

Calculations for

Molecular Biology and Biotechnology

A Guide to Mathematics in the Laboratory



Second Edition

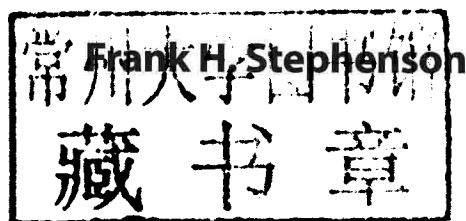
Frank H. Stephenson Ph.D



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Second Edition



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Scientific notation and metric prefixes

■ INTRODUCTION

There are some 3 000 000 000 base pairs (bp) making up human genomic DNA within a haploid cell. If that DNA is isolated from such a cell, it will weigh approximately 0.000 000 000 003 5 grams (g). To amplify a specific segment of that purified DNA using the polymerase chain reaction (PCR), 0.000 000 000 01 moles (*M*) of each of two primers can be added to a reaction that can produce, following some 30 cycles of the PCR, over 1 000 000 000 copies of the target gene.

On a day-to-day basis, molecular biologists work with extremes of numbers far outside the experience of conventional life. To allow them to more easily cope with calculations involving extraordinary values, two shorthand methods have been adopted that bring both enormous and infinitesimal quantities back into the realm of manageability. These methods use scientific notation and metric prefixes. They require the use of exponents and an understanding of significant digits.

1.1 SIGNIFICANT DIGITS

Certain techniques in molecular biology, as in other disciplines of science, rely on types of instrumentation capable of providing precise measurements. An indication of the level of precision is given by the number of digits expressed in the instrument's readout. The numerals of a measurement representing actual limits of precision are referred to as **significant digits**.

Although a zero can be as legitimate a value as the integers one through nine, significant digits are usually nonzero numerals. Without information on how a measurement was made or on the precision of the instrument used to make it, zeros to the left of the decimal point trailing one or more nonzero numerals are assumed not to be significant. For example, in stating that the human genome is 3 000 000 000 bp in length, the only significant digit in the number is the 3. The nine zeros are not significant. Likewise, zeros to the right of the decimal point preceding a set of nonzero numerals are assumed not to be significant. If we determine that the DNA within a

sperm cell weighs 0.000 000 000 003 5 g, only the 3 and the 5 are significant digits. The 11 zeros preceding these numerals are not significant.

Problem 1.1 How many significant digits are there in each of the following measurements?

- a) 3 001 000 000 bp
- b) 0.003 04 g
- c) 0.000 210 liters (L) (volume delivered with a calibrated micropipettor).

Solution 1.1

- a) Number of significant digits: 4; they are: 3001
- b) Number of significant digits: 3; they are: 304
- c) Number of significant digits: 3; they are: 210

1.1.1 Rounding off significant digits in calculations

When two or more measurements are used in a calculation, the result can only be as accurate as the least precise value. To accommodate this necessity, the number obtained as solution to a computation should be rounded off to reflect the weakest level of precision. The guidelines in the following box will help determine the extent to which a numerical result should be rounded off.

Guidelines for rounding off significant digits

1. When adding or subtracting numbers, the result should be rounded off so that it has the same number of significant digits to the right of the decimal as the number used in the computation with the fewest significant digits to the right of the decimal.
2. When multiplying or dividing numbers, the result should be rounded off so that it contains only as many significant digits as the number in the calculation with the fewest significant digits.

Problem 1.2 Perform the following calculations, and express the answer using the guidelines for rounding off significant digits described in the preceding box

- a) $0.2884 \text{ g} + 28.3 \text{ g}$
- b) $3.4 \text{ cm} \times 8.115 \text{ cm}$
- c) $1.2 \text{ L} \times 0.155 \text{ L}$

Solution 1.2

a) $0.2884 \text{ g} + 28.3 \text{ g} = 28.5884 \text{ g}$

The sum is rounded off to show the same number of significant digits to the right of the decimal point as the number in the equation with the fewest significant digits to the right of the decimal point. (In this case, the value 28.3 has one significant digit to the right of the decimal point.)

28.5884 g is rounded off to 28.6 g

b) $3.4 \text{ cm} \times 8.115 \text{ cm} = 27.591 \text{ cm}^2$

The answer is rounded off to two significant digits since there are as few as two significant digits in one of the multiplied numbers (3.4 cm).

27.591 cm² is rounded off to 28 cm²

c) $1.2 \text{ L} \div 0.155 \text{ L} = 7.742 \text{ L}$

The quotient is rounded off to two significant digits since there are as few as two significant digits in one of the values (1.2 L) used in the equation.

7.742 L is rounded off to 7.7 L

1.2 EXPONENTS AND SCIENTIFIC NOTATION

An **exponent** is a number written above and to the right of (and smaller than) another number (called the **base**) to indicate the power to which the base is to be raised. Exponents of base 10 are used in scientific notation to express very large or very small numbers in a shorthand form. For example, for the value 10^3 , 10 is the base and 3 is the exponent. This means that 10 is multiplied by itself three times ($10^3 = 10 \times 10 \times 10 = 1000$). For numbers less than 1.0, a negative exponent is used to express values as a reciprocal of base 10. For example,

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \times 10 \times 10} = \frac{1}{1000} = 0.001$$

1.2.1 Expressing numbers in scientific notation

To express a number in scientific notation:

1. Move the decimal point to the right of the leftmost nonzero digit. Count the number of places the decimal has been moved from its original position.

2. Write the new number to include all numbers between the leftmost and rightmost significant (nonzero) figures. Drop all zeros lying outside these integers.
3. Place a multiplication sign and the number 10 to the right of the significant integers. Use an exponent to indicate the number of places the decimal point has been moved.
 - a. For numbers greater than 10 (where the decimal was moved to the left), use a positive exponent.
 - b. For numbers less than one (where the decimal was moved to the right), use a negative exponent.

Problem 1.3 Write the following numbers in scientific notation

- a) 3 001 000 000
- b) 78
- c) 60.23×10^{22}

Solution 1.3

- a) Move the decimal to the left nine places so that it is positioned to the right of the leftmost nonzero digit.

$$3\,001\,000\,000$$

Write the new number to include all nonzero significant figures, and drop all zeros outside of these numerals. Multiply the new number by 10, and use a positive 9 as the exponent since the given number is greater than 10 and the decimal was moved to the left nine positions.

$$3\,001\,000\,000 = 3.001 \times 10^9$$

- b) Move the decimal to the left one place so that it is positioned to the right of the leftmost nonzero digit. Multiply the new number by 10, and use a positive 1 as an exponent since the given number is greater than 10 and the decimal was moved to the left one position.

$$78 = 7.8 \times 10^1$$

- c) 60.23×10^{22}

Move the decimal to the left one place so that it is positioned to the right of the leftmost nonzero digit. Since the decimal was moved one position to the left, add 1 to the exponent ($22 + 1 = 23 =$ new exponent value).

$$60.23 \times 10^{22} = 6.023 \times 10^{23}$$

Problem 1.4 Write the following numbers in scientific notation

- a) 0.000 000 000 015
- b) 0.000 050 004 2
- c) 437.28×10^{-7}

Solution 1.4

- a) Move the decimal to the right 11 places so that it is positioned to the right of the leftmost nonzero digit. Write the new number to include all numbers between the leftmost and rightmost significant (nonzero) figures. Drop all zeros lying outside these numerals. Multiply the number by 10, and use a negative 11 as the exponent since the original number is less than 1 and the decimal was moved to the right by 11 places.

$$0.000000000015 = 1.5 \times 10^{-11}$$

- b) Move the decimal to the right five positions so that it is positioned to the right of the leftmost nonzero digit. Drop all zeros lying outside the leftmost and rightmost nonzero digits. Multiply the number by 10 and use a negative 5 exponent since the original number is less than 1 and the decimal point was moved to the right five positions.

$$0.0000500042 = 5.00042 \times 10^{-5}$$

- c) Move the decimal point two places to the left so that it is positioned to the right of the leftmost nonzero digit. Since the decimal is moved two places to the left, add a positive 2 to the exponent value ($-7 + 2 = -5$).

$$437.28 \times 10^{-7} = 4.3728 \times 10^{-5}$$

1.2.2 Converting numbers from scientific notation to decimal notation

To change a number expressed in scientific notation to decimal form:

1. If the exponent of 10 is positive, move the decimal point to the right the same number of positions as the value of the exponent. If necessary, add zeros to the right of the significant digits to hold positions from the decimal point.
2. If the exponent of 10 is negative, move the decimal point to the left the same number of positions as the value of the exponent. If necessary, add zeros to the left of the significant digits to hold positions from the decimal point.