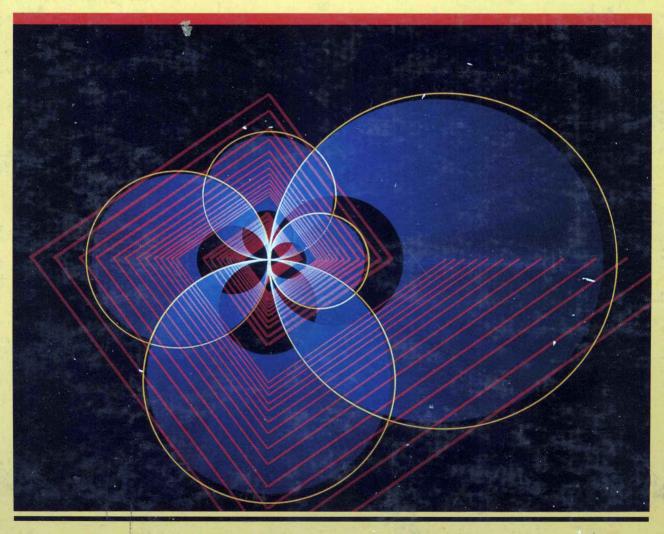
# STATISTICS FOR BUSINESS AND ECONOMICS



Paul Newbold

# SECOND EDITION

# STATISTICS FOR BUSINESS AND ECONOMICS

# Paul Newbold

University of Illinois, Urbana-Champaign



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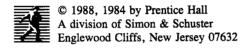
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# **PREFACE**

Statistical literacy is so important to the manager that statistics courses are offered in virtually all college business programs and are required in the great majority. Such a requirement often generates reactions ranging from terror to nausea: At least at the outset, many students suspect that the course will be both difficult and boring. I would like to reassure the minority of students who actually read prefaces by asserting that the subject matter of business statistics is both easily grasped and invariably entertaining. However, honesty prohibits my doing so.

In preparing the second edition of this text, I have kept in mind that problems in immediately grasping some of the material will inevitably arise. Many new procedures for the analysis of business data will be introduced in the chapters that follow. It is tempting to try to memorize the ingredients of each ("cookbook" style), but this inevitably leads to confusion. A better strategy is to attempt to grasp the reasons that these procedures are appropriate, so I have tried to set out carefully the rationale for each, keeping in mind that the vast majority of readers of this book will become consumers rather than producers of statistical information. The end product of an introductory statistics course should be students who are well-informed consumers, with some critical insight into statistical claims that they will later meet. To facilitate understanding, I have included a large number of numerical examples. However, in the final analysis, statistics is a subject in which "learning by doing" is crucially important. Since this is the case, the text contains a great many exercises. The student's understanding of the material will best be sharpened by working through several exercises on each topic.

In writing this text, I have been conscious of the importance of persuading students to invest the time and effort needed to absorb the methodology of statistical analysis. My strategy has been to include many real and realistic examples and exercises taken from the business literature, including the fields of accounting, economics, finance, marketing, industrial organization, and organizational behavior. It is hoped that the accumulation of this illustrative material will convince the reader of the relevance of statistics to the modern business environment and thus heighten his or her interest in the subject.

This text is suitable for a one- or two-semester introductory course aimed at business or economics majors. I have included more than enough material for a two-semester course, and many instructors will not want to cover every chapter in detail, particularly if a large amount of time is spent on project work. All, or parts,

of Chapters 10, 11, and 14–19 can be omitted without loss of continuity. Also, Chapter 19 can be covered at any stage after Chapter 4, and Chapter 18 at any time after Chapter 8. One of the many possibilities for a one-semester course is to cover Chapters 1–9, 12, and perhaps 13. In such a course, instructors may wish to omit some of the following sections: 2.6, 4.6, 4.7, 5.8, 7.3, 9.9, and 12.2.

This book developed from a two-semester course at the University of Illinois, covering most of the material in Chapters 1–9, 11–14, 17, and 18. I am very grateful to my colleagues, teaching assistants, and, most of all, students, for helpful discussions, suggestions, and criticisms. Many of these prompted changes from the first edition. New topics covered in this edition include stem-and-leaf diagrams (Chapter 2), the exponential distribution (Chapter 5), a test for normality (Chapter 11), partial correlation (Chapter 13), autocorrelated errors in models with lagged dependent variables (Chapter 14), and sensitivity analysis of optimal decision solutions (Chapter 19). In addition, several sections, where the quality of exposition fell embarrassingly short of perfection, have been rewritten. However, I have tried to maintain and enhance the two central features of the first edition. These were care and clarity in explaining, using no mathematics beyond elementary algebra, how and why the various procedures work, and the provision of large numbers of examples and exercises involving real business and economic data.

I continue to acknowledge the many people without whose help the first edition of this book would have been less successful. In addition, in preparing the second edition, thanks also go to my Prentice Hall editors Dennis Hogan and Cheryl Smith. Finally, I am grateful for the expert typing of Jerry Rowley.

To any student who has read this far through the preface, I wish you success in your studies of business statistics. In particular, I hope that I will be able to convince you of the importance and relevance of statistics in many branches of business.

Paul Newbold

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# WHAT IS STATISTICS?

Since the majority of readers tend to skip the introductory chapter of a text, it is tempting to answer the question of the title with "Statistics is what statisticians do" and to proceed immediately to the next chapter. Unfortunately, my editor argued forcibly that rather more might be expected. In the end, I decided to compromise with convention and to keep these introductory remarks as brief as possible.

Compulsory statistics courses have received a bad press. Indeed, on the typical college campus, enrollment in such a course ranks somewhere in popularity between laws establishing a minimum drinking age and mandatory draft registration. In part, this reputation is deserved. The concepts involved are not always easy to grasp at first, and it is certainly necessary to work hard to keep up with any worthwhile course in the subject. However, one complaint heard occasionally is definitely unjustified. Statistics is not irrelevant. The remainder of this chapter is devoted to an expansion of this point, particularly with respect to business and economic problems.

Statistics is what statisticians do. What is remarkable is the range of activities in which statisticians are involved. These activities impinge on virtually every aspect of daily business and economic life. We will group these activities under six broad headings.

# 1.1 MAKING SENSE OF NUMERICAL INFORMATION

Any manager operating in the business environment requires as much information as possible about the characteristics of that environment. In the modern era, thanks in part to the massive information storage capacities of computer systems, much of the

available information is **quantitative**. For example, it may be necessary to assimilate movements in interest rates, stock market prices, money supply, or unemployment. Market research surveys are carried out to determine the strength of product demand. An auditor is concerned about the number and size of errors found in accounts receivable. A personnel manager may be able to use aptitude test scores, in addition to subjective judgment of candidates for employment. The list is virtually endless.

The common features of these examples are that the information to be absorbed is *numerical*, and that the sheer amount of that information renders it, in its raw form, virtually impossible to comprehend fully. The statistician's role involves the extraction and synthesis of the important features of a large body of numerical information. One objective is to try to make sense of numerical data by summarizing it in such a way that a readily understood picture emerges while little of importance is lost.

Many issues are involved in a comprehensive analysis and synthesis of numerical data. The most appropriate method will depend on the nature of the numerical information and how it is to be used. In some circumstances, it will be desirable to employ some of the heavy artillery of formal techniques to be discussed in the later chapters of this book. On other occasions, a relatively straightforward numerical or graphical summary may be sufficient and should, in any event, provide a good basis for a deeper analysis. In Chapter 2, we will consider some useful techniques for summarizing numerical information.

## 1.2 DEALING WITH UNCERTAINTY

A second answer to the question posed at the beginning of this chapter is "Statistics is the science of uncertainty." In statistics, we do not deal with questions of what is but of what could be, what might be, or what probably is. Consider the following statements:

"The price of IBM stock will be higher in six months than it is now."

"If the federal budget deficit is as high as predicted, interest rates will remain high for the rest of the year."

"If a bid of this level is submitted, it will be lower than competitors' bids and the contract will be secured."

"The best opportunities for improvement in market share for this product lie in an advertising campaign aimed at the 18-to-25-year-old age group."

Each of these statements contains language suggesting a spurious amount of certainty. At the time the assertions were made, it would have been impossible to be *sure* of their truth. Although an analyst may believe that anticipated developments over the next few months are such that the price of IBM stock is likely to rise over the period, he or she will not be certain of this. Thus, from a purely semantic point

of view, the statements should be modified, as indicated by the following examples:

"The price of IBM stock is likely to be higher in six months than it is now."

"If a bid of this level is submitted, it is probable that it will be lower than competitors' bids and the contract will be secured."

However, our concern about uncertainty is not merely semantical. All we have done so far is to replace unwarrantedly precise statements by unnecessarily vague statements. After all, what is meant by "is likely to be" or "it is probable that"? Perhaps the two modified statements could be interpreted as assertions that the events of interest are more likely than not to occur. But how much more likely? The English language is rich in words that describe uncertainty, and, indeed, some of these suggest a gradation from the impossible to the certain. Nevertheless, language alone is inadequate to provide a satisfactory description of the degree of uncertainty attached to the occurrence of a particular event. Rather, we need a more formal structure for this purpose.

In the majority of this book, we will be discussing procedures for attacking problems where the conclusion will necessarily be couched in the formal language of uncertainty. As a prelude, that language—**probability**—is introduced in Chapter 3.

### 1.3 SAMPLING

Before bringing a new product to market, a manufacturer wants to arrive at some assessment of the likely level of demand, and a market research survey may be undertaken. The manufacturer is, in fact, interested in the **population** of all potential buyers. However, it is prohibitively expensive, if not impossible, for a typical market research survey to contact every member of that population. Rather, a small subset—or **sample**—of population members will be contacted, and any conclusions about the population will be based on information obtained from the sample.

The technique of sampling large populations is commonly used in business. For example, decisions about whether a production process is operating correctly are based on the quality of a sample from its output. Again, an audit of accounts receivable will generally be based on a sample of all accounts.

When we have information on a sample from a population, it is generally straightforward to summarize the numerical sample data. However, taking a sample is merely a means to an end. The objective is not to make statements about the sample but, rather, to draw conclusions about the wider population. Thus, an important problem for the statistician involves the extent to which it is possible to generalize about a population, based on results obtained from a sample.

Of course, if a sample is taken from a population, we will not be able to learn *precisely* the population characteristics. For example, suppose that a sample of accounts receivable is examined, and it is found that 8.2% of these are in error. It does not follow that *exactly* 8.2% of all the accounts receivable in the population are in

error. We will have learned something about this population percentage, but we will not know its exact value. Some uncertainty will remain. Hence, in making inferences about a population based on the results of a sample, any conclusions will naturally involve the language of uncertainty, as discussed in the preceding section.

We begin our exploration of procedures for the analysis of sample data in Chapter 6, postponing to Chapter 18 a comprehensive discussion of methods for selecting samples.

### 1.4 ANALYZING RELATIONSHIPS

Does the rate of growth of the money supply influence the inflation rate?

If General Motors increases the price of subcompact cars by 5%, what will be the effect on the sales of these cars?

Are companies whose dividends are a high percentage of total cash flow viewed as high or low risk?

Are utilities more profitable in areas where they have local monopoly power than where they are subject to competition?

Does minimum wage legislation affect the level of unemployment?

Each of these questions is concerned with the possibility and nature of a relationship between two or more variables of interest. For example, how might we begin to answer the question about the effect on the demand for automobiles of a 5% increase in prices? Simple economic theory tells us that, all other things being equal, an increase in price will lead to a decrease in demand. However, such theory is purely qualitative. It does not tell us by how much demand will fall. Subject matter theory is extremely valuable in suggesting the influential factors for such quantities of interest as product demand. To proceed further, we must collect quantitative information in order to assess how demand has responded to price changes in the past. We would then base our assessment on the premise that what happened in the past is likely to be repeated after the proposed current price increase.

In the automobile example, the objective is to use numerical information to learn something about the relationship between the variables of interest. Procedures for analyzing relationships are discussed in Chapters 11–14.

### 1.5 FORECASTING

The desire to be able to foretell the future is a very human characteristic. However, the need for reliable predictions in business goes far beyond curiosity. Investment decisions must be made well ahead of the time at which a new product can be brought to market, and forecasts of likely market conditions some years into the future would obviously be desirable. For established products, short-term sales forecasts are important in the setting of inventory levels and production schedules. Pre-

dictions of future interest rates are important to a company deciding whether to issue new debt. In formulating a coherent economic policy, the government requires forecasts of the likely outcomes for variables such as gross domestic product, unemployment, and inflation under various policy options.

Essentially, forecasts of future values are obtained through the discovery of regularities in past behavior. Thus, data are collected on the past behavior of the variable to be predicted, and on the behavior of other related variables. The analysis of this information may then suggest likely future trends.

Some of the methods of business forecasting are introduced in Chapter 17.

# 1.6 DECISION-MAKING IN AN UNCERTAIN ENVIRONMENT

In any business, decisions are made regularly in an environment where the decisionmaker cannot be certain of the future behavior of those factors that will eventually affect the outcomes following from the various options under consideration.

In submitting a bid for a contract, a manufacturer will not be completely certain of the total future cost involved in fulfilling it. Moreover, he will not know the levels of the bids to be submitted by his competitors. In spite of this uncertainty, a decision as to where to pitch the bid must be made. An investor, deciding how to balance her portfolio among stocks, bonds, and money market instruments, must make this decision when future market movements are unknown. She may take some view on probable future developments, but she will not be able to predict the future with perfect accuracy.

These examples demonstrate that in order to think about possible options when business decisions are to be made, it is inevitable that techniques for dealing with uncertainty will be relevant. Some useful procedures will be outlined in Chapter 19.

In the remainder of this book we present an array of techniques useful in the analysis of numerical information. Their goal is to help in the understanding of an uncertain environment so that better decisions are likely to be made. It should be emphasized, however, that these techniques are simply useful tools for the manager. They are not intended as substitutes for the familiarity with the business environment that develops through years of study and accumulated experience but, rather, as aids to the sharpening of that familiarity. Thus, even though a careful technical analysis of numerical information will often be of considerable value, that analysis may not be very profitable unless it is allied with the expertise that comes from studying the qualitative characteristics of the relevant environment.