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# Modern Quantum Field Theory: A Concise Introduction

# 现代量子场论简引

（影印版）

〔美〕班克斯（T. Banks） 著

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# 序　　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学者的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套“中外物理学精品书系”的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会　主任

中国科学院院士，北京大学教授

**王恩哥**

2010 年 5 月于燕园

# Modern Quantum Field Theory

## A Concise Introduction

Tom Banks
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CAMBRIDGE
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# 1 Introduction

## 1.1 Preface and conventions

This book is meant as a quick and dirty introduction to the techniques of quantum field theory. It was inspired by a little book (long out of print) by F. Mandl, which my advisor gave me to read in my first year of graduate school in 1969. Mandl's book enabled the smart student to master the elements of field theory, as it was known in the early 1960s, in about two intense weeks of self-study. The body of field-theory knowledge has grown way beyond what was known then, and a book with similar intent has to be larger and will take longer to absorb. I hope that what I have written here will fill that Mandl niche: enough coverage to at least touch on most important topics, but short enough to be mastered in a semester or less. The most important omissions will be supersymmetry (which deserves a book of its own) and finite-temperature field theory. Pedagogically, this book can be used in three ways. Chapters 1–6 can be used as a text for a one-semester introductory course, the whole book for a one-year course. In either case, the instructor will want to turn some of the starred exercises into lecture material. Finally, the book was designed for self-study, and can be assigned as a supplementary text. My own opinion is that a complete course in modern quantum field theory needs 3–4 semesters, and should cover supersymmetric and finite-temperature field theory.

This statement of intent has governed the style of the book. I have tried to be terse rather than discursive (my natural default) and, *most importantly, I have left many important points of the development for the exercises. The student should not imagine that he/she can master the material in this book without doing at least those exercises marked with a* *. In addition, at various points in the text I will invite the reader to prove something, or state results without proof. The diligent reader will take these as extra exercises. This book may appear to the student to require more work than do texts that try to spoon-feed the reader. I believe strongly that a lot of the material in quantum field theory can be learned well only by working with your hands. Reading or listening to someone's explanation, no matter how simple, will not make you an adept. My hope is that the hints in the text will be enough to let the student master the exercises and come out of this experience with a thorough mastery of the basics.

The book also has an emphasis on theoretical ideas rather than application to experiment. This is partly due to the fact that there already exist excellent texts that concentrate on experimental applications, partly due to the desire for brevity, and partly to increase the shelf life of the volume. The experiments of today are unlikely to be of intense interest even to experimentalists of a decade hence. The structure of quantum field theory will exist forever.

Throughout the book I use natural units, where $\hbar = c = 1$. Everything has units of some power of mass/energy. High-energy experiments and theory usually concentrate on the energy range between $10^{-3}$ and $10^3$ GeV and I will often use these units. Another convenient unit of energy is the natural one defined by gravitation: the Planck mass, $M_P \approx 10^{19}$ GeV, or reduced Planck mass, $m_P \approx 2 \times 10^{18}$ GeV. The GeV is the natural unit for hadron masses. Around 0.15 GeV is the scale at which strong interactions become strong. Around 250 GeV is the natural scale of electro-weak interactions, and $\sim 2 \times 10^{16}$ GeV appears to be the scale at which electro-weak and strong interactions are unified.

I will use non-relativistic normalization, $\langle p|q\rangle = \delta^3(p - q)$, for single-particle states. Four-vectors will have names which are single Latin letters, while 3-vectors will be written in bold face. I will use Greek mid-alphabet letters for tensor indices, and Latin early-alphabet letters for spinors. Mid-alphabet Latin letters will be 3-vector components. I will stick to the van der Waerden dot convention (Chapter 5) for distinguishing left- and right-handed Weyl spinors. As for the metric on Minkowski space, I will use the West Coast, *mostly minus*, convention of most working particle theorists (and of my toilet training), rather than the East Coast (mostly plus) convention of relativists and string theorists.

Finally, a note about prerequisites. The reader must begin this book with a thorough knowledge of calculus, particularly complex analysis, and a thorough grounding in non-relativistic quantum mechanics, which of course includes expert-level linear algebra. Thorough knowledge of special relativity is also assumed. Detailed knowledge of the mathematical niceties of operator theory is unnecessary. The reader should be familiar with the Einstein summation convention and the totally anti-symmetric Levi-Civita symbol $\epsilon^{a_1 \ldots a_n}$. We use the convention $\epsilon^{0123} = 1$ in Minkowski space. It would be useful to have a prior knowledge of the theory of Lie groups and algebras, at a physics level of rigor, although we will treat some of this material in the text and Appendix G. I have supplied some excellent references [1–4] because this math is crucial to much that we will do. As usual in physics, what is required of your mathematical background is a knowledge of terminology and how to manipulate and calculate, rather than intimate familiarity with rigor and formal proofs.

### 1.1.1 Acknowledgements

I mostly learned field theory by myself, but I want to thank Nick Wheeler of Reed College for teaching me about path integrals and the beauties of mathematical physics in general. Roman Jackiw deserves credit for handing me Mandl's book, and Carl Bender helped me figure out what an instanton was before the word was invented. Perhaps the most important influence in my grad school years was Steven Weinberg, who taught me his approach to fields and particles, and everything there was to know about broken

symmetry. Most of the credit for teaching me things about field theory goes to Lenny Susskind, from whom I learned Wilson's approach to renormalization, lattice gauge theory, and a host of other things throughout my career. Shimon Yankielowicz and Eliezer Rabinovici were my most important collaborators during my years in Israel. We learned a lot of great physics together. During the 1970s, along with everyone else in the field, I learned from the seminal work of D. Gross, S. Coleman, G. 't Hooft, G. Parisi, and E. Witten. Edward was a friend and a major influence throughout my career. As one grows older, it's harder for people to do things that surprise you, but my great friends and sometimes collaborators Michael Dine, Willy Fischler, and Nati Seiberg have constantly done that. Most of the field theory they've taught me goes beyond what is covered in this book. You can find some of it in Michael Dine's recent book from Cambridge University Press.

Field theory can be an abstract subject, but it is physics and it has to be grounded in reality. For me, the most fascinating application of field theory has been to elementary particle physics. My friends Lisa Randall, Yossi Nir, Howie Haber, and, more recently, Scott Thomas have kept me abreast of what's important in the experimental foundation of our field.

In writing this book, I've been helped by M. Dine, H. Haber, J. Mason, L. Motl, A. Shomer, and K. van den Broek, who've read and commented on all or part of the manuscript. The book would look a lot worse than it does without their input. Chapter 10 was included at the behest of A. Strominger, and I thank him for the suggestion. Chris France, Jared Rice, and Lily Yang helped with the figures. Finally, I'd like to thank my wife Ada, who has been patient throughout all the trauma that writing a book like this involves.

## 1.2 Why quantum field theory?

Students often come into a class in quantum field theory straight out of a course in non-relativistic quantum mechanics. Their natural inclination is to look for a straightforward relativistic generalization of that formalism. A fine place to start would seem to be a covariant classical theory of a single relativistic particle, with space-time position variable $x^\mu(\tau)$, written in terms of an arbitrary parametrization $\tau$ of the particle's path in space-time.

The first task of a course in field theory is to explain to students why this is not the right way to do things.[1] The argument is straightforward.

Consider a classical machine (an emission source) that has probability amplitude $J_{\mathrm{E}}(x)$ of producing a particle at position $x$ in space-time, and an absorption source, which has amplitude $J_{\mathrm{A}}(x)$ to absorb the particle. Assume that the particle propagates

[1] Then, when they get more sophisticated, you can show them how the particle path formalism can be used, with appropriate care.

Fig. 1.1. **Boosts can reverse causal order for $(x-y)^2<0$.**

freely between emission and absorption, and has mass $m$. The standard rules of quantum mechanics tell us that the amplitude (to leading order in perturbation theory in the sources) for the entire process is (remember our natural units!)

$$A_{\mathrm{AE}} = \int \mathrm{d}^4x\,\mathrm{d}^4y \langle x|\mathrm{e}^{-\mathrm{i}H(x^0-y^0)}|y\rangle J_{\mathrm{A}}(x)J_{\mathrm{E}}(y), \tag{1.1}$$

where $|x\rangle$ is the state of the particle at spatial position $x$. This doesn't look very Lorentz-covariant. To see whether it is, write the relativistic expression for the energy $H = \sqrt{p^2+m^2} \equiv \omega_p$. Then

$$A_{\mathrm{AE}} = \int \mathrm{d}^4x\,\mathrm{d}^4y\, J_{\mathrm{A}}(x)J_{\mathrm{E}}(y) \int \mathrm{d}^3p |\langle 0|p\rangle|^2 \mathrm{e}^{-\mathrm{i}p(x-y)}. \tag{1.2}$$

The space-time set-up is shown in Figure 1.1. In writing this equation I've used the fact that momentum is the generator of space translations[2] to evaluate position/momentum overlaps in terms of the momentum eigenstate overlap with the state of a particle at the origin. I've also used the fact that $(\omega_p, p)$ is a 4-vector to write the exponent as a Lorentz scalar product. So everything is determined by quantum mechanics, translation invariance and the relativistic dispersion relation, up to a function of 3-momentum. We can determine this function up to an overall constant, by insisting that the expression is Lorentz-invariant, if the emission and absorption amplitudes are chosen to transform as scalar functions of space-time. An invariant measure for 4-momentum integration, ensuring that the mass is fixed, is $\mathrm{d}^4p\,\delta(p^2-m^2)$. Since the momentum is then forced to be time-like, the sign of its time component is also Lorentz-invariant (Problem 2.1). So we can write an invariant measure $\mathrm{d}^4p\,\delta(p^2-m^2)\theta(p^0)$ for positive-energy particles of mass $m$. On doing the integral over $p^0$ we find $\mathrm{d}^3p/(2\omega_p)$. Thus, if we choose the normalization

$$\langle 0|p\rangle = \frac{1}{\sqrt{(2\pi)^3 2\omega_p}}, \tag{1.3}$$

then the propagation amplitude will be Lorentz-invariant. The full absorption and emission amplitude will of course depend on the Lorentz frame because of the coordinate dependence of the sources $J_{\mathrm{E,A}}$. It will be covariant if these are chosen to transform like scalar fields.

[2] Here I'm using the notion of the infinitesimal generator of a symmetry transformation. If you don't know this concept, take a quick look at Appendix G, or consult one of the many excellent introductions to Lie groups [1–4].

This equation for the momentum-space wave function of "a particle localized at the origin" is not the same as the one we are used to from non-relativistic quantum mechanics. However, if we are in the non-relativistic regime where $|p| \ll m$ then the wave function reduces to $1/m$ times the non-relativistic formula. When relativity is taken into account, the localized particle appears to be spread out over a distance of order its Compton wavelength, $1/m = \hbar/(mc)$.

Our formula for the emission/absorption amplitude is thus covariant, but it poses the following paradox: *it is non-zero when the separation between the emission and absorption points is space-like.* The causal order of two space-like separated points is not Lorentz-invariant (Problem 2.1), so this is a real problem.

The only known solution to this problem is to impose a new physical postulate: every emission source could equally well be an absorption source (and vice versa). We will see the mathematical formulation of this postulate in the next chapter. Given this postulate, we define a total source by $J(x) = J_{\mathrm{E}}(x) + J_{\mathrm{A}}(x)$ and write an amplitude

$$\begin{aligned} A_{\mathrm{AE}} &= \int \mathrm{d}^4x\, \mathrm{d}^4y\, J(x)J(y) \int \frac{\mathrm{d}^3p}{2\omega_p(2\pi)^3}[\theta(x^0 - y^0)\mathrm{e}^{-\mathrm{i}p(x-y)} \\ &\qquad + \theta(y^0 - x^0)\mathrm{e}^{\mathrm{i}p(x-y)}] \\ &\equiv \int \mathrm{d}^4x\, \mathrm{d}^4y\, J(x)J(y)D_{\mathrm{F}}(x-y), \end{aligned} \tag{1.4}$$

where $\theta(x^0)$ is the Heaviside step function which is 1 for positive $x^0$ and vanishes for $x^0 < 0$. From now on we will omit the 0 superscript in the argument of these functions. This formula is manifestly Lorentz-covariant when $x - y$ is time-like or null. When the separation is space-like, the momentum integrals multiplying the two different step functions are equal, and we can add them, again getting a Lorentz-invariant amplitude. It is also consistent with causality. In any Lorentz frame, the term with $\theta(x^0 - y^0)$ is interpreted as the amplitude for a positive-energy particle to propagate forward in time, being emitted at $y$ and absorbed at $x$. The other term has a similar interpretation as emission at $x$ and absorption at $y$. Different Lorentz observers will disagree about the causal order when $x - y$ is space-like, but they will all agree on the total amplitude for any distribution of sources.

Something interesting happens if we assume that the particle has a conserved Lorentz-invariant charge, like electric charge. In that case, one would have expected to be able to correlate the question of whether emission or absorption occurred to the amount of charge transferred between $x$ and $y$. Such an absolute definition of emission versus absorption is not consistent with the postulate that saved us from a causality paradox. In order to avoid it we have to make another, quite remarkable, postulate: every charge-carrying particle has an anti-particle of exactly equal mass and opposite charge. If this is true we will not be able to use charge transfer to distinguish between emission of a particle and absorption of an anti-particle. One of the great triumphs of quantum field theory is that this prediction is experimentally verified. The equality of particle and anti-particle masses has been checked to one part in $10^{18}$ [5].

Now let's consider a slightly more complicated process in which the particle scatters from some external potential before being absorbed. Suppose that the potential is

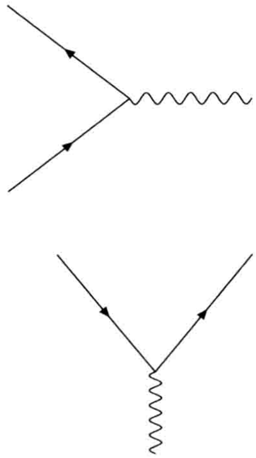

Fig. 1.2. Scattering in one frame is production amplitude in another.

short-ranged, and is turned on for only a brief period, so that we can think of it as being concentrated near a space-time point $z$. The scattering amplitude will be approximately given by propagation from the emission point to the interaction point $z$, some interaction amplitude, and then propagation from $z$ to the absorption point. We can draw a space-time diagram like Figure 1.2. We have seen that the propagation amplitudes will be non-zero, even when all three points are at space-like separation from each other. Then, there will be some Lorentz frame in which the causal order is that given in the second drawing in the figure. An observer in this frame sees particles created from the vacuum by the external field! Scattering processes inevitably imply particle-production processes.[3]

We conclude that a theory consistent with special relativity, quantum mechanics, and causality must allow for particle creation when the energetics permits it (in the example of the previous paragraph, the time dependence of the external field supplies the energy necessary to create the particles). This, as we shall see, is equivalent to the statement that a causal, relativistic quantum mechanics must be a theory of quantized local fields. Particle production also gives us a deeper understanding of why the single-particle wave function is spread over a Compton wavelength. To localize a particle more precisely we would have to probe it with higher momenta. Using the relativistic energy–momentum relation, this means that we would be inserting energy larger than the particle mass. This will lead to uncontrollable pair production, rather than localization of a single particle.

Before leaving this introductory section, we can squeeze one more drop of juice from our simple considerations. This has to do with how to interpret the propagation amplitude $D_F(x - y)$ when $x - y$ is space-like, and we are in a Lorentz frame where

[3] Indeed, there are quantitative relations, called *crossing symmetries*, between the two kinds of amplitude.