	Davidon-Fletcher-Powell algorithm
DSM	direct search method(s)
EDSM	evolving direct search method(s)
ELSM	extended least squares methods
ET	estimation theory
FIR	finite impulse response
GM	gradient(-based) methods
GNM	Gauss-Newton method
I/O	Input-Output
IIR	infinite impulse response
ITaaS	Information Technology as a Service
ITC	Information Technology and Communications
IVM	instrumental variables method
KBF	Kalman-Bucy filter
KBP	Kalman-Bucy predictor
LDSM	linear direct search method(s)
LOP	linear optimization problem(s)
LQ	linear quadratic solution or order

LQG	linear quadratic generalized solution or order
LSM	least squares method
LTR	loop transfer recovery
NLOP	nonlinear optimization problem(s)
NRM	Newton-Raphson method
OT	optimization theory
RIO	class of rational input-output identification models
SaaS	Software as a Service
SI	systems identification
SOP	separable optimization problem(s)
TM	transformation methods

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Chapter 1

Linear Programming

1.1. Objective of linear programming

The purpose of linear programming [MUR 83, MEL 04, VAN 08] is to optimize a linear function $J(\mathbf{x}) = \mathbf{f}^T\mathbf{x}$ of a set of variables grouped in vector $\mathbf{x} \in \mathbb{R}^n$ in the presence of linear constraints. This is one of the rare cases where an iterative algorithm converges into a finite number of iterations, by only using elementary manipulations.

1.2. Stating the problem

Consider a polyhedron in $\mathbb{R}^n$ (with $n \geq 2$), defined by a system of linear inequalities $\mathbf{Ax} \leq \mathbf{b}$. To each point of polyhedron, a value defined by linear function $J(\mathbf{x}) = \mathbf{f}^T$ is assigned. Here, $\mathbf{f} \in \mathbb{R}^n$ is a constant vector, initially known. By *linear programming* we understand a procedure, which enables us to solve the problem of finding a point $\mathbf{x} \in \mathbb{R}^n$ of the polyhedron that minimizes or maximizes J function. Since the maximization problem is similar to the minimization one. This problem reads as follows:

$$\left[\begin{array}{l} \min\limits_{\mathbf{x}\in\mathbb{R}^n} \mathbf{f}^T\mathbf{x} \\ \text{with}: \begin{cases} \mathbf{Ax} \leq \mathbf{b} \\ \mathbf{x} \geq \mathbf{0}, \end{cases} \end{array}\right. \qquad [1.1]$$

where "min" means *minimize* and: $\mathbf{A} \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^{m}$, with $m < n$. Usually, J is referred to as *economic function* or *objective function* or simply *criterion* (of linear optimization). The inequalities $\mathbf{Ax} \leq \mathbf{b}$ define the constraints of the problem, while "s.t." stands for "subject to". Matrix $\mathbf{A}$ is by nature of maximum rank (i.e. *epic*), in order to make the constraints independent of each other.

To illustrate the corresponding geometric problem [1.1], consider the case of a polygon (in the Euclidean plane), as shown in Figure 1.1.

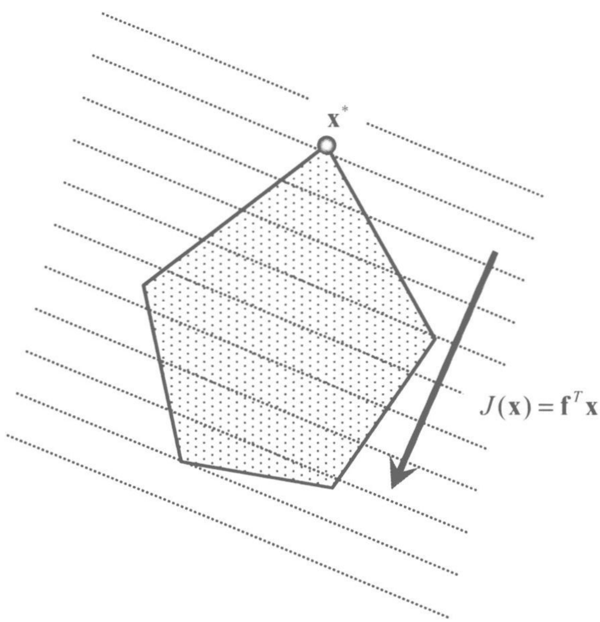

Figure 1.1. *Geometrical representation of the linear optimization problem*

The set of parallel lines is generated by considering $\mathbf{f}^T\mathbf{x}$ equal to various constants, hence the name *linear programming problem*. In this context, a result of mathematics states that the minimum can only be obtained at one of the polyhedron vertices (e.g. $\mathbf{x}^*$ in the figure). If the lines are also parallel to a side of the polyhedron, then all the points of this side correspond to an extreme of the objective function. More generally, a non-vertex polyhedron point can correspond to an optimal solution, only if there is an optimum side of the polyhedron that includes it.