PAUL J. OSSENBRUGGEN

FUNDAMENTAL PRINCIPLES OF SYSTEMS ANALYSIS AND DECISION-MAKING

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Preface

This book deals with fundamental issues of infrastructure that modern industrialized societies have to address and solve. This is accomplished by using systems analysis and modeling techniques for decision-making. Systems analysis is more than an exercise in mathematics; it offers a logical framework for organizing studies dealing with complex issues that face our society.

The student is required to learn five basic concepts:

- The Systems Approach: How to address complex problems in an orderly and thorough manner.
- Optimization: How to formulate and analyze mathematical models for finding the best solution subject to constraints, such as physical, economic, legal, and other societal restrictions.
- Economic Evaluation: How to effectively measure and compare the performance of alternatives offered as solutions to a problem.
- Variability and Uncertainty: How to analyze processes involving chance and uncertainty.
- Model Calibration: How to extract information from data.

This book is intended for engineering, mathematics, and science students interested in technological, social, and economic issues. A prerequisite course in freshmen calculus is essential as well as an elementary course in probability/statistics, and physics or chemistry is desirable. Depending on the students' background, the material can be covered in one or two semesters. I have taught the material given in this book in a two-semester sequence—at the second-semester junior level, and at the first-semester senior years.

Exercises

Exercises are chosen to reinforce fundamental concepts and to enhance practical skills. There are two types of questions:

1. *Questions for Discussion* are designed to stimulate thought about a particular topic and reinforce the idea that real world problems and answers are not "right" or "wrong" but are judged by rules of adequacy. Some problems deal with personal opinion or controversial issues. Sometimes, students are asked to take a position on an issue and support it with factual evidence.

Since knowledge of the vocabulary of a subject is so important to understanding and communicating about it, every section of this book first asks for the definitions of terms. Terms can be defined by word definitions but, in some instances, it might be clearer to use mathematical definitions, sketches, or diagrams. When possible, the student is urged to give an example to illustrate the meaning of the term.

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2. *Questions for Analysis* are designed to reinforce understanding of the material and enhance problem-solving skills. These problems generally require more than plugging numbers into a formula. The student might be asked to formulate a problem as a mathematical model and then to solve it. In many cases, the real challenge is in the formulation.

An instructor's manual, which includes detailed solutions to problems and the author's opinions and comments to open-ended questions, is available to instructors through the publisher.

Computer Programs

The computer plays an important role in the quantitative analysis and the decision-making processes. It is particularly helpful in performing sensitivity analyses or answering "what if" questions and solving other problems where tables and graphic output can help interpret the results. Assignments that are most easily solved with the aid of a computer are indicated with an asterisk (*).

Students are expected to have a working knowledge of how to use a spreadsheet program. No other computer programming skill is necessary. Students can use a spreadsheet program of their choice. For Chapters 2 and 3, it is necessary for students to have access to the simplex method for solving linear programming problems. The program called "simplex," written for the Macintosh, is available from the author and publisher. Other programs written for personal and mainframe computers can be used.

Case Study

The methods of analysis presented in this book deal with planning, design, and management problems, and so have broad application. In order to truly appreciate the methods being introduced, we must know how the system works and understand the scope of problems and the consequences associated with an alternative that is offered as a solution. Technology alone cannot solve a societal problem. With systems modeling and analysis, various alternatives are investigated and critically evaluated, and solutions are sought. Systems analysis and modeling are not offered as a panacea, but as a way to achieve more informed decision-making.

To stimulate interest and motivate understanding, a case study of collection, treatment, and disposal of municipal solid waste (MSW) is used in parts of the book. It is a real-world, multidisciplinary problem involving economic, technical, social, and political issues. It must be emphasized that the MSW case study has been chosen to bridge the gap between mathematical modeling and real-world application. Of more than 250 examples and exercises in this book, fewer than 20 percent of the problems deal with the MSW case study. The same approaches used to address it have direct application to other fields. Let us consider the breadth and depth of this case study.

The MSW case study examines a variety of issues including transportation, construction, economic, and other important problems. For example, incineration, recycling, and waste source reduction and disposing of MSW in landfills are offered as solutions to the problem (Chapter 1). As communities seek a solution to this problem in an environmentally safe and cost effective manner, they are finding that no one method of disposal is proving to be completely acceptable. As the cost for disposal has become increasingly expensive and new demands for environmental quality are required, solid waste management has become a

major social and political issue in many communities. A comprehensive plan should take into consideration the monetary and social benefits and costs of each alternative. Clearly, one must have more than superficial understanding of the problem to appreciate the consequences of a solution. Data and other pertinent information have been collected and is used in examples and exercises to give deeper insight and meaning to the issues at hand.*

This same philosophy used for addressing resource allocation and capital investment problems (Chapters 1 through 4) is carried forward to discuss issues dealing with capital investment, design, and planning problems under conditions of uncertainty (Chapters 5 through 8). Since a comprehensive MSW management plan should consider the long-term public health and environmental impacts associated with incineration and landfilling, analysis and discussion of these systems are used to answer questions of technological risk associated with incineration, to the design of a landfill, and to the operation of incinerators. Of course, problems from other fields are used to illustrate the principles and applications of systems analysis.

I thank the following people for their assistance: Paul C. Ossenbruggen, for the development of the computer programs; John Bonneville, Marie Gaudard, Árpád Horváth, Scott Nerny, Karoly Palovits, and David Tolli, for their support and assistance, particularly in evaluating the manuscript and its pedagogical integrity; Nickolas Buckeridge, Christopher Dwinal, Rita McCarroll, Jonathan Hilt, and other members of the junior level classes of 1991, 1992 and 1993, for their patience in using this material during its development.

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*A user-friendly computer program, SWAP (Solids Waste Analysis Program), has been developed to aid in the analysis of the MSW management problems (Section 3.3). This program, written for the Macintosh, uses computer graphics to establish a network diagram, formulates a linear programming model to minimize cost of disposal, and uses the simplex method to determine the least cost solution. It is available from the author and publisher. While desirable, it is not an imperative that this program be used with this book.

Notation

The following units are generally given in parentheses within the chapter text:

$$T \ = \ \text{time}$$

$$E \ = \ \text{energy}$$

$$L \ = \ \text{length}$$

$$M \ = \ \text{mass}$$

$$\# \ = \ \text{unitless number}$$

anw = annual net worth as a uniform payment series ($/T where T in year),

a = ash content of a material (% of dry mass)

c = unit cost ($/#)

d = distance (L)

e = efficiency (% or $0 \leq e \leq 1$)

h = unit heat content of a material (E/M of dry mass)

f = annual inflation rate (% or $0 \leq f \leq 1$)

i = discount as an annual interest rate (% or $0 \leq i \leq 1$)

i_b = bank loan interest rate (% or $0 \leq i \leq 1$)

i_f = risk free, rate of return interest rate on investment (% or $0 \leq i \leq 1$)

i' = internal rate of return interest rate (% or $0 \leq i \leq 1$)

$k, k(t)$ = total cost or amortized debt in year t ($/T)

m = moisture content of a material (%)

$$n \;=\; \text{project life (T in years)}$$

$$nw \;=\; \text{net worth (\$),}$$

$$pw \;=\; \text{present worth of discounted dollars where } t = \text{year 0 (\$)}$$

$$npw \;=\; \text{net present worth expressed in year zero } t = 0 \text{ dollars (\$)}$$

$$p \;=\; \text{price (\$/\#)}$$

$$q \;=\; \text{consumer demand (\#)}$$

$$r, r' \;=\; \text{real and corrected real interest rate where } 0 \leq r \leq 1$$

$$s \;=\; \text{shipping fee (\$/M-L)}$$

$$t \;=\; \text{time (T)}$$

$$u \;=\; \text{tipping fee (\$/M)}$$

$$v \;=\; \text{fraction of people who volunteer to recycle, (\%)}$$

$$w \;=\; \text{total worth such as a benefit, revenue, or savings (\$)}$$

$$x \;=\; \text{control variable (M), or number of items produced (\#)}$$

Vectors and Matrix Notation

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix} \quad \text{where } j = 0,1,2,\ldots,m$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad \text{where } j = 0, 1, 2, \ldots, n$$

$$A^T, \mathbf{x}^T = \text{transpose of } \mathbf{x} \text{ and } A$$

$$\mathbf{A} = m \times n \ \ \text{matrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

For example, $\mathbf{A}\,\mathbf{x} = \mathbf{b}$.

For Optimization Models

x^o = critical point, $f'(x^o) = 0$

x = vector of control variables

x^* = location of optimum solution

j = subscript index, 0, 1, 2, . . . ,

$f(\mathbf{x})$ = performance function = function of the control variables of $\mathbf{x}$

b_j = right-hand side of the constraint function, for example, $f_j(\mathbf{x}) \le b_j$

y = value of $f(x)$

y^* = optimum value of $f(x) = f(x^*)$

For Probability Models

$a(t)$ = availability at time t (% or $0 \le a(\text{t}) \le 1$)

$f(y)$ = probability function of Y, when Y is a discrete variable or a probability density function of Y, when Y is a continuous variable

$F(y)$ = $P(Y \le y)$ = cumulative probability function of Y

h = hazard or damage cost as a consequence of system failure (\$)

L_Y, U_Y = lower and upper bound confidence or prediction endpoints for Y

$r(t)$ = system reliability at time t (% or $0 \leq r(t) \leq 1$)

$R(t)$ = risk as an expected annual cost = $h\theta(t)$

R = risk as an expected total cost = $h\theta$

S = failure event

S' = complement of S or success event or working state.

Y = random variable

y = observed outcome of Y

Z = sample space, for example, $Z = \{S, S'\}$ where S = event and S' = complementary event with $P(S) + P(S') = 1$ and $0 \leq P(S), P(S') \leq 1$

θ = lifetime risk probability = $n\theta(t)$ where $\theta(t)$ is generally assumed to be constant for all t and n = project life or human lifespan (% or $0 \leq \theta \leq 1$)

$\theta(t)$ = annual risk probability in year t (% or $0 \leq \theta(t) \leq 1$)

$\theta(d)$ = risk probability for intake dosage d (% or $0 \leq \theta(d) \leq 1$)

$\hat{\theta}, \hat{\theta}(t), \hat{\theta}(d)$ = estimate of θ, $\theta(t)$ and $\theta(d)$

μ = $E(Y)$ = mean or expectation of Y

σ^2 = $E[(Y - \mu)^2]$ = variance of Y

σ = $\sqrt{\sigma^2}$ = standard deviation of Y

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Chapter 1

The Systems Approach

Why does this magnificent applied science which saves work and makes life easier bring us so little happiness? The simple answer runs: we have not yet learned how to make sensible use of it.

<div align="right">Albert Einstein (1879–1955)</div>

A *system* is an organized, integrated unit that serves a common purpose. Typically, it is formed from many, often diverse components. This definition implies that the components act in unison and in accordance with a coherent plan for smooth and orderly operation.

Systems analysis is a systematic methodology for defining problems and evaluating solutions. It requires that a clearly defined project objective be established, that alternative solutions be proposed and carefully analyzed in light of this objective, and that the best solution be selected. It is a general, logical method of analysis that is used to investigate physical, social, and political systems.

This book introduces approaches that lead to the creation of successful systems and discusses issues that surround their implementation, particularly those issues that have an impact on society.

1.1 AN OPTIMIZATION PROCESS

Systems analysis uses mathematics to determine how a set of interconnected components behave in response to a given set of inputs. It is a goal-oriented discipline, so it is imperative that a clearly stated project objective be established. In this book, considerable attention is given to the *problem definition*, the establishment of an objective that properly reflects the needs and desires of the client. The *client* is a generic term used to describe the individual or individuals who pay for and/or receive benefits from the system.

Systems analysis can be used for *decision making*. After the project objective is established, the benefits and cost of each alternative solution are identified, analyzed, and weighed; then a final choice is made by the client. Since the analysis is structured in a

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mathematical framework, a suitable measure of effectiveness or merit must be defined. A *measure of effectiveness* or *merit* is a standard or benchmark that serves as a basis for judging how well a solution achieves the project objective. All alternatives are evaluated using the same criteria and are given an equal chance of being selected; therefore, the process is considered to be inclusive, rational, and fair.

Typically, systems analysis is applied to processes that are complex, interactive, and expensive to build, maintain, and operate. Our task in this book is to demonstrate how to address and solve decision-making problems. Hence, we illustrate

- How information is gathered and used
- How mathematical models are derived and analyzed
- How solutions are obtained

Planning, design, and management are all facilitated by system analysis. Planning involves the establishment of goals, policies, and strategies for social and economic units. Design, on the other hand, deals with the arrangement of parts to make an engineered object work in a desired manner, whereas management deals with the control and operation of a system. Depending on the problem definition and boundaries placed on it, a problem can be considered one of planning, design, or management. The differences between them can be subtle, and the distinction is not that important. Whatever the application, the approach is the same; it requires innovation and creativity.

Problems associated with systems analysis can be broadly classified as those of (1) resource allocation, (2) capital investment, (3) probability, and (4) statistics. Regardless of the classification, the aim is the same—to allocate resources in a manner that best satisfies a defined goal.

Money is needed to purchase, operate, and maintain system components. In the context of systems analysis, money, labor, and material are considered to be primary *resources*. Our task is to determine the best way to use them to achieve the stated objective. We are free to use resources within certain restrictions. In a problem treated as one of a resource allocation, we introduce these restrictions as constraint equations, which are derived from laws of nature, from component performance and resource limitations, and from regulations imposed by society. The combination of resources, an alternative solution, that best meets the stated objective and satisfies all constraint equations is called an *optimum solution*.
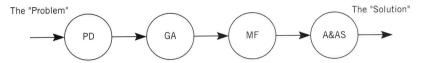
The methods introduced in this book are general and can be applied to a broad range of problems, including structural design, water resources planning, air quality management, and other important economic concerns.

The Systems Approach

Problem analysis involves the following stages:

- Problem definition (PD)
- Generation of alternatives (GA)
- Model formulation (MF)
- Analysis and alternative selection (A&AS)

The overall strategy, called the *systems approach*, endeavors to simplify a problem by reducing it into these stages. The overall process is depicted in Figure 1.

Figure 1   The systems approach as a flow of information.

Problem Definition

In the problem definition stage, the most important stage of the systems approach, we seek to identify the client's single most important goal. In this book, optimization models are limited to mathematical models of resource allocation problems with a single objective function. This may appear to be a major limitation, but in most situations, it is not as severe as we might first imagine.

Closely coupled with identifying the objective function is defining a measure of effectiveness or merit. These two factors have a major impact on the analysis and solution; therefore, they must be chosen with care.

The following steps are recommended for establishing the project objective and its measure of effectiveness:

- Determine your client.
- Establish the needs of the client.
- Identify the client's single most important objective.
- Choose a measure of effectiveness.
- Discuss the project objective with the client.
- Insure that the client clearly understands and agrees with it.

If the client is a group, then the last two steps should be presented to key representatives of the group.

When these seemingly simple steps are overlooked, considerable effort, time, and money can be wasted and the plan or design can be rejected because the recommended solution does not meet the client's need.

Generation of Alternatives

Alternative solutions that can satisfy the project objective should be identified. As much information about each alternative as possible should be obtained at this point. Gathering information may require searching the literature, obtaining technical and cost data on equipment, operation, and maintenance, and finding other pertinent information. Through this information, mathematical models can be formulated that accurately simulate the performance of each alternative.

It is also recommended that the availability of resources and anything that may impose a constraint on the system be established. In this book, constraint equations are typically derived from mass balance considerations, limitations on labor, materials, and budget. Codes, laws, and other regulations may impose additional constraints on the system.

Model Formulation

A primary task is to develop a mathematical model that reflects reality. This stage usually involves two steps:

- Model decoupling
- Model integration

Model decoupling is a simplifying step whereby system components are modeled and analyzed as subsystems. Simplifying in this manner often leads to a better understanding of how the whole system works. In the model integration step, the interaction that was lost by decoupling is regained.

The primary purpose of this two-step procedure is to simplify the overall model while maintaining reality. A delicate balance exists between model detail and the ability to effectively and efficiently analyze the model. Modeling detail may offer better reality at increased computational expense. Under certain circumstances, a simple model may prove to be more valuable than a complex one.

Analysis and Alternative Selection

The aim here is to find an optimum solution, the alternative that best achieves the client's objective and satisfies all constraint conditions. The analysis and alternative selection stages must be conducted in a fair and unbiased manner. In other words, all alternatives are given an equal chance of being selected, and the final choice is based on its merits. Analytical techniques are introduced in Chapter 2; in this chapter, the qualitative aspects of systems analysis are stressed.

Feedback and Sensitivity Analyses

An initial analysis may lead not to a final solution but to more questions. Nevertheless it is useful because it leads to an improved understanding of the problem and ultimately, a better solution. The systems approach is flexible, allowing the model to be modified and stages to be repeated. This kind of analysis is called a *feedback analysis*.

Through feedback analysis, the problem is often redefined, additional resources are found, models reformulated, and analyses repeated. Figure 2, for example, shows a feedback loop indicating a generation of new alternatives. It indicates that the mathematical model is reformulated and that a new solution is obtained.
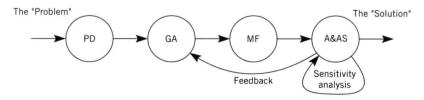

Figure 2 The concepts of feedback and sensitivity analyses as a flow of information.