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国外
优秀数学
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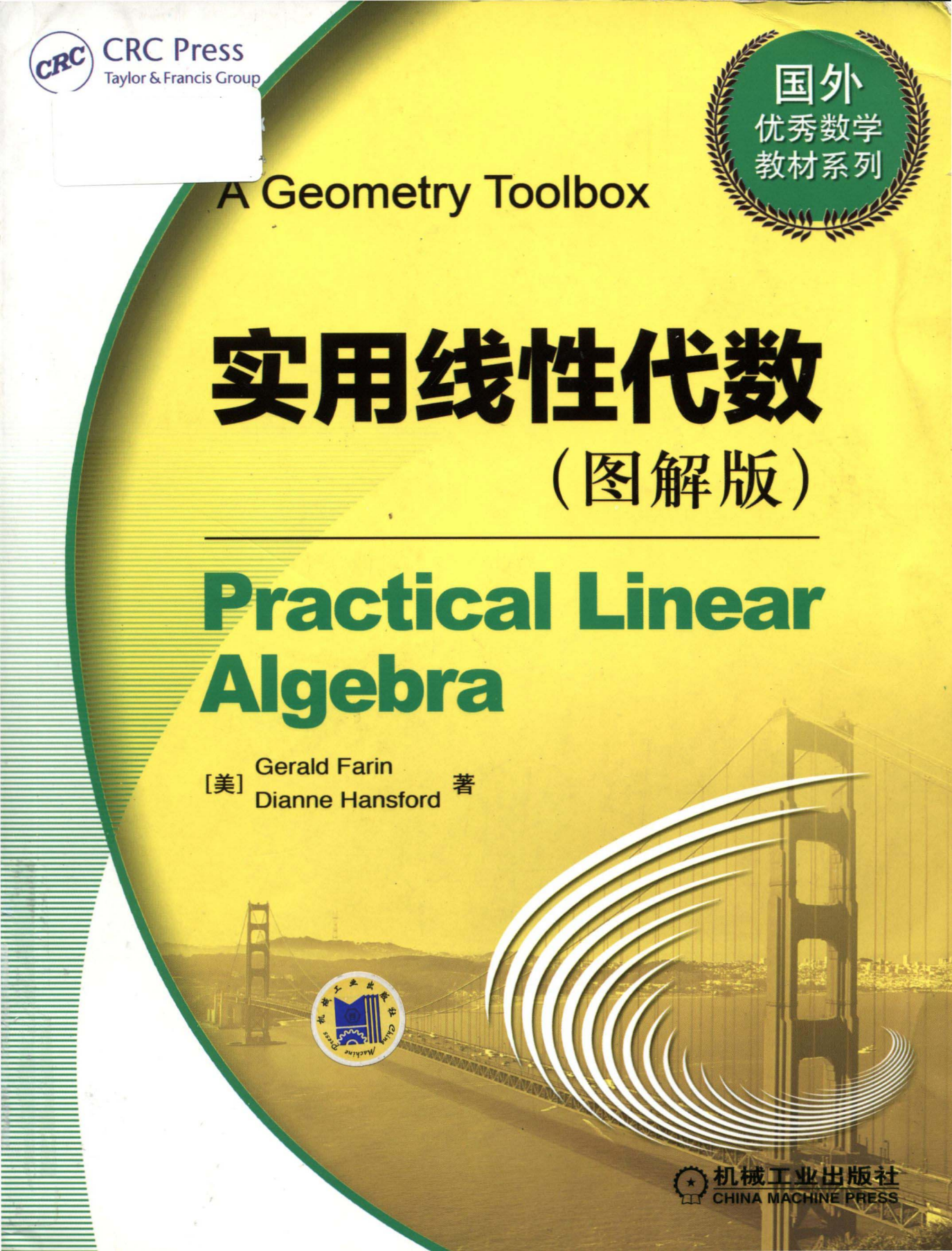
A Geometry Toolbox

# 实用线性代数
## （图解版）

# Practical Linear Algebra

[美] Gerald Farin
Dianne Hansford 著



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# 实用线性代数（图解版）

[美] Gerald Farin
Dianne Hansford 著

李红玲 注释



机械工业出版社

本书区别于以往线性代数的书籍，内容新颖，编排独特。作者以几何视角讲述线性代数，通过二维平面和三维空间中的例子解释线性代数中的各种概念和性质。本书强调直观性以及知识点的背景，结合计算机中各种图形的变换来理解线性变换，在注重可读性的同时突出数学的基本思想，将直观图形与数学证明进行了巧妙的结合。作者在书页侧边空白处手绘200余幅示意图并给出了相关概念的解释，以便更好地帮助读者理解。

本书可供非数学专业的学生及数学爱好者使用，亦可作为数学专业学生和教师的参考用书。

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# 注释者的话

对于工科与经济类的大学生而言，“线性代数”是一门公共基础课，也是必修的课程之一。常见的线性代数教科书大致有这样几个章节——行列式、线性方程组、矩阵、向量空间、特征值与特征向量、二次型。翻开教材，迎面而来的就是计算与证明，鲜有知识点产生的缘由及其在实际生活中的应用。诚然，数学是一门抽象的科学，它具有高度的概括性，但这不代表数学教材就应该这样“斩头去尾烧中段”，干巴巴得毫无吸引力，冷冰冰得让人生畏。数学教育研究者们一直在呼吁数学文化的渗透，那么对于具体的一本教材而言，渗透什么内容？如何进行渗透？渗透到什么程度？我想，最基本地来说，至少要将知识的来龙去脉说清楚。比如常见教科书开篇就是行列式的计算，那么学生肯定想要知道“行列式的本质是什么？为什么要学行列式？它在实际应用中的作用是什么？”在缺乏理解的基础上，就算学会了计算与证明，对知识的把握也是稀里糊涂的。这是件让人遗憾的事情。

本书，是一本弥补遗憾的书，是一本不同视角的书，是一本呈现知识点来龙去脉的书。本书按照先二维后三维的顺序呈现知识，使得知识形象化，便于理解。全书共分 18 章，第 1 章到第 9 章是二维情形，以独特的顺序与适宜的方式介绍了线性代数的基本知识点；第 10 章到第 13 章是三维情形，因此这四章是前九章的推广，但并不重复，各有侧重；第 14 章到第 18 章是高维情形，呈现了许多实际生活中的应用，同时也有助于读者抽象思维的发展。本书采用了非常规却符合认知的知识呈现顺序，以直观的、几何的叙述方式呈现内容，以大量的实际例证呈现应用。如果你不曾学过线性代数，阅读本书，会让你兴趣盎然地沉浸在其中，顺理成章地掌握所有应掌握的知识点；如果你曾经学过线性代数，阅读本书，会让你不断地恍然大悟：“哦！原来这个知识点是这么来的！原来这个知识点是这么用的！原来这两个知识点之间竟然有这层关系！”如果你想要看一本有趣且有用的线性代数书籍，那么本书就是一个不可错过的上好选择！

作为注释者，必须要兼顾到两个方面：第一是作者方面——注释者必须要掌握书中的内容，理解作者的意图，跟随作者的思路，体会各种安排的妙处；第二是读者方面——注释者必须站在读者的角度去学习、去思考、去询问、去解答，才会知道哪些点是需要注释的。通过与编辑的交流，我们统一了标准：既要对读者有疑惑的地方给予注释，同时又要注意到不可以越线——注释不应该代替读者的学习，所以注释中尽量只提出解决疑惑的方法与工具，具体的过程仍然留给读者自行完成。如有读者想要讨论该书的注释或内容，请发邮件至 sxlslhl@126.com，愿与君同思考共探索！

李红玲

# Preface

We assume just about everyone has watched animated movies, such as *Toy Story* or *Shrek*, or is familiar with the latest three-dimensional computer games. Enjoying 3D entertainment sounds like more fun than studying a Linear Algebra book, right? But it is because of Linear Algebra that those movies and games can be brought to a TV or computer screen. When you see a character move on the screen, it's animated using some equation straight out of this book. In this sense, Linear Algebra is a driving force of our new digital world: it is powering the software behind modern visual entertainment and communication.

But this is not a book on entertainment. We start with the fundamentals of Linear Algebra and proceed to various applications. So it doesn't become too dry, we replaced mathematical proofs with motivations, examples, or graphics. For a beginning student, this will result in a deeper level of understanding than standard theorem-proof approaches. The book covers all of undergraduate-level linear algebra in the classical sense—except it is not delivered in a classical way. Since it relies heavily on examples and pointers to applications, we chose the title *Practical Linear Algebra*, or *PLA* for short.

The subtitle of this book is *A Geometry Toolbox*; this is meant to emphasize that we approach linear algebra in a geometric and algorithmic way. This book grew out of a previous one, namely *The Geometry Toolbox for Graphics and Modeling*. Our goal was to bring the material of that book to a broader audience, motivated in a large part by our observations of how little engineers and scientists (non-math majors) retain from classical linear algebra classes. Thus, we set out to fill a void in the linear algebra textbook market. We feel

that we have achieved this, as well as maintaining the spirit of our first effort: present the material in an intuitive, geometric manner that will lend itself to retention of the ideas and methods.

## Review of Contents

As stated previously, one clear motivation we had for writing PLA was to present the material so that the reader would retain the information. In our experience, approaching the material first in two and then in three dimensions lends itself to visualizing and then to understanding. Incorporating many illustrations, Chapters 1–7 introduce the fundamentals of linear algebra in a 2D setting. These same concepts are revisited in Chapters 10–13 in a 3D setting. The 3D world lends itself to concepts that do not exist in 2D, and these are explored there too.

Higher dimensions, necessary for many real-life applications and the development of abstract thought, are visited in Chapters 14–16. The focus of these three chapters includes linear system solvers (Gauss elimination, LU decomposition, Householder's method, and iterative methods), determinants, inverse matrices, revisiting "eigen things," linear spaces, inner products, and the Gram-Schmidt process.

Conics, discussed in Chapter 9, are such a fundamental geometric entity, and since their development provides a wonderful application for affine maps, "eigen things," and symmetric matrices, they really shouldn't be missed. Triangles in Chapter 8 and polygons in Chapter 17 are discussed because they are fundamental geometric entities and are important in generating computer images. The basics of generating curves are presented in Chapter 18; this, too, is a nice example of how linear algebra may be applied.

The illustrations in the book come in two forms: figures and sketches. The figures are computer generated and tend to be complex. The sketches are hand-drawn and illustrate the core of a concept. Both are great teaching and learning tools! We made all of them available on the book's website (http://vidya.prism.asu.edu/~farin/pla). Many of the figures were generated using PostScript, an easy-to-use geometric language. We have provided a tutorial to this language in Appendix A. This brief tutorial gives enough information for the reader to modify the figures in this book, as well as create their own. However, this book can certainly be used without getting involved with PostScript.

At the end of each chapter, we have included a list of topics, *What You Should Know* (*WYSK*), marked by the icon on the left. This list

is intended to encapsulate the main points of each chapter. It is not uncommon for a topic to appear in more than one chapter. We have made an effort to revisit some key ideas more than once. Repetition is useful for retention!

Exercises are listed at the end of each chapter. Solutions to selected exercises are given in Appendix B. More solutions may be found on the book's website. PostScript figures are also used as a basis for some exercises—PostScript offers an easy way to get hands-on experience with many geometry concepts. Often these exercises are labeled *PS* to indicate that they involve PostScript.

## Classroom Use

PLA is meant to be used at the freshman/sophomore undergraduate level. It serves as an introduction to Linear Algebra for engineers or computer scientists, as well as a general introduction to geometry. It is also an ideal preparation for Computer Graphics and Geometric Modeling.

As a one-semester course, we recommend choosing a subset of the material that meets the needs of the students. In the table below, LA refers to an introductory Linear Algebra course and CG refers to a course tailored to those planning to work in Computer Graphics or Geometric Modeling. We have created shortened chapter titles to make the table more readable.

| | Chapter | LA | CG |
|---|---|---|---|
| 1 | 2D Coordinate Systems | • | • |
| 2 | 2D Points & Vectors | • | • |
| 3 | 2D Lines | | • |
| 4 | 2D Linear Maps | • | • |
| 5 | 2×2 Linear Systems | • | • |
| 6 | 2D Affine Maps | • | • |
| 7 | 2D Eigen Things | • | |
| 8 | Triangles | | • |
| 9 | Conics | • | |
| 10 | 3D Geometry | • | • |
| 11 | 3D Interactions | | • |
| 12 | 3D Linear Maps | • | • |

| | Chapter | LA | CG |
|---|---|---|---|
| 13 | 3D Affine Maps | • | • |
| 14 | Linear Systems | • | • |
| 15 | Linear Spaces | • | |
| 16 | Numerical Methods | • | |
| 17 | Polygons | | • |
| 18 | Curves | | • |

## Website

*Practical Linear Algebra, A Geometry Toolbox* has a website: http://vidya.prism.asu.edu/~farin/pla.

This website provides:

- teaching materials,
- additional solutions to exercises,
- the PostScript files illustrated in the book,
- data files referred to in the text,
- errata,
- and more!

A K Peters, Ltd. also maintains a website: http://www.akpeters.com.

## Acknowledgements

We certainly appreciate the positive feedback that we received on *The Geometry Toolbox for Graphics and Modeling.* Some of that feedback may be found on 'The Toolbox's' website.

A few 3D images were generated using GEOMVIEW from The Geometry Center at the University of Minnesota, http://www.geom.uiuc.edu/. (It is now closed.)

Lastly, thanks to the great team at A K Peters! They are a pleasure to work with.

Gerald Farin
Dianne Hansford

August 2004
Arizona State University

# 第 1 章

# 笛卡儿的发现

## 引　读

本章介绍了局部坐标与整体坐标之间的互化。先介绍如何由局部坐标求出整体坐标，再介绍如何从整体坐标求出局部坐标。从介绍二维平面中的映射，再推广至三维空间中的映射。先介绍框内一点的映射，再推广至框外一点的映射。最后介绍了如何通过仪器将实物数字化映射在电脑中。内容介绍由局部到整体，由平面到空间，由一推多，由正及反，由浅入深。

# 目 录

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