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Geometric Methods in Elastic Theory of Membranes in Liquid Crystal Phases

液晶相生物膜弹性理论的几何方法

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序　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，“中外物理学精品书系”力图完整呈现近现代世界和中国物理

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科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任

中国科学院院士，北京大学教授

王恩哥

2010 年 5 月于燕园

Preface

Since the first edition was published twenty years ago, a lot of theoretical results on geometric theory of membrane elasticity have been achieved. In particular, the shape equation and boundary conditions for open lipid membranes were obtained. The main changes in this second edition of the present book, we add a chapter (Chapter 4) to explain how to calculate variational problem on a surface with a free edge by using a new mathematical tool–moving frame method and exterior differential forms, and how to derive the shape equation and boundary conditions for open lipid membranes through this new method. In addition, we also include the recent concise work on chiral lipid membranes as a section in Chapter 5. In Chapter 6, we also mention some topics that we have not fully investigated but they are also important to geometric theory of membrane elasticity.

We owe our sincere gratitude to our colleagues such as Profs. Udo Seifert, Jemal Guven, Ivailo Mladenov, Qiang Du and so on since many supplemental results in this edition stem from their work. We also thank Pan Yang and Yang Wang for their patient help in typing this manuscript. Finally, Zhan-Chun Tu, Zhong-Can Ou-Yang and Ji-Xing Liu would like to dedicate this edition to Prof. Yu-Zhang Xie who passed away on May 29, 2011.

Preface for the First Edition

Liquid crystal state was discovered by F. Reinitzer in 1888. A review article written by G. Friedel in 1921 summarizes the general properties of liquid crystals known up to that time. In that article the correlation between liquid crystal and differential geometry was noticed for the first time. The focal conic texture of smectic liquid crystals was shown to have its geometrical origin from Dupin cyelides. However, for quite some time, geometrical studies on liquid crystals were in a stagnant state. Based on the structural similarity between smectic liquid crystal and fluid membrane, in 1973, W. Helfrich offered an elastic theory of membranes in analogy with the curvature elasticity theory of liquid crystals. Since then the study on the correlation between liquid crystals, biomembranes and differential geometry attracts great attention to both physicists and mathematicians. A review on the aspect of differential geometry in the study of biomembranes was given by mathematician J. C. C. Nitsch in 1993 [Q. Appl. Math. 51, 363 (1993)]. The present book is intended to serve as an introduction to those who are interested in this subject.

This book gives a comprehensive treatment on the conditions of mechanical equilibrium and the deformation of membranes as a surface problem in differential geometry. It is aimed at readers engaging in the field of investigation of the shape formation of membranes in liquid crystalline state with differential geometry. It does not offer a compiled survey of all the results obtained in this field. Since the pioneer publications by D. P. B. Canham and W. Helfrich on the geometrical form of membrane elasticity in the early 1970's, a lot of works on this cross disciplinary subject in physics, biology and mathematics have been published. The literatures concerning this field increase continuously. Many important results in which the shape problem of membrane vesicles being treated numerically have been published by authors such as W. Helfrich, H. Deuling, M. A. Peterson, S. Svetina, M. Wortis, R. Lipowski, U. Seifert, and others. However, under the personal interests of the present authors, the material chosen in this book is mainly limited in analytical results. They apologize to those authors whose beautiful results are not included here.

One of us (Z.-C. Ou-Yang) wishes to express his grateful thanks to Prof.

W. Helfrich for his kind guidance in leading him into this interesting field. We thank many colleagues for their collaborating research in this field. Special thanks are due to Profs. Peng Huan-wu, Hao Bai-Lin, Yu Lu and Su Zhao-Bin for their great support of our working in this cross-disciplinary field of theoretical physics and biology. Also we owe our sincere gratitude to many of our collaborators Profs. Zheng Wei-mou, H. Naito, S. Komura, only mentioned a few, with whom many results presented in this book were obtained. We thank Dr. Haijun Zhou for his patient help in typing this manuscript. We also thank World Scientific for waiting patiently for the long delayed submission of the final edition of the manuscript.

Institute of Theoretical Physics
Beijing, China
December, 1998

Ou-Yang Zhong-Can
Liu Ji-Xing
Xie Yu-Zhang

Contents

1 Introduction to Liquid Crystal Biomembranes 1

1.1 Liquid Crystals 1

1.1.1 Mysterious Matter 1

1.1.2 Orientational Order 3

1.1.3 Classification of Thermotropics 4

1.1.4 Classification of Lyotropics 7

1.2 Amphiphiles and Lyotropic Liquid Crystals 8

1.2.1 Amphiphile 8

1.2.2 Monolayer 9

1.2.3 Micelle 10

1.2.4 Phase Diagram 11

1.3 Phase Transitions in Biomembranes 12

1.3.1 Fluid Mosaic Model 12

1.3.2 Lipid Bilayer 13

1.3.3 Phase Transitions in Bilayer 14

1.3.4 Classification of Lipid Bilayer Phase 16

1.4 Simple Biochemistry of LC Biomembranes 18

1.4.1 Effect of Chain Length 18

1.4.2 Double Bond Effect 19

1.4.3 Effect of Ionic Condition 20

1.4.4 Cholesterol Effect 20

1.4.5 Complexity in Biomembrane 22

1.5 Artificial Bilayers and Vesicles 22

1.5.1 Lipids for Artificial Vesicles 23

1.5.2 Multilamellar Vesicles 23

1.5.3 Single-Bilayered Vesicles 24

References 24

2 Curvature Elasticity of Fluid Membranes 26
2.1 Shape Problem in Red Blood Cell 26
2.1.1 Membranes in Cell 26
2.1.2 High Deformability of Cell Membranes 28
2.1.3 Difficulty in the Explanation of Discocyte Shape 29
2.2 Classic Differential Geometry of Surface 30
2.2.1 Lipid-Bilayer Vesicle Viewed as a Closed Surface 30
2.2.2 Space Curve 31
2.2.3 Surface and Parametric Curves 33
2.2.4 First Fundamental Form 33
2.2.5 Area 35
2.2.6 The Normal and the Tangent Plane 36
2.2.7 Second Fundamental Form 36
2.2.8 Christoffel Symbols 37
2.2.9 Curves and Directions on a Surface 38
2.2.10 Normal Curvature of a Curve on a Surface 40
2.2.11 Principal Directions, Line of Curvature and Principal Curvatures 41
2.2.12 The Mean Curvature and the Gaussian Curvature 42
2.3 Differential Invariants on a Surface 45
2.3.1 Gradient of a Scalar Field 45
2.3.2 Divergence of a Vector Field 47
2.3.3 Laplace-Beltrami Operator on a Scalar Function 48
2.3.4 Two-Dimensional Curl of a Vector 48
2.3.5 Other Differential Invariants 49
2.4 Curvature Elasticity of Fluid Membranes in Liquid Crystal Phase 50
2.4.1 Fluid Membranes Viewed as Liquid Crystals 50
2.4.2 Helfrich's Approach 51
2.4.3 A Derivation by Way of 2D Differential Invariants 56
2.4.4 The Spontaneous Curvature Viewed from Landau-de Gennes Theory 60
2.4.5 Discussion of Helfrich Bending Energy on Liquid Crystal Point of View 64
2.4.6 Spontaneous Curvature and Flexoelectric Effect 66

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References ... 68
3 Shape Equation of Lipid Vesicles and Its Solutions ... 70
3.1 Mathematical Preliminary ... 70
3.2 General Shape Equation ... 81
3.3 Spherical Vesicles ... 86
3.4 Nearly Spherical Vesicles and Third Order Energy Variation ... 90
3.5 Circular Cylindrical Vesicles ... 94
3.6 Noncircular Cylindrical Vesicles ... 96
3.7 Clifford Torus ... 117
3.8 Dupin Cyclide ... 123
3.9 Shape Equation for Axisymmetric Vesicles ... 129
3.10 Circular Biconcave Discoid ... 138
3.11 Surfaces of Revolution with Constant Mean Curvature and Extended Surfaces ... 142
3.12 Challenge ... 148
References ... 148
4 Governing Equations for Open Lipid Membranes and Their Solutions ... 151
4.1 Mathematical Preliminary ... 151
4.1.1 Surface Theory Based on Moving Frame Method ... 151
4.1.2 Hodge Star, Stokes Theorem and some Important Geometric Relations ... 153
4.1.3 Variational Theory Based on the Moving Frame Method ... 155
4.2 Governing Equations for Open Lipid Membranes ... 162
4.2.1 Governing Equations ... 162
4.2.2 Compatibility Conditions ... 164
4.3 Analytic Solutions ... 165
4.4 Quasi-Exact Solutions ... 167
4.5 Challenge ... 168
4.5.1 Neck Condition of Two-Phase Vesicles in the Budding State ... 168
4.5.2 Minimal Surface with Boundary Curve ... 170
References ... 170
5 Theory of Tilted Chiral Lipid Bilayers ... 172
5.1 Theory of Tilted Chiral Lipid Bilayers with Strong Chirality ... 173
5.1.1 Geometric Description and Free Energy ... 173

5.1.2 Tilt-Equilibrium and Surface-Equilibrium Equations in Case of Strong Chiral Effect ··· 177
5.1.3 Wound Ribbon Helix ··· 179
5.1.4 Twisted Ribbon ··· 185
5.1.5 Spherical Vesicle ··· 189
5.2 General Theory for TCLB ··· 192
5.2.1 General Formula of Chiral Free Energy ··· 192
5.2.2 The Effect of Other Elastic Constants ··· 197
5.2.3 High- and Low-Pitch Helical Structures of TCLBs ··· 201
5.3 Concise theory for Chiral Lipid Membranes ··· 208
5.3.1 Brief Introduction to Moving Frame Method and Exterior Differential Forms ··· 209
5.3.2 Constructing the Free Energy ··· 212
5.3.3 Governing Equations to Describe Equilibrium Configurations of CLMs without Free Edges ··· 213
5.3.4 Governing Equations to Describe Equilibrium Configurations of CLMs with Free Edges ··· 215
5.3.5 Solutions to Governing Equations of CLMs without Free Edges ··· 218
5.4 Challenge ··· 231
References ··· 232
6 Some Untouched Topics ··· 234
6.1 Nonlocal Theory of Membrane Elasticity ··· 234
6.1.1 Area-difference Elasticity ··· 234
6.1.2 Membrane with Nonlocal Interactions ··· 236
6.2 Numerical Simulations ··· 237
6.2.1 Surface Evolver ··· 238
6.2.2 Phase Field ··· 248
6.3 Effects of Membrane Skeleton ··· 249
6.3.1 Composite Shell Model of Cell Membranes ··· 250
6.3.2 Stability of Cell Membranes and the Function of Membrane Skeleton ··· 252
6.4 Application in Elasticity of Low-Dimensional Carbon Materials ··· 254
6.4.1 Revised Lenosky Model ··· 254

6.4.2 Carbon Nanotubes 256
References 259
Appendix A Tensor Calculus 262
Appendix B The Gradient Operator 268
Reference 270
Appendix C Elastic Theory of Membranes Viewed from Force Balance 271
Appendix D A Different Viewpoint of Surface Tension 273
Reference 274

1
Introduction to Liquid Crystal Biomembranes

In this chapter, firstly we are going to give a general description on liquid crystalline states, states which are different from the commonly known three states of matter. The essential characteristic of liquid crystal, unlike ordinary liquid state, is that the constituent molecules although have no positional order but possess orientational order. Next, the two component system lyotropic liquid crystal will be introduced. Then follows with discussions on the amphiphile-water system, especially the liquid crystalline state of biomembranes, mainly bilayers. Detailed discussion on the phase transitions in biomembranes will be introduced next. The part of biochemistry of biomembrane on the molecular factors affecting the phase transitions in lipid bilayers will also be included in our discussion. Finally, a comprehensive introduction of the methods of preparing artificial vesicles will be given, which may be helpful for experimentalists in investigating the shape transitions of vesicles. The subject matter in this chapter is mainly taken from several well known books[1], especially, many schematic figures are based on the book written by Datta.

1.1 Liquid Crystals

1.1.1 Mysterious Matter

Since 1960s there are several nice books which serve for teaching us a new state of matter—liquid crystal[2, 3]. In his famous book[2], Nobel Prize Laureate de Gennes remarked that liquid crystals are beautiful and mysterious. This state shows different feature from our common knowledge that matter exists in three states: solid, liquid, and gas. We are all confused by the name

of liquid crystal when we encounter it for the first time: how can some thing be in liquid state and in crystalline state at the same time?

To answer the question, let us start with a bit of history. The liquid crystalline state of matter was discovered by botanist Reinitzer in 1888 who observed that cholesteryl benzoate has "two melting points". Crystals of cholesteryl benzoate melt at 145°C to become a turbid liquid and turn clear at 179°C. This turbid state in a pure substance was unknown to people at that time. At first Reinitzer thought it could be caused by the presence of impurities. However, even under high purification the persistence of "two constant melting points" makes the "impurity point of view" very suspicious. With consideration of Reinitzer's suggestion, Lehmann, a physicist, with the collaboration of several chemists studied systemically and found a large number of substances with "two constant melting points". They confirmed that the turbid liquids, like crystals, are optically birefringent. Since then, the state between the "two melting points" has been called variously as "anisotropic liquid", "paracrystal", "mesomorphous state", and by now commonly accepted as liquid crystal, a name first used by Lehmann. The relation of the new state with the common states of matter is illustrated in Fig. 1.1.

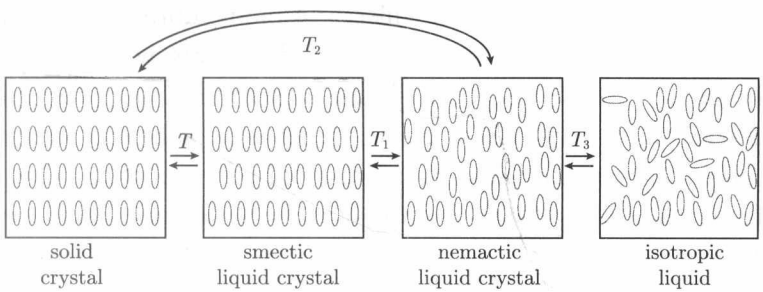

Fig. 1.1 Two liquid crystal states and their relation with solid and liquid states

Experimental study revealed that not all molecules can achieve the liquid crystalline state[2, 3]. The liquid crystal phase can be observed in certain organic compounds composed of elongated molecules with an axial ratio around 4–8:1. But one should notice that some discotic molecules can also form liquid crystals[3]. Basically, the anisotropic geometry of the molecular shape is the origin of the formation of liquid crystalline state. A liquid crystal can flow like an ordinary liquid but its other properties are strongly anisotropic.

Among them the optical anisotropy, such as birefringence and optical activity, is reminiscent of the crystal phase.

1.1.2 Orientational Order

In order to understand the nature of the liquid crystalline state, let us recall the lattice structure of a crystal: the molecules are located on three-dimensional periodic lattice sites, a state with long-range positional order. It is easy to recognize that X-ray diffraction patterns in crystalline state show point-like Bragg reflections. In contrast, molecules in isotropic liquid have only short-range positional order. Their X-ray diffraction patterns show only broad and diffuse Bragg reflections. In liquid crystals, the molecular long-range positional order disappears, while their orientational order is a long-range one (i.e., on macroscopic scale the directions of the long axes of the molecules are the same in thermal average). Thereby the X-ray diffraction patterns in liquid crystals appear to be different from both crystals (without point-structure) and isotropic liquids (with some ring-like structure coming from cylindrical symmetry).

To specify the orientational order in liquid crystalline phase we need two quantities: the local preferred direction, $\mathbf{n}(\mathbf{r})$, and the amount of ordering, $S(\mathbf{r})$. $\mathbf{n}(\mathbf{r})$ is a unit vector called the *director* and $S(\mathbf{r})$ is called the *order parameter*[2]. Here $\mathbf{r}$ is the position vector in liquid crystal. The existence of both the director-field and the order parameter-field reflects the long-range orientational order in the liquid crystalline state. The order parameter S is a measure of the degree of alignment of the long axes of the molecules and is usually defined by

$$S = 2\pi \int_0^{\pi} P_2(\cos\theta) f(\theta) \sin\theta \mathrm{d}\theta, \tag{1.1}$$

where θ is the angle between the temporary direction of the molecular long-axis and the director $\mathbf{n}$, $f(\theta)$ is a distribution function of the molecular orientation corresponding to the local temperature, and $P_2(\cos\theta) = (3/2)\cos^2\theta - 1/2$. Eq. (1.1) is simply the average value of $P_2(\cos\theta)$ over the orientation of all molecules, since $2\pi f(\theta)\sin\theta \mathrm{d}\theta$ is just the fraction of molecules in a cone making an angle between θ and $\theta + \mathrm{d}\theta$ with $\mathbf{n}$. Here, we should remind our reader that in discotics the director $\mathbf{n}$ may be taken as the average direction of their short axes (normal to the surface of the molecules).

In isotropic phase, the distribution of the long axes of the molecules is at

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