



# **BASICS OF ENGINEERING TURBULENCE**

*David S-K. Ting*



# BASICS OF ENGINEERING TURBULENCE

**DAVID S-K. TING**

Turbulence & Energy Laboratory  
Centre for Engineering Innovation  
University of Windsor  
Windsor, Ontario, Canada



Amsterdam • Boston • Heidelberg • London  
New York • Oxford • Paris • San Diego  
San Francisco • Singapore • Sydney • Tokyo  
Academic Press is an imprint of Elsevier



Academic Press is an imprint of Elsevier  
125 London Wall, London EC2Y 5AS, UK  
525 B Street, Suite 1800, San Diego, CA 92101-4495, USA  
50 Hampshire Street, 5th Floor, Cambridge, MA 02139, USA  
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, UK

Copyright © 2016 Elsevier Inc. All rights reserved.

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or any information storage and retrieval system, without permission in writing from the publisher. Details on how to seek permission, further information about the Publisher's permissions policies and our arrangements with organizations such as the Copyright Clearance Center and the Copyright Licensing Agency, can be found at our website: [www.elsevier.com/permissions](http://www.elsevier.com/permissions).

This book and the individual contributions contained in it are protected under copyright by the Publisher (other than as may be noted herein).

### Notices

Knowledge and best practice in this field are constantly changing. As new research and experience broaden our understanding, changes in research methods, professional practices, or medical treatment may become necessary.

Practitioners and researchers must always rely on their own experience and knowledge in evaluating and using any information, methods, compounds, or experiments described herein. In using such information or methods they should be mindful of their own safety and the safety of others, including parties for whom they have a professional responsibility.

To the fullest extent of the law, neither the Publisher nor the authors, contributors, or editors, assume any liability for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions, or ideas contained in the material herein.

### Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress

### British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library

ISBN: 978-0-12-803970-0

For information on all Academic Press publications  
visit our website at <http://store.elsevier.com/>



Working together  
to grow libraries in  
developing countries

[www.elsevier.com](http://www.elsevier.com) • [www.bookaid.org](http://www.bookaid.org)

# **BASICS OF ENGINEERING TURBULENCE**



# DEDICATION

The meaning of an endeavor is found in the process, more so than the final outcome. This book is dedicated to those who attempt to make the best out of everyday turbulence.

*'Yesterday is history, tomorrow is a mystery, today is a gift of God, which is why we call it the present.'* — Bil Keane



# ACKNOWLEDGMENTS

In the absent of fore- and corunners, this book would have been but an anxious dream of starting a marathon without the stamina to cross the finish line. The author is particularly grateful to the strength from above and many individuals who eased this challenging endeavor, giving him the fuel needed to make it through the finish line. These instrumental individuals include the following:

Prof Dr D.J. Wilson, by whom the author was culturally shocked by flow turbulence in 1989. Some parts of this book have been written based on his lecture notes (“Mec E 632 Turbulent Fluid Dynamics,” University of Alberta, 1989).

The numerous engineering artists who have supplied the beautiful figures. While their helping hands are explicitly recognized in the figure caption, a general heartfelt thank you goes to the Turbulence and Energy (T&E) Laboratory. Everyone who has contributed, one way or another, is a T&E-er at heart, even though some graduated before the official establishment of the T&E Lab. Also, thanks to many of my T&E colleagues – even Dr Rupp Carriveau, who stole the spotlight with his eloquent exaggerations in the foreword.

The Elsevier publishing team who is responsible for many sleepless nights. It started with Chelsea Johnston and Joseph P. Hayton. The nightmare became particularly real with Carrie L. Bolger. The extremely supportive reviewers are also responsible. In particular, Prof Dr Pierre Sullivan, Prof Dr Himanshu Tyagi, and the overly positive Prof Dr Alain deChamplain and Alain Fossi.

The Turbulent flow graduate classes of 2012, 2014, and 2015, especially those who picked up the many less-than-obvious errors scattered over the two versions of self-published lecture notes. During the final round, Jamie C. Smith went through each sentence and objected to countless “leap-of-faith” wordings, Sai Praneeth Mupparapu checked every tilde in the equations, and Hao Wu combed out whatever debris he came across.

Mom, dad, sisters, and brother, and Uncle Mitchell and the other founding members of the Allinterest Research Institute; fluid dynamics is still turbulent my heart after all these years.



Naomi, Yoniana, Tachelle, and Zarek Ting for their unfailing love and continual encouragements, especially those conveyed via many philosophical sarcasms. They were there from the groundbreaking days of the first (self-published) version of this book with Naomi Ting's Books. It has been a long but rewarding journey together.

# FOREWORD

Turbulence can be a very beautiful thing, a dynamic cascade of scales that connects us to each other and our environment. Occasions to study turbulence are abundant; examples flow from our bodies to the heavens. Opportunities for true understanding are considerably less ample. Sir Horace Lamb himself once famously quipped that on reaching heaven he hoped for divine enlightenment on just two matters: quantum electrodynamics and turbulence. He said he was “rather optimistic of the former.” One barrier that has challenged more widespread understanding has been the lack of a true bridge to the topic. From the seminal works to most contemporary texts, the treatment of the subject is detailed and advanced. This is perfectly appropriate for select scholars of the science and sufficiently discouraging for the beginner, enthusiast, or cross-disciplinarian looking for application-level understanding. Subsequently, the ranks of the well-informed remain somewhat exclusive.

Enter David Ting. I have had the pleasure and challenge of working with David for the last 11 years. While David’s turbulence publication record is impressive in its own right, I have always been more impressed by his dedication and concern for students. More than anyone I have ever known, he is able to simplify, rearrange, and relate complex matters to those lost sheep keen to join the flock of the initiated. When his conventional teaching toolset is not reaching the students, it is his unparalleled faith in them that inspires their personal development. I trust you will enjoy the bridge David has built with this textbook. If you will not subscribe to my endorsement, then please have faith; in the pursuit of turbulence enlightenment, it would seem a minimum requirement.

**Rupp Carriveau**

*Dr Rupp Carriveau* is the Associate Professor at the University of Windsor, Lumley Centre for Engineering Innovation. He is the coordinator in the Centre for Energy and Water Advancement and Director in the Turbulence and Energy Laboratory. Dr Carriveau serves on the Editorial Boards of *Wind Engineering*, *Advances in Energy Research*, and the *International Journal of Sustainable Energy*. He is the current President of the Underwater Energy Storage Society. He was recently designated as the University Scholar and has served as the Research Ambassador for the Council of Ontario Universities.



# PREFACE

This book is intended for keen minds interested in flowing fluids. Specifically, it aims at removing the “fear of water” from those who are new to flow turbulence. The basic background on everyday flow turbulences, especially those encountered in engineering applications, forms the crux of the book. Some undergraduate knowledge of fluid mechanics and statistics is needed to best appreciate the material covered.

David S-K. Ting  
August 14, 2015



# CONTENTS

|                        |             |
|------------------------|-------------|
| <i>List of Figures</i> | <i>xi</i>   |
| <i>List of Tables</i>  | <i>xv</i>   |
| <i>Acknowledgments</i> | <i>xvii</i> |
| <i>Foreword</i>        | <i>xix</i>  |
| <i>Preface</i>         | <i>xxi</i>  |

## **PART 1. Some Basics of Flow Turbulence 1**

### **1. Introducing Flow Turbulence 3**

|  |    |
|--|----|
| 1.1 Introduction                       | 4  |
| 1.1.1 Irregular or Random              | 6  |
| 1.1.2 Rich in Scales of Eddying Motion | 6  |
| 1.1.3 Large Reynolds Number            | 7  |
| 1.1.4 Dissipative                      | 8  |
| 1.1.5 Highly Vortical                  | 9  |
| 1.1.6 Three-Dimensional                | 9  |
| 1.1.7 Highly Diffusive                 | 9  |
| 1.1.8 Turbulent Flows are Flows        | 10 |
| 1.1.9 Continuum                        | 10 |
| 1.2 A Brief Historic Account           | 10 |
| 1.2.1 Leonardo da Vinci (1452–1519)    | 10 |
| 1.2.2 Lord Rayleigh (1878, 1880)       | 11 |
| 1.2.3 Osborne Reynolds (1842–1912)     | 11 |
| 1.2.4 Henri Bénard (1900)              | 12 |
| 1.2.5 Taylor (1915, 1923, 1935, 1938)  | 14 |
| 1.2.6 Prandtl (1925)                   | 15 |
| 1.3 Organization of the Book           | 16 |
| Problems                               | 17 |
| References                             | 17 |

### **2. Equations of Fluid in Motion 19**

|  |    |
|--|----|
| 2.1 Introduction                         | 20 |
| 2.2 Eulerian and Lagrangian Frames       | 22 |
| 2.2.1 Eulerian                           | 22 |
| 2.2.2 Lagrangian                         | 22 |
| 2.2.3 Eulerian-Lagrangian Transformation | 23 |
| 2.3 Common Equations in Fluid Mechanics  | 24 |
| 2.3.1 Conservation of Mass               | 25 |
| 2.3.2 Momentum Equation                  | 28 |
| 2.4 Reynolds Decomposition               | 33 |

|   |           |
|---|-----------|
| 2.5 Conservation of Mass from Laminar to Turbulent Flow                     | 37        |
| 2.6 Momentum Equation in Turbulent Flow                                     | 39        |
| Problems  | 45        |
| References  | 46        |
| <b>3. Statistical Description of Flow Turbulence</b>                        | <b>47</b> |
| 3.1 Introduction  | 48        |
| 3.2 Probability   | 50        |
| 3.3 Moments   | 54        |
| 3.4 Joint Statistics and Correlation Functions                              | 59        |
| 3.5 Additional Considerations   | 64        |
| 3.5.1 Fourier Series and Coefficients                                       | 65        |
| 3.5.2 Fourier Transforms and Characteristic Functions                       | 66        |
| Problems  | 67        |
| References  | 68        |
| <b>4. Turbulence Scales</b>   | <b>69</b> |
| 4.1 Introduction  | 70        |
| 4.2 Velocity and Key Length Scales in Laminar and Turbulent Boundary Layers | 72        |
| 4.2.1 Laminar Boundary Layer  | 74        |
| 4.2.2 Turbulent Boundary Layer  | 75        |
| 4.3 Molecular Versus Turbulent Diffusion                                    | 77        |
| 4.4 Kolmogorov Microscales of Dissipation                                   | 79        |
| 4.5 An Inviscid Estimate for Dissipation Rate                               | 83        |
| 4.6 The Energy Cascade – Scales from Production-Dissipation Energy Balance  | 86        |
| 4.6.1 Production, Dissipation, and Local Equilibrium                        | 87        |
| 4.6.2 Approximate Scaling of Production and Dissipation                     | 88        |
| 4.6.3 Relating Production and Dissipation Scales                            | 89        |
| 4.7 Refined Estimates for Turbulence Dissipation and Integral Scales        | 90        |
| 4.7.1 Dissipation Microscales in Isotropic Turbulence                       | 91        |
| 4.7.2 Integral Scales   | 93        |
| 4.8 Turbulent Kinetic Energy Spectrum                                       | 95        |
| Problems  | 97        |
| References  | 98        |
| <b>5. Turbulence Simulations and Modeling</b>                               | <b>99</b> |
| 5.1 Introduction  | 100       |
| 5.2 The Closure Problem in Turbulent Flows                                  | 103       |
| 5.2.1 Zero-Order Closures (Algebraic Models)                                | 104       |
| 5.2.2 One-Equation Models   | 108       |
| 5.2.3 Two-Equation Models   | 110       |
| 5.3 Large Eddy Simulation   | 113       |
| 5.4 Direct Numerical Simulation   | 115       |
| Problems  | 116       |
| References  | 117       |

|  |                |
|--|----------------|
| <b>6. Wall Turbulence</b>                                | <b>119</b>     |
| 6.1 Introduction   | 120            |
| 6.2 Common Types of Boundary-Layer Thickness             | 122            |
| 6.3 Flat-Plate Boundary Layer                            | 123            |
| 6.3.1 Laminar Boundary Layer                             | 126            |
| 6.3.2 Transition to Turbulent                            | 126            |
| 6.3.3 Turbulent Boundary Layer                           | 127            |
| Problems   | 135            |
| References   | 137            |
| <br><b>7. Grid Turbulence</b>                            | <br><b>139</b> |
| 7.1 Introduction   | 140            |
| 7.2 Homogeneous and Isotropic Turbulence                 | 141            |
| 7.3 Characteristics of Grid Turbulence                   | 142            |
| 7.3.1 Initial Turbulence Developing Region               | 147            |
| 7.3.2 Power-Law Decay Region                             | 148            |
| 7.3.3 Dominating Large-Scale Region                      | 151            |
| 7.3.4 Final Decay Region                                 | 151            |
| 7.4 Decay of Homogeneous Isotropic Turbulence            | 151            |
| 7.5 Estimating the Integral Scale Variation              | 154            |
| 7.6 Kolmogorov Scale in Decaying Grid Turbulence         | 156            |
| 7.7 Spectral Space                                       | 158            |
| Problems   | 162            |
| References   | 162            |
| <br><b>8. Vortex Dynamics</b>                            | <br><b>165</b> |
| 8.1 Introduction   | 166            |
| 8.2 Vorticity  | 167            |
| 8.3 Kelvin's Circulation Theorem                         | 170            |
| 8.4 Evolution of Vorticity                               | 175            |
| 8.5 Interpreting Tangential Velocity as Turbulence       | 177            |
| 8.5.1 Stretching of Vortex Tubes by Flow Acceleration    | 179            |
| 8.5.2 Oblique Vortex Tubes Passing Through a Contraction | 180            |
| 8.5.3 Compressing a Vortex Tube                          | 181            |
| 8.5.4 Vortex Tube Distortion by an Expanding Sphere      | 181            |
| Problems   | 184            |
| References   | 185            |

## **PART 2. Examples of Engineering Problems Involving Flow Turbulence** **187**

|  |            |
|--|------------|
| <b>9. Sphere and Circular Cylinder in Cross Flow</b> | <b>189</b> |
| 9.1 Introduction                                     | 190        |
| 9.2 Flow Over a Smooth Sphere                        | 190        |



|            |  |            |
|------------|--|------------|
| 9.3        | Smooth Flow Across a Circular Cylinder                         | 194        |
| 9.4        | A Circular Cylinder in Turbulent Cross Flow                    | 196        |
| 9.5        | Turbulent Flow Over a Heated Circular Cylinder                 | 198        |
| 9.6        | Some Comments on Flow Over a Bluff Body                        | 200        |
|            | Problems   | 200        |
|            | References   | 201        |
| <b>10.</b> | <b>Premixed Turbulent Flame Propagation</b>                    | <b>203</b> |
| 10.1       | Introduction   | 204        |
| 10.1.1     | Premixed Laminar Flame   | 205        |
| 10.1.2     | Premixed Turbulent Flame                                       | 206        |
| 10.2       | Relative Scales of Flow and Combustion                         | 209        |
| 10.3       | Categorization of Premixed Turbulent Flame Regimes             | 211        |
| 10.4       | Turbulent Length Scale and the Flame Surface Area              | 213        |
| 10.4.1     | A Saturated Wrinkled Flame Front                               | 214        |
| 10.4.2     | An Unsaturated Wrinkled Flame Front                            | 215        |
| 10.4.3     | Comments on Turbulent Length Scale in Combustion               | 216        |
| 10.5       | Turbulent Flame Acceleration and the Driving Mechanisms        | 216        |
| 10.5.1     | Progressive Flame-Turbulence Interaction (Evolution Mechanism) | 217        |
| 10.5.2     | Relative Flame/Eddy Size Development                           | 217        |
| 10.5.3     | Volume Expansion Effect (Expanding-Pushing Mechanism)          | 219        |
| 10.5.4     | Darrieus-Landau Instability                                    | 220        |
| 10.5.5     | Attenuation of Flame Front Wrinkling                           | 220        |
| 10.5.6     | Some Progressive Turbulent Flame Growth Evidence               | 221        |
|            | Problems   | 223        |
|            | References   | 224        |
|            | <b>Subject Index</b>   | <b>227</b> |