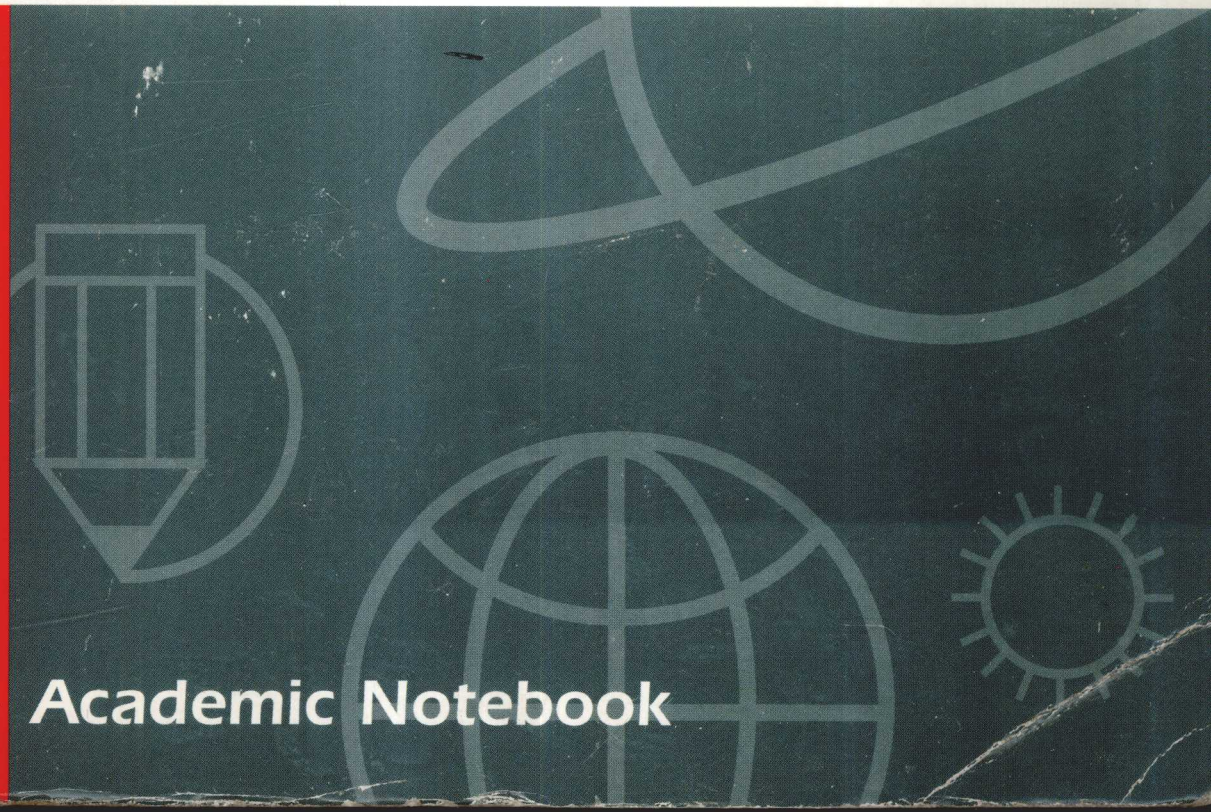
ACADEMIC SYSTEMS

Interactive Mathematics

Elementary Algebra



Personal Academic Notebook

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TOPIC EI ESSENTIALS — PREPARING FOR ALGEBRA

Much of what you have learned in arithmetic will be useful in your study of algebra. In particular, you will use what you know about fractions and about signed numbers.

This topic begins with a brief review of the basic operations on fractions — multiplication, division, addition, and subtraction. The topic continues with the arithmetic of positive and negative numbers, which we refer to as signed numbers.

Lesson EI.A Fractions

Concept 1: Multiplying and Dividing

- Introduction to Fractions
- Equivalent Fractions
- Prime Factorization
- Greatest Common Factor (GCF)
- Reducing to Lowest Terms
- Multiplying Fractions
- Dividing Fractions

Concept 2: Adding and Subtracting

- Adding and Subtracting Fractions With the Same Denominator
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Lesson EI.B Signed Numbers

Concept 1: Adding and Subtracting

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- Multiplying and Dividing Signed Numbers
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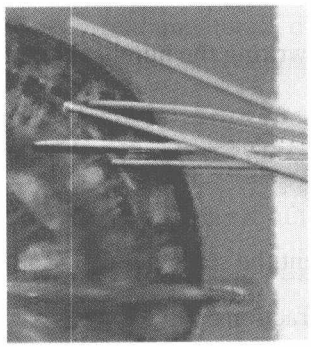
- Properties of Real Numbers
- Distributive Property
- Order of Operations

LESSON EI.A FRACTIONS

Overview



This lesson provides a review of fractions — how to change their form and how to combine them by adding, subtracting, multiplying, and dividing.

You may already understand some of the concepts in this lesson. In many of the examples, you may be able to find the answer in a way that differs from the given solution.

If you have a method that works well for you, keep using it. But, also try to understand the notation and the procedures you see here. It will help you as you study algebra.

Explain

Concept 1 has sections on

- **Introduction to Fractions**
- **Equivalent Fractions**
- **Prime Factorization**
- **Greatest Common Factor (GCF)**
- **Reducing to Lowest Terms**
- **Multiplying Fractions**
- **Dividing Fractions**

CONCEPT 1: MULTIPLYING AND DIVIDING

Introduction to Fractions

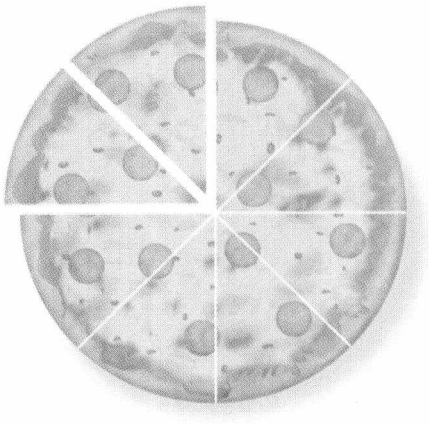
A **fraction** is the quotient of two numbers written one over the other and separated by a horizontal bar, called a **fraction bar**.

For example, we write the fraction two-eighths as follows: $\frac{2}{8}$.

To save space, we sometimes write a fraction using a slanted bar, /.

Using this notation, we write two-eighths as 2/8.

We often use a fraction to represent the number of parts selected from a total number of equal-size parts. For example, we can use the fraction $\frac{2}{8}$ to represent 2 slices of a pizza cut into 8 equal slices.



The number in the bottom of the fraction is called the **denominator**; it can indicate the number of equal parts into which a whole has been divided.

In the pizza example, the 8 in $\frac{2}{8}$ indicates there are 8 equal-size slices.

The number in the top of the fraction is called the **numerator**; it can indicate the number of parts selected.

In the pizza example, the 2 in $\frac{2}{8}$ indicates 2 of the 8 pizza slices.

We can write a whole number as a fraction by writing the number over 1.

For example: $6 = \frac{6}{1}$; $9 = \frac{9}{1}$

Equivalent Fractions

Equivalent fractions are fractions that represent the same number.

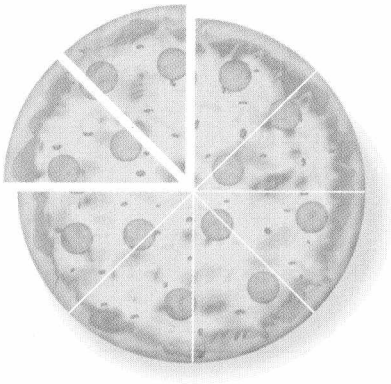
For example, $\frac{1}{4}$, $\frac{2}{8}$, $\frac{6}{24}$, and $\frac{17}{68}$ are equivalent fractions.
To verify this, divide each numerator by its denominator.

$1 \div 4 = 0.25$ $\quad 2 \div 8 = 0.25$ $\quad 6 \div 24 = 0.25$ $\quad 17 \div 68 = 0.25$

In each case, the result is the decimal 0.25.

To picture why $\frac{1}{4}$ and $\frac{2}{8}$ represent the same number, look at the pizza.

You can see that 2 pieces out of 8 represents $\frac{1}{4}$ of the pizza.



For example, before we add $\frac{1}{4}$ and $\frac{1}{8}$, we find a fraction that has denominator 8 and is equivalent to $\frac{1}{4}$

To add or subtract fractions, we will often need to convert a given fraction to an equivalent fraction that has a larger denominator.

Here is a procedure for finding equivalent fractions.

*In some cases, the old denominator does not divide evenly into the new given denominator. Then we often choose a **common** denominator and rewrite **both** fractions.*

— Procedure — To Find an Equivalent Fraction With a Given Larger Denominator	
Step 1	Check whether the old denominator divides evenly into the new (larger) denominator.
Step 2	If it does, divide the new denominator by the old denominator.
Step 3	Multiply the old numerator by this result. This is the new numerator.
Step 4	Use the new numerator and the new denominator to write the equivalent fraction.

Example EI.A.1

Find a fraction equivalent to $\frac{3}{5}$ with denominator 40.

Solution

Step 1 Check whether the old denominator divides evenly into the new denominator.

$$\frac{3}{5} = \frac{?}{40}$$

The old denominator, 5, divides evenly into the new denominator, 40.

Step 2 If it does, divide the new denominator by the old denominator.

New denominator ↘ Old denominator ↙

Divide 40 by 5. The result is 8.

$$40 \div 5 = 8$$

Step 3 Multiply the old numerator by this result.

Old numerator ↘ New numerator ↙

Multiply the old numerator, 3, by the result of the previous step, 8.

$$3 \cdot 8 = 24$$

Step 4 Use the new numerator and the new denominator to write the equivalent fraction.

The result of the previous step, 24, is the new numerator. The denominator is 40.

So, $\frac{3}{5} = \frac{24}{40}$.

Here is a compact way to show how to write $\frac{3}{5}$ with denominator 40:

$$\frac{3}{5} = \frac{3 \cdot 8}{5 \cdot 8} = \frac{24}{40}$$

Example EI.A.2

Find a fraction equivalent to $\frac{5}{12}$ with denominator 72.

Solution

Step 1 Check whether the old denominator divides evenly into the new denominator.

$$\frac{5}{12} = \frac{?}{72}$$

The old denominator, 12, divides evenly into the new denominator, 72.

Step 2 If it does, divide the new denominator by the old denominator.

Divide 72 by 12. The result is 6.

$$72 \div 12 = 6$$

Step 3 Multiply the old numerator by this result.

Multiply the old numerator, 5, by the result of the previous step, 6.

$$5 \cdot 6 = 30$$

Step 4 Use the new numerator and the new denominator to write the equivalent fraction.

The result of the previous step, 30, is the new numerator. The denominator is 72.

$= \frac{30}{72}$

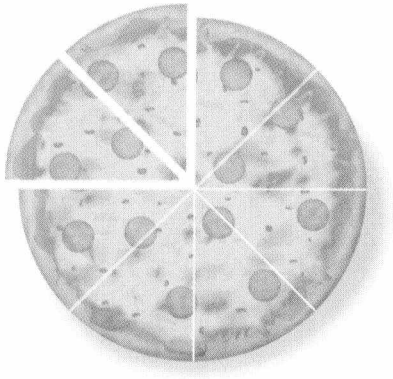
So, $\frac{5}{12} = \frac{30}{72}$.

Here is a compact way to show how to write $\frac{5}{12}$ with denominator 72:

$$\frac{5}{12} = \frac{5 \cdot 6}{12 \cdot 6} = \frac{30}{72}$$

In the last two examples, we started with a fraction and found an equivalent fraction with a larger denominator.

We will also need to do the reverse. That is, we will want to find a new fraction equivalent to a given fraction with a *smaller* denominator. This process is called **reducing** a fraction.



Sometimes you may be able to reduce a fraction by inspection. For example, using the pizza, we see that $\frac{2}{8} = \frac{1}{4}$.

Often, however, a fraction cannot be reduced this easily. In such cases we use a method that involves **prime factorization**. Now we will discuss prime factorization. Then we will use it to reduce fractions.

Prime Factorization

When two numbers are multiplied, each number is called a **factor**.

For example, in the product $2 \cdot 3$, both 2 and 3 are factors.

To find a **factorization** of a number means to write the number as a product.

For example, here are two factorizations of 6: $6 = 2 \cdot 3$ $6 = 1 \cdot 6$

A **prime number** is a whole number greater than 1 and divisible only by 1 and itself.

The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29

To find the prime factorization of a number, we write the number as the product of prime numbers.

— Procedure —
To Find the Prime Factorization of a Number

Step 1 Check to see if 2 is a factor of the number.
(That is, divide the number by 2. If the remainder is 0, then 2 is a factor).

If 2 is a factor, write the number as the product of 2 and another factor.

Step 2 If 2 is not a factor, try other prime numbers (3, 5, 7, 11, ...) until you find a prime number which is a factor of the original number. Write the original number as a product of this prime number and another factor.

Step 3 Check the other factor using the same procedure as in Steps 1 and 2.

Step 4 Continue this process until you can no longer factor. That is, continue dividing by prime numbers until you recognize the quotient as a prime number.

You can stop when the number you are dividing by is greater than the square root of the number you are dividing into.

Step 5 Write the original number as a product of the prime factors.

Example EI.A.3

Find the prime factorization of 60.

Solution

Step 1 Check to see if 2 is a factor of the number.

Divide 60 by 2.
The remainder is 0, so 2 is a factor of 60.
Write 60 as the product $2 \cdot 30$. $60 = \mathbf{2} \cdot 30$

Step 2 If 2 is not a factor, try other prime numbers (3, 5, 7, 11, ...).

Since 2 *is* a factor, we skip this step and proceed to Step 3.

Step 3 Check the other factor using the same procedure as in Steps 1 and 2.

Divide 30 by 2.
The remainder is 0, so 2 is a factor of 30.
Write 30 as the product $2 \cdot 15$. $60 = 2 \cdot \mathbf{2} \cdot \mathbf{15}$