Studies in Systems, Decision and Control 34

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(时滞复杂神经网络的定性分析与控制)

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Science Press, Beijing
ISBN: 978-7-03-045218-4

ISSN 2198-4182 ISSN 2198-4190 (electronic)
Studies in Systems, Decision and Control
ISBN 978-3-662-47483-9 ISBN 978-3-662-47484-6 (eBook)
DOI 10.1007/978-3-662-47484-6

Springer Heidelberg New York Dordrecht London

Studies in Systems, Decision and Control

Volume 34

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Preface

Background of This Book With the development of neural networks theory, many neural network models and stability concepts have been extended and upgraded. For example, it is well known that recurrent neural networks (RNNs) can be used to realize associate memory and information storage. The fundamental explanation of the statement is based on the fact that artificial neural networks (ANNs) are from the biological neural networks (BNNs). In fact, this explanation seems too far-fetched. Although ANNs models are the reduction of the real biological neural network models in function, this does not mean that ANNs have the same characteristics as BNNs in nature. In other words, ANNs are an implementation of partial functions of BNNs in engineering applications. Therefore, based on the existing research results of dynamical systems, we can regard RNNs as special dynamical systems that have the fading memory function. In this way, there will be an excellent explanation that RNNs are dynamical systems with memory and storage function. Moreover, with the assembly of many RNNs, the traditional neural network models are coupled together and a new name called complex neural networks (CNNs) emerges, which can be regarded as an upgraded version of RNNs. CNNs have more complex dynamics than RNNs due to the different coupling strengths and topology structures. For such kind of CNNs, under some restrictions on the coupling matrices and topology structures, synchronization problem now being hot research, is an upgraded version of the classical stability conception.

As one of the most important qualitative characteristics of a dynamical system, stability problem is a long-term continuous research topic in many fields such as mechanics, mathematics, control theory and neural networks. Stability characteristic or stability concept may have different meanings due to different application problems, such as stability of fixed point, structure stability, stability in the sense of Lyapunov, input to output stability, input to state stability and invariant sets. Every stability definition conforms to the requirements of the engineering practice or reality. Meanwhile, different requirements of the engineering practice or reality may produce or promote the development of stability theory, in which some different branches of stability theory may come into the world. Every theory research has its own practical background, and the real-life world provides the opportunities of emergence and establishment of new theory. From this point of view, almost all the hot academic topics in the present are in accordance with the current requirement of the economic development and science and technology innovation. That is, the hot academic topic is to solve the present problems encountered in reality.

Along this line, this book aims to investigate some stability problems for recurrent neural networks with delays, in which the main purpose of the research is to reduce the conservativeness of the stability criteria. Specifically, the book is mainly focused on the qualitative stability analysis and synthesis of continuous-time real-valued recurrent neural networks with delays (a special case of additive neural network models). The discussed stability concept is in the sense of Lyapunov, and naturally the proof method is based on Lyapunov stability theory. For a fixed/specified activation function and external constant input, the concerned stability problems fall into the fields of Lyapunov stability. If the activation functions belong to a special class of functions, the concerned stability problems, strictly speaking, fall into the scope of absolute stability. In this sense, most of the existing stability results for RNNs are absolutely stable for any activation function belonging to a specified function class. Therefore, different stability definitions can lead to different understandings on the dynamics of RNNs. Meanwhile, some other qualitative characteristics of RNNs such as passivity, dissipativity, invariant set and synchronization are also discussed. Based on the qualitative analysis, two kinds of control schemes are designed to realize the stabilization and controlled synchronization for the concerned complex dynamical networks.

Why to Write This Book? The first author Zhanshan Wang began formally to contact stability theory of RNNs at the end of 2002. It was an opportunity for him to join the teams to translate the excellent monograph "A.N. Michel and D. Liu, *Qualitative Analysis and Synthesis of Recurrent Neural Networks*, New York: Marcel Dekker, 2002" into a Chinese book. The Chinese version was published by Science Press of China in 2004. It was in early 2005 that Zhanshan Wang began to dedicate his life to the stability research of RNNs. Zhenwei Liu in 2007 and Chengde Zheng in 2009 began to engage in the stability research of RNNs under the direction of Prof. Huaguang Zhang, who is a professor at Northeastern University of China, under the assistance of Zhanshan Wang. Through 10-years' research on the stability theory of RNNs, we have cooperated on many academic treatises and achieved some valuable results, and especially improved a lot in the aspects of understanding the meaning of stability theory. Therefore, it is necessary to write a book to introduce our results.

Although a number of monographs on stability and neural networks have appeared, this book has its untouchable features which distinguishes it from others.

First, the historical and logical development of stability theory of RNNs involved in this monograph is rather complete. From the point of view of system, readers can find not only the origin of stability theory of RNNs and some well-known models of RNNs, additive neural networks, Hopfield neural networks and Cohen-Grossberg neural networks, but also some new insights into the relations among these models of RNNs and the different evolutions of stability analysis. From the point of view of practicability, one can find many relations among different stability definitions, stability theory and stability analysis methods.

Second, since the monograph is a summary of the study results of the authors, the methods proposed here for stability analysis, stabilization and synchronization to a great degree benefit from the theory of nonlinear control systems and dynamical systems, and are more advanced than those appearing in other introductory books. We only mention a few of them as examples: To present a detailed review of the stability research of Cohen-Grossberg-type RNNs (i.e., a kind of additive RNNs), at least 17 aspects of RNNs have been introduced, which will be helpful for readers to further investigate. How to study the effects of delay on the stability of RNNs, weighting-delay method and secondary delay partitioning are proposed, which forms the novel delay partitioning method. To demonstrate an evolution of stability method, stability results have been studied for RNNs from fixed point to invariant set, from global stability to local stability, from absolute stability to relative stability, and from self-stability to controlled-stability. Some insightful comments are presented for different kinds of stability definitions. Because there are many excellent papers scattered in books, conferences and Internet, we have collected a lot of classical and excellent books and papers for further reading on the stability research of RNNs in a systematic manner in this monograph, which may show their great roles in the development of stability theory of RNNs.

Last but not the least, some rather unique contributions are included in this monograph. For example, the relations and meanings between Hopfield neural networks and Cohen-Grossberg neural networks are discussed; the reasons why Lyapunov stability theory is significantly popular in the scientific community are presented; some comparisons among absolute stability, complete stability and global stability are provided. These statements are first discussed by the authors and their merits both in neural network system theory and stability theory will be interesting.

The Audiences of This Book The book is suitable for a formal graduate course in stability theory of neural networks or dynamical systems, or for self-study by researchers and practitioners with an interest in system theory in the following areas: all engineering disciplines, stability theory, control engineering, dynamical system, computer science and applied mathematics. It is assumed that the reader of this book has some background in neural networks, ordinary differential equations, matrix theory and automatic control theory.

The Content of This Book This book is divided into 12 chapters. Chapter 1 provides the background knowledge on the origin of artificial neural networks, especially the relations among the associate memory networks, Hopfield neural networks and Cohen-Grossberg neural networks. Furthermore, as some dynamical systems have information processing capability, it is reasonable to understand that, as a special case of dynamical systems, RNNs have some computation and storage ability. For dynamical neural networks, one of the fundamental qualitative properties is stability, which is not only related to the external structure of the networks, but also related to the signal transmission delay. In this case, some summaries about the delay effects on the stability of neural networks and the linear matrix inequality (LMI)-based analysis method are provided, which are the strategic insights of the authors' research in the past 10 years.

Chapter 2 reviews the history of dynamical systems and stability theory. Different kinds of definitions of dynamical systems are compared. The well-known Lyapunov stability theory is revisited, and the general stability theory is also introduced. Meanwhile, the applications of dynamical system theory are simply summarized. Finally, some comments on the evolution of different stability definition are provided, which will help readers to understand the stability definition in a different sense. This chapter mainly presents the preliminaries of dynamical systems and the corresponding stability theory. Looking through these preliminaries, one can find the evolutionary trajectory of the research on the dynamical system and the stability theory, from which one can find some meaningful inspiration and excite someone to further extend the cognitive ability on the stability concept.

The literature on the stability research of recurrent neural networks is presently scattered throughout journals and conference proceedings. Consequently, to become reasonably proficient in the stability analysis of recurrent neural networks may require considerable investment of time. This book aims to fill this void. To accomplish this, Chap. 3 presents a detailed review of the development of stability of Cohen-Grossberg type neural networks (a special kind of RNNs). The contents include the research directions of stability of RNNs, stability analysis for Cohen-Grossberg type RNNs, and the sufficient and necessary stability conditions of RNNs. In each section, there are many insightful comments on the concerned problems by the authors.

Chapters 4 and 5 present two kinds of delay-dependent stability results for RNNs with time-varying delay on the basis of delay partitioning method. The main method in Chap. 4 is, in the case of fixed interval terminal of time-delay, to insert many virtual points in this interval, and by optimizing these dynamical subintervals partitioned by the virtual points, some novel delay-dependent stability criteria are established. In contrast, the main method in Chap. 5 is, in the case of flexible interval terminal of time delay, to adjust the variable terminal parameter to change the length of the subinterval, and by constructing some novel Lyapunov functions with variable upper and lower integral term, some delay-dependent stability criteria are established. These two methods of partitioning the delay interval are different, which are based on different insight into the flexible change in delay interval.

In Chap. 6, delay-dependent exponential stability criteria for delayed static neural networks (a special kind of RNNs) are established on the basis of LMI method. Static neural networks are often used in the optimization problem. Therefore, the established stability result plays an important role in judging the convergence of designed optimization neural networks.

In Chap. 7, local stability or multiple stability criteria are established for a class of RNNs with discontinuous activation functions and time-varying delay, in which multiple equilibrium points exist. Such RNNs are usually used in the associative memory and pattern recognition. The present local stability result has large basin of attraction of the fixed point, and this feature can keep the stable memory of RNNs longer.

Some stability analysis methods of RNNs have been extended to the more general qualitative cases of RNNs, such as passivity, dissipativity and invariant sets, and synchronization, which have formed Chaps. 8, 9 and 10, respectively. These results will provide profound insight into the dynamics of RNNs with delays.

Based on the above qualitative analysis results, controller design problems are considered for the stabilization and controlled synchronization of RNNs, which form Chaps. 11 and 12, respectively. From these two chapters, one can see that stability analysis is the fundamental of the controller synthesis, and makes the controller design more convenient.

The fundamental knowledge of artificial neural networks in Chap. 1 is mainly cited from the classical textbook (Rojas, Springer-Verlag, 1996). The background materials in Chap. 2 are mainly from Wikipedia–the free encyclopedia on the Internet. However, some remarkable comments on the evolution of Hopfield model and Cohen-Grossberg model, and stability conception in Chaps. 1 and 2 are presented by the authors, which make the contents of this book more systematic and complete. Without the background materials on the history of artificial neural networks and dynamical systems, many insightful comments cannot exhibit their powerful effectiveness. The materials from Chaps. 3 to 12 are from the combined research results of Zhanshan Wang, Zhenwei Liu and Chengde Zheng, respectively.

Acknowledgments The authors would like to acknowledge all of the help and encouragement received in the development of this book. Many our graduate students and colleagues have contributed to the materials of the book. They are Y. Huang, Q. Shan, S. Ding, H. Tian, Z. Huang, B. Huang, J. Wang, Z. Shen, J. Wang, H. Liang, C. Cai and L. Liu. Some of them are the coauthors of some cited papers, which form several chapters of this book. Some of them helped to collect the materials. Moreover, there are many books and academic papers cited and referred, therefore, the authors of the book would like to thank so many excellent scholars and experts, R. Rojas, J. Hale, X. Liao, T. Chen, J. Park, J. Jian, D. Xu, A. Michel, M. Forti, J. Wang, D. Liu, H. Zhang, S. Grossberg, G. Chen, L. Pecora, S. Strogatz, L. Chua and so on for their excellent books and papers on the theory of dynamical systems, neural networks and stability. The entire contents of the book were mainly compiled by Zhanshan Wang. Finally, Zhanshan Wang, Zhenwei Liu and Chengde Zheng have checked the whole book carefully.

We are grateful to the National Natural Science Foundation of China (Grant Nos. 61473070, 61433004, 61273022, 61074073, 61203046), the Fundamental Research Funds for the Central Universities of China (Grant Nos. N130504002, N110504001, N140406001), and SAPI Fundamental Research Funds of China (Grant No. 2013ZCX01), Program for New Century Excellent Talents in University of China under Grant NCET-10-0306, which gave the best supports for the writing of this book.

The authors are indebted to Haina Zhang, China Science Press's Editor, for the consideration, support, and professionalism that she rendered during the preparation and production of this book. The authors would also like to thank their families for their understanding during the writing of this book.

Shenyang
April 2015

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Contents

Chapter 1
Introduction to Neural Networks

Analog circuits have played a very important role in the development of modern electronic technology. Even in our digital computer era, analog circuits still dominate such fields as communications, power, automatic control, audio, and video electronics because of their real-time signal processing capabilities. Conventional digital computation methods have run into a serious speed bottleneck due to their serial nature. To overcome this problem, a new computation model as an alternative, called "neural networks" has been proposed, which is based on some aspects of neurobiology and adapted to integrated circuits. The key features of neural networks are asynchronous parallel processing, continuous-time dynamics, and global interaction of network elements. Some encouraging if not impressive applications of neural networks have been proposed for various fields such as optimization, linear and nonlinear programming, associative memory, pattern recognition, and computer vision. Since 1943, when Warren McCulloch and Walter Pitts [1] presented the first model of artificial neurons, new and more sophisticated proposals have been made from decade to decade. Mathematical analysis has solved some of the mysteries posed by the new models but has left many questions open for future investigations. Needless to say, the study of neurons, their interconnections, and their role as the brain's elementary building blocks is one of the most dynamic and important research fields in modern biology. It is not an exaggeration to say that researchers have learned more about the nervous system in the past 70 years than ever before. In this chapter we will deal with artificial neural networks, and therefore the first question to be clarified is their relation to the biological paradigm. What do we abstract from real neurons for our models? What is the link between neurons and artificial computing units? In the following we will present a preliminary answer to these important questions, in which some contents are mainly cited from the book [2].

Z. Wang et al., *Qualitative Analysis and Control of Complex Neural Networks with Delays*, Studies in Systems, Decision and Control 34,
DOI 10.1007/978-3-662-47484-6_1

1.1 Natural and Artificial Neural Networks

Artificial neural networks are an attempt at modeling the information processing capabilities of nervous systems. Thus, one needs to consider the essential properties of *biological neural networks* from the viewpoint of information processing. This will allow us to design abstract models of artificial neural networks, which can then be simulated and analyzed. Although the models were proposed to explain the structure of the brain and the nervous systems of some animals are different in many respects, there is a general consensus that the essence of the operation of neural ensembles is "control through communication" [3]. Animal nervous systems are composed of thousands or millions of interconnected cells. Each of them is a very complex arrangement that deals with incoming signals in many different ways. However, neurons are rather slower when compared to electronic logic gates. These can achieve switching times of a few nanoseconds, whereas neurons need several milliseconds to react to a stimulus. Nevertheless, the brain is capable of solving problems that no digital computer can yet efficiently deal with.

Massive and hierarchical networking of the brain seems to be the fundamental precondition for the emergence of consciousness and complex behavior [4]. So far, however, biologists and neurologists have concentrated their research on uncovering the properties of individual neurons. Today, mechanisms for the production and transport of signals from one neuron to the other are well-understood physiological phenomena, but how these individual systems cooperate to form complex and massively parallel systems capable of incredible information processing feats has not yet been completely elucidated. Mathematics, physics, and computer science can provide invaluable help in the study of these complex systems. It is not surprising that the study of the brain has become one of the most interdisciplinary areas of scientific research in recent years. However, we should be careful with the metaphors and paradigms commonly introduced when dealing with the nervous system. It seems to be a constant in the history of science that the brain has always been compared to the most complicated contemporary artifact produced by human industry [5]. In ancient times the brain was compared to a pneumatic machine, in the Renaissance to a clockwork, and to the telephone network. There are some today, who consider computers the paradigm par excellence of a nervous system. It is rather paradoxical that when John von Neumann wrote his classical description of future universal computers, he tried to choose terms that would describe computers in terms of brains, not brains in terms of computers. The nervous system of an animal is an information processing totality. The sensory inputs, i.e., signals from the environment, are coded and processed to evoke the appropriate response. Biological neural networks are just one of many possible solutions to the problem of processing information. The main difference between neural networks and conventional computer systems is the massive parallelism and redundancy they exploit in order to deal with the unreliability of the individual computing units. Moreover, biological neural networks are self-organizing systems and each individual neuron is also a delicate self-organizing structure capable of processing information in many different ways.