



中国科学院教材建设专家委员会规划教材  
全国高等医药院校规划教材

# MEDICAL PHYSICS

## 医学物理学

(英文版)

Chief Editor (主编) Chen Yanxia (陈艳霞)



科学出版社

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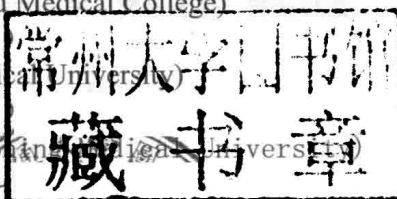
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## Contents Synopsis

This textbook is compiled primarily for overseas students in China who plan to develop a career in some field of medicine. There are twelve chapters that determined by the needs of medicine majors including the basis of biomechanics; vibration, wave and sound; the motion of fluid; phenomena on liquid surfaces; electric field; magnetic field, direct current; geometric optics; wave optics; laser; x-rays; and nuclear physics.

Although this book is aimed at overseas medical students, we believe it can be used by Chinese long-year program medical students and biology major students as bilingual teaching materials. And it also serves as an excellent reference book for teachers and students in the related profession.

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## Preface

Have you ever wondered about the relationship between an ultrasonic image and your body? How to correct vision defect? How does a medical X-ray work? How much dosage could you probably receive from a radiation? These questions and the like involving applications of physics to medicine are answered in this book.

This textbook is compiled primarily for overseas students in China who plan to develop a career in some field of medicine. We have a try to provide a text book in English that is comparable with content to the Chinese textbook on physics normally used in medical university. And at the same time, considering that the basis of physics and mathematics are different in the overseas students and China student, we combine the advantages of domestic and foreign textbooks. Even though most students have an introductory physics course, they could hardly understand relationship between physics and medicine. This gap is due primarily to the large amount of material in a traditional year-long physics course that precludes an adequate treatment of physics applied to medicine. We describe the usefulness of physics in understanding the behavior of the body in a simple way. In the book, we add some contents in Chinese in order to arouse student's interest to Chinese culture and the students can consult and use Chinese document materials conveniently.

Since medical physics is an elective course for overseas students in some medical universities and the class hours are limited, the book covers only a few areas of medical physics. There are twelve chapters that determined by the needs of medicine majors including the basis of biomechanics; vibration, wave and sound; the motion of fluid; phenomena on liquid surfaces; electric field; magnetic field; direct current; geometric optics; wave optics; Laser; X-rays and nuclear physics. The level of mathematics in this book is majorly at the algebra and a introduction to calculus level. International System (SI) units are used most often. Appendix A gives information on physical constants. Appendix B gives a general references for this book. All editors express sincerely thanks to the author of the references. We obtain invaluable materials from these books.

Although this book is aimed at overseas medical students, we believe it can be used by Chinese long-year program medical students and biology major students as bilingual teaching materials. And it also serves as an excellent reference book for teachers and students in the related profession.

It is the editors' greatest honor to be in charge of compiling this textbook of the first edition. Actually for Chinese editors to compile the Medical Physics in English is not easy because we have not had such experience before. The textbook has gone only the first draft, it is certain to contain some shortages. We apologize for the shortages and hope that the readers will bring up your views and proposals so that we can correct them in the next edition.

Finally, we must sincerely express our deep gratitude to the comrades of the International Education College of Dalian Medical University and Science Press. Their assistance and suggestions have made the publication of this textbook possible. We also thank Song Xudong at Dalian Jiaotong University, Sun Yingping and Song Bo at Dalian Medical University, Wang Yaping at Liaoning Medical University who contributed to this book by offering many helpful suggestions and reference materials. We are particularly thankful to Li Xiang at Dalian Medical University for help in reviewing and correcting the English for part of the textbook.

Chen Yanxia  
May, 2014





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## CHAPTER 1 THE BASIS OF BIOMECHANICS

### • Newton's Laws of Motion

Newton's First Law of Motion

Newton's Second Law of Motion

Newton's Third Law of Motion

### • Rotation of Rigid Bodies

Angular Displacement, Angular Velocity and Angular Acceleration

Rotation of Rigid Bodies

The Moment of Inertia

### • Elastic Properties of Materials

Stress

Strain

Modulus of Elasticity

### • The Mechanical Properties of Bone

Tensile and Compression

Shear

Torsion

Bending

Composite Load

### • The Mechanical Properties of Muscle

Elongate Pinch of Skeletal Muscle

Equal Tensile Pinch of Skeletal Muscle

Equal Length Pinch of Skeletal Muscle

In this chapter, we will review Newton's laws of motion, analyze the rotation of rigid body, discuss elastic properties of materials, and explore mechanical features of bone and muscle.

## 1.1 Newton's Laws of Motion

We know from experience that an object at rest never starts to move by itself. In order to move a body, a push or pull must be exerted on it by some other body. Similarly, a force is required to slow down or stop a body already in motion, and to make a moving body deviate from straight line motion requires a sideways force. All these processes (speeding up, slowing down, or changing direction) involve a change in either the magnitude or direction of the velocity. Thus in each case the body accelerates, and an external force must act on it to produce the acceleration.

### 1.1.1 Newton's First Law of Motion

Any object remains at rest or in motion along a straight line with constant speed unless acted upon by a net force (or resultant force  $\Sigma F$ ). This is Newton's first law of



**motion(牛顿第一运动定律).**

Newton's first law describes the motion of an isolated object and there is no net force acting on it. In the most general case, a single force acting on a body produces a change in motion. However, when several forces act on a body simultaneously, their effects can compensate one another, with the result that there is no change in motion. When this is the case, the body is said to be in equilibrium. Mathematically, this means  $a=0$ , when  $F_{\text{net}}=0$ . Where  $F_{\text{net}}$  is the vector sum of all the forces acting on the body.

This property of matter, that its motion will not change unless a net force acts on it, is what we call **inertia(惯性)**. Inertia is the property of an object that resists acceleration. And Newton's first law is often called the **law of inertia(惯性定律)**.

### 1.1.2 Newton's Second Law of Motion

The product of the mass of any object times its acceleration is equal to the net force acting on the object. This is **Newton's second law of motion(牛顿第二运动定律)**.

$$F_{\text{net}}=ma \quad (1.1)$$

That is, if the sum of all forces acting on an object is not zero, then it will be accelerated. The acceleration depends on the net force and on the mass of the object as well.

Notice that this equation says the acceleration is always in the same direction as the net force, although they are very different quantities.

If you think of inertia as the qualitative term for the property of a body that resists acceleration, then mass (a scalar quantity) is the quantitative measure of inertia. If the mass is large, the acceleration produced by a given force will be small.

### 1.1.3 Newton's Third Law of Motion

For every force, or action, there is an equal but opposite force, or reaction. This is **Newton's third law of motion(牛顿第三运动定律)**.

This law is true for any type of force, including frictional, gravitational, electrical, and magnetic forces. The important thing to realize about this law is that the action force is on one object and the reaction force is on the other. These two forces always act on different objects, so they can never balance each other, or cancel. Only when equal and opposite forces act on the same object can you add them together and then they do balance one another. So in a playground collision, the force on one child can't cancel the force on the other.

## 1.2 Rotation of Rigid Bodies

The **rigid body(刚体)**, a body with a perfectly definite and unchanged shape regardless of the external force, is an idealized model. Therefore, the distance between any two points on a rigid body is always the same.

**Rotation(转动)** of a rigid body, each point on the rigid body is in circular motion around the same straight line. This line is called the **axis of rotation(转轴)**. If the axis of rotation is fixed, the rotation is called **fixed-axis rotation(定轴转动)**. Fixed-axis rotation of rigid body is the most simple form of rotation.

### 1.2.1 Angular Displacement, Angular Velocity and Angular Acceleration

We usually use **angular displacement** (角位移)  $\theta$ , **angular velocity** (角速度)  $\omega$ , **angular acceleration** (角加速度)  $\alpha$  and other physical quantity to describe the rotation of rigid body. The relationship among them is as follows

$$\omega = \frac{d\theta}{dt} \quad (1.2)$$

For motion a circle, there is a simple relation between  $\omega$  and the velocity  $v$  along the circumference. As we know

$$s = R\theta \quad (1.3)$$

Now differentiating the both sides with respect to  $t$ , we have

$$\frac{ds}{dt} = R \frac{d\theta}{dt}$$

So

$$v = R\omega$$

The velocity  $v$  is the distance traveled in one second. So

$$v = 2\pi R\nu$$

$\nu$  is the number of revolution

$$2\pi R\nu = R\omega$$

Therefore we have the very important relation between the frequency and the angular velocity

$$v = R\omega \quad (1.4)$$

$$\omega = 2\pi\nu \quad (1.5)$$

The letter  $\nu$  stands for frequency in revolution per second and  $\omega$  is radius per second. The acceleration

$$\alpha = \frac{d\omega}{dt} = R \frac{d\nu}{dt}$$

The angular acceleration

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} \quad (1.6)$$

So, we have

$$a = R\alpha \quad (1.7)$$

Obviously, the acceleration  $a$  is the tangential acceleration. This relation is not only valid for the circular motion but also for any kind of curve motion. The only difference is the radius  $R$  is changeable at different moment.

### 1.2.2 Rotation of Rigid Bodies

For translational motion, we can find the acceleration from the forces using  $F_{\text{net}} = ma$ . For rotational motion, it is not the forces but the **torques** (力矩) exerted by these forces that determine the angular acceleration. Fig.1.1 shows a mass  $m$  at the end of a string swinging on a frictionless plane in a horizontal circle. The mass is subjected to two forces, the tension  $T$  in the

string directed toward the center  $O$ , and the force  $F_a$  applied at right angles to the string. These two forces produce radial acceleration  $a_r$  and tangential acceleration  $a_T$ .

$$T = ma_r$$

$$F_a = ma_T$$

The line of action of  $T$  passes through point  $O$ , so it produces no torque about that point and has no effect on the rotation. On the other hand,  $F_a$  does produce a torque  $M$  about point  $O$ .  $M = F_a r$ . From Newton's second law,  $F_a = ma_T$ , and  $a_T = r\alpha$ . Thus the torque due to  $F_a$  can be written as  $M = F_a r = mr\alpha r = mr^2\alpha$ . The quantity  $mr^2$  is the **moment of inertia**(转动惯量)  $I$ . The vector form of the torque equation is then

$$\mathbf{M} = I\boldsymbol{\alpha} \quad (1.8)$$

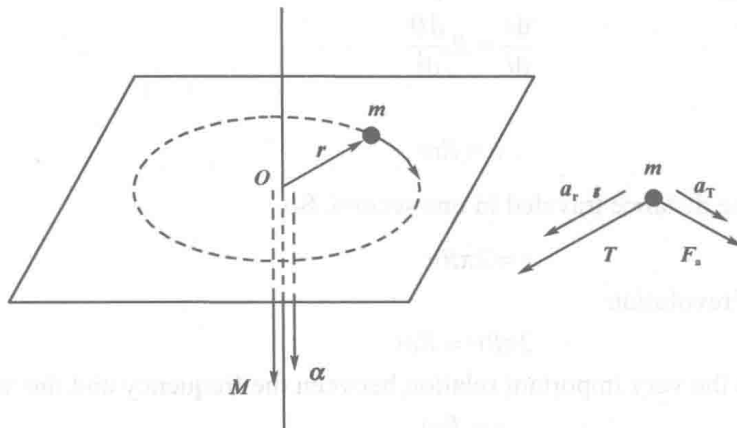


Fig.1.1 Torques

For any object rotating about a fixed axis, the net torque on the object is equal to the moment of inertia of the object times the angular acceleration. This is the rotational version of Newton's second law.

### 1.2.3 The Moment of Inertia

Despite the fact that  $\mathbf{M} = I\boldsymbol{\alpha}$  is similar in form to  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ , it is important to realize that both the torque  $\mathbf{M}$  and the moment of inertia  $I$  depend on the position of the axis of rotation. We will also find that  $I$  depend on the shape and mass of the rotating object.

To calculate the moment of inertia of a complex object, we must mentally separate the object into  $N$  small pieces of mass  $m_1, m_2, \dots, m_n$ . Then each piece is a distance  $r_1, r_2, \dots, r_n$  from the axis of rotation. The moment of inertia of the first piece is  $m_1 r_1^2$ , that of the second is  $m_2 r_2^2$ , and so on. The net moment of inertia is the sum of all such terms

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2 \quad (1.9)$$

It is obvious that the moment of inertia is large when the pieces are far from the rotation axis. The calculation of the moment of inertia is illustrated in the following example.

**Example 1.1** Two equal point masses  $m$  are at the ends of a massless thin bar of length  $l$ , as shown in Fig. 1.2. Find the moment of inertia for an axis, perpendicular to the bar through: ①the center, and ②an end.

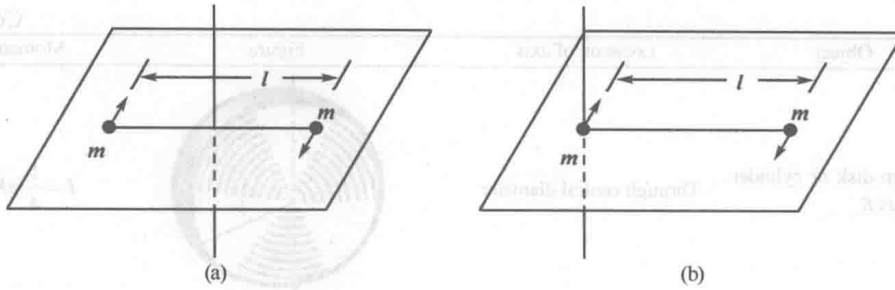


Fig.1.2 Example 1.1

**Solution** ①For an axis through the center, each mass is a distance  $l/2$  from the axis. Summing the  $mr^2$  terms for each of the two masses,

$$I = m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{2}ml^2$$

②For an axis through an end, the mass at that end has  $r=0$ , while the other mass is at a distance  $l$ , so we have

$$I = 0 + ml^2 = ml^2$$

Thus we see that the moment of inertia depends on the position of the rotation axis.

The moment of inertia is often required for objects such as rods or cylinders whose mass is distributed in a continuous fashion. If this distribution is known in detail, then  $I$  can be calculated mathematically. In this case, Eq. (1.9) defining moment of inertia becomes

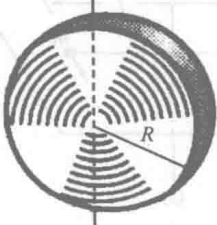
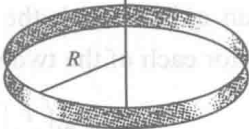
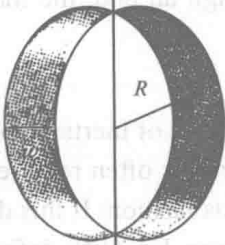
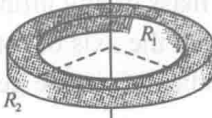
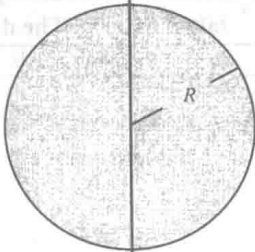
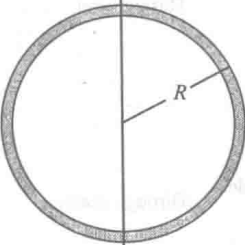
$$I = \int r^2 dm \quad (1.10)$$

Where  $dm$  represents the mass of any infinitesimal particle of the body, and  $r$  is the perpendicular distance of the particle from the axis of rotation. The integral is taken over the whole body. This is easily done only for bodies of simple geometric shape. Some of the results obtained are listed in Table 1.1.

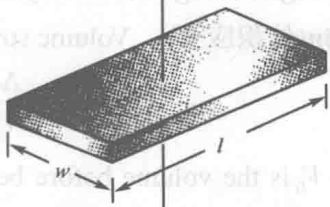
**Table 1.1 Moments of inertia for various objects of uniform composition (The mass of each object is taken to be  $m$ . The dotted line is the rotation axis.)**

No	Object	Location of axis	Figure	Moments of inertia
1	Long uniform rod of length $l$	Through center		$I = \frac{1}{12}ml^2$
2	Long uniform rod of length $l$	Through end		$I = \frac{1}{3}ml^2$
3	Uniform disk or cylinder of radius $R$	Through center		$I = \frac{1}{2}mR^2$

Continue

No	Object	Location of axis	Figure	Moments of inertia
4	Uniform disk or cylinder of radius $R$	Through central diameter		$I = \frac{1}{2} mR^2$
5	Thin ring or cylindrical shell of radius $R$	Through center		$I = mR^2$
6	Thin hoop of radius $R$ and width $w$	Through central diameter		$I = \frac{1}{2} mR^2 + \frac{1}{12} mw^2$
7	Hollow cylinder of inner radius $R_1$ and outer radius $R_2$	Through center		$I = \frac{1}{2} m(R_1^2 + R_2^2)$
8	Uniform sphere of radius $R$	Through center		$I = \frac{2}{5} mR^2$
9	Spherical shell of radius $R$	Through center		$I = \frac{2}{3} mR^2$



No	Object	Location of axis	Figure	Continue
				Moments of inertia
10	Rectangular thin plate, of length $l$ and width $w$	Through center		$I = \frac{1}{12} m(l^2 + w^2)$

### 1.3 Elastic Properties of Materials

Elasticity of materials is used often in biology and medicine, for example, elastic blood vessels, properties of bone, etc. In this section, the stress, strain and modulus will be discussed when different forces acting on an object, as shown in Fig.1.3.

#### 1.3.1 Stress

The definition of the **stress**(应力) is

$$\text{Stress} = \frac{dF}{dA} \quad (1.11)$$

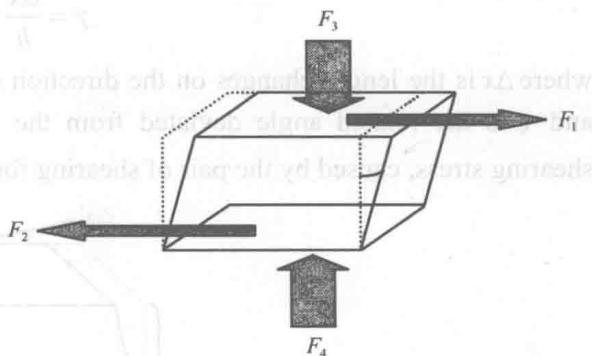


Fig.1.3 Different forces acting on an object

where  $dF$  is the element of force suffered by material on  $dA$  area. The SI unit of stress is Newton per square meter ( $\text{N} \cdot \text{m}^{-2}$ ). This unit is also given the special name, the Pascal (abbreviated Pa), ( $1\text{N} \cdot \text{m}^{-2} = 1\text{Pa}$ ) Stress can be classified into two different types. One is called normal stress or stretching stress, the other is called shearing stress. The normal (stretching) stress is perpendicular to the surface exerted by a force. It is expressed by

$$\sigma = \frac{dF}{dA}$$

Of course, it is equal to  $F/A$  if the force is uniform on the area. Shearing stress is parallel to the acting area, expressed by

$$\tau = \frac{dF}{dA}$$

Same as above, it is equal to  $F/A$  if the area is uniformly exerted by the force.

#### 1.3.2 Strain

The **strain**(应变) refers to the relative change in dimensions or shape of a body that is subjected to stress. There are three kinds of strains, which are stretching, volume and shearing strains. The definition of the three strains is given below respectively.

**1. Stretching (tensile) strain**(拉伸应变) Stretching (tensile) strain is defined by

$$\varepsilon = \frac{\Delta L}{L_0} \quad (1.12)$$

where  $L_0$  is the original length of the object, and  $\Delta L = L_0 - L$  denotes the length changes.

**2. Volume strain(体积应变)** Volume strain, expressed by  $\theta$ , is defined by

$$\theta = -\frac{\Delta V}{V_0} \quad (1.13)$$

where  $\Delta V = V_0 - V$ ,  $V_0$  is the volume before being depressed and  $V$  is the volume under strain. The minus sign means that the bulk of object is always depressed and becomes smaller. Volume strain is a pure number with no units because it is a ratio of two volumes.

**3. Shearing strain(剪切应变)** Fig.1.4 illustrates the nature of the deformation when shear stresses act on the faces of a block. Shearing strain, denoted by  $\gamma$ , is defined as

$$\gamma = \frac{\Delta x}{h} = \tan \varphi \quad (1.14)$$

where  $\Delta x$  is the length changes on the direction of acting force, and  $h$  is the height of the object and  $\varphi$  is the related angle deviated from the vertical line. Shearing strain is related to the shearing stress, caused by the pair of shearing forces.

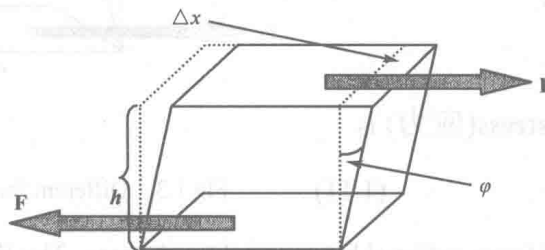


Fig.1.4 The shearing strain related elements

In practice,  $\Delta x$  is nearly always much smaller than  $h$ , so  $\tan \varphi$  is very nearly equal to  $\varphi$  and the strain is simply the angle. Like all strains, it also is a pure number.

**4. Poisson ratio** When an object is elongated, its cross-section will be getting smaller (contractive). The two sides of the cross-section are originally labeled as  $a_0$ , and  $b_0$  respectively, and its length is labeled as  $L_0$ . After elongation, there are some changes along the three edges. The **Poisson ratio(泊松比)** is defined as

$$\mu = \frac{\Delta a}{a_0} \bigg/ \frac{\Delta L}{L_0} = \frac{\Delta b}{b_0} \bigg/ \frac{\Delta L}{L_0} \quad (1.15)$$

If the material is incompressible, then

$$\mu = \frac{1}{2}$$

It is also for other materials,  $\mu < 1/2$ .

### 1.3.3 Modulus of Elasticity

**1. Stretch modulus** The stress required to produce a given strain depends on the nature of the material under stress. The ratio of stress to strain, or the stress per unit strain, is called an

**elastic modulus**(拉伸模量) of the material. The larger the elastic modulus, the greater the stress needed for a given strain.

A straight wire doesn't stretch as easily as a coiled wire, but its length does increase when a force is applied. The wire made of different material will have a different increase. How far the wire stretches depends on the stress and strain and finally depends on the material properties. In order to describe such a property, the concept of modulus is introduced.

A stretch modulus  $E$  of a straight wire, for example, is the ratio of stress to strain when the length of the wire changes. It is also called the elastic modulus or **Young's modulus**(杨氏模量) and denoted by  $Y$  or  $E$ .

$$E = \frac{\sigma}{\varepsilon} \quad (1.16)$$

where  $\sigma$  denotes normal stress,  $\varepsilon$  is tensile strain. We can also express it as

$$\text{Stretch modulus} = \frac{\text{stretching stress}}{\text{stretching strain}} = \frac{dF/dA}{\Delta L/L_0} \quad (1.17)$$

If the proportional limit is not exceeded, the ratio of stress to strain is constant, and **Hooke's law**(胡克定律) is therefore equivalent to the statement that within the proportional limit, the elastic modulus of a given material is constant, depending only on the nature of the material.

Since a strain is a pure number, the units of Young's modulus are the same as those of stress, namely, force per unit area.

**2. Shear modulus** Under certain conditions, a force applied to a solid object can change the shape of the object. Fig. 1.4 illustrates the result of applying a large horizontal force to the top of a rectangular block welded to a horizontal steel plane. A **shearing strain**(剪切模量) is the displacement divided by height. If the force is not great enough to produce a permanent distortion of the block, the block will return to its original shape when the force is removed. The shearing stress Force/Area is the applied force divided by the area. The shear modulus of an object is the ratio of shearing stress to strain when the shape of the object changes.

$$\text{Shear modulus } G = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{dF/dA}{\Delta x/h} = \frac{\tau}{\gamma} \quad (1.18)$$

The shear modulus has a significance for solid materials only. A liquid or gas flows under the influence of a shearing stress, and cannot permanently support such a stress.

**3. Bulk modulus** A solid object, such as a steel cylinder or a copper block, subjected to a high pressure decreases slightly in volume. For a given substance, the relative change in volume is proportional to the applied pressure. The **bulk modulus**(体积弹性模量)  $K$  of an object is the ratio of stress to strain when the volume of the object changes. That is the bulk modulus is equal to the volume stress divided by volume strain.

$$K = -\frac{\Delta P}{\theta} = -V_0 \frac{\Delta P}{\Delta V} \quad (1.19)$$

The minus sign is included in the definition because an increase of pressure always causes a decrease in volume. This is, if  $\Delta P$  is positive,  $\Delta V$  is negative. By including a minus sign in its definition, we make the bulk modulus itself a positive quantity.

The units of bulk modulus are the same as those of pressure. Some typical values of elastic

constants are listed in Table 1.2.

**Table 1.2 Elastic constants of some typical materials**

Materials	Young's modulus $E$ / $\times 10^{11}$ Pa	Shear modulus $S$ / $\times 10^{11}$ Pa	Bulk modulus $K$ / $\times 10^{11}$ Pa	Poisson's ratio $\mu$
Aluminum	0.70	0.30	0.70	0.16
Brass	0.91	0.36	0.61	0.26
Copper	1.1	0.42	1.4	0.32
Glass	0.55	0.23	0.37	0.19
Iron	1.9	0.70	1.0	0.27
Lead	0.16	0.056	0.077	0.43
Nickel	2.1	0.77	2.6	0.36
Steel	2.0	0.84	1.6	0.19
Tungsten	3.6	1.5	2.0	0.20

## 1.4 The Mechanical Properties of Bone

Bone plays a very important role in people's life. Its main function is to support the body, movement and the various organ protection etc.. Skeletal deformation, damage or destruction are related to the stress state. There are four basic forms of force acting on human skeleton, they are **tension and compression**(拉伸与压缩), **shear**(剪切), **bending**(弯曲), **torsion**(扭转), called **basic load**(基本载荷). If the skeleton is subject to two or more than two kinds of basic load, it is called **composite load**(复合载荷). Composite load can be considered as the combination of two or more than two kinds of basic load.

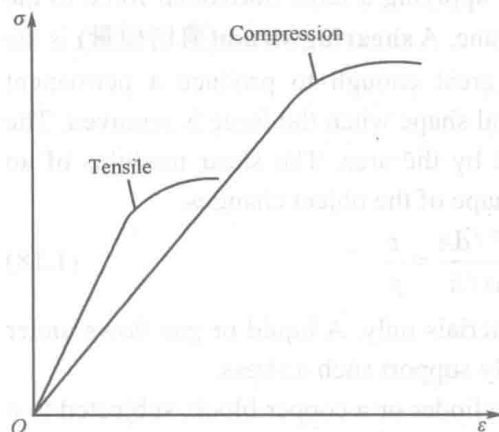


Fig.1.5 The stress strain curve of long bone

Fig.1.5 is the experimental curve of axial tensile and compression of wet bone, the stress and the strain curve. Tensile and compression curves are of similar shape, line segments are longer. Within this area, the stress is proportional to the strain, obeys the Hooke's Law. It can be considered that the bone has elasticity, but tensile and compressive bone with different Young's modulus. For example, adult femur when stretched, the Young's modulus is  $1.46 \times 10^4 \text{ MN} \cdot \text{m}^{-2}$  ( $1 \text{ MN} = 10^6 \text{ N}$ ), when compressed, the Young's modulus is  $8 \times 10^3 \text{ MN} \cdot \text{m}^{-2}$ . In addition, different from the general metal material, the skeleton will show different characteristics in different directions, this property is called **anisotropy**(各向异性).

### 1.4.2 Shear

Shear is the load applied in parallel to the direction of the cross section of skeleton. The stress on the cross section of bone is **shear stress**(切应力). Shear load skeleton bears is much