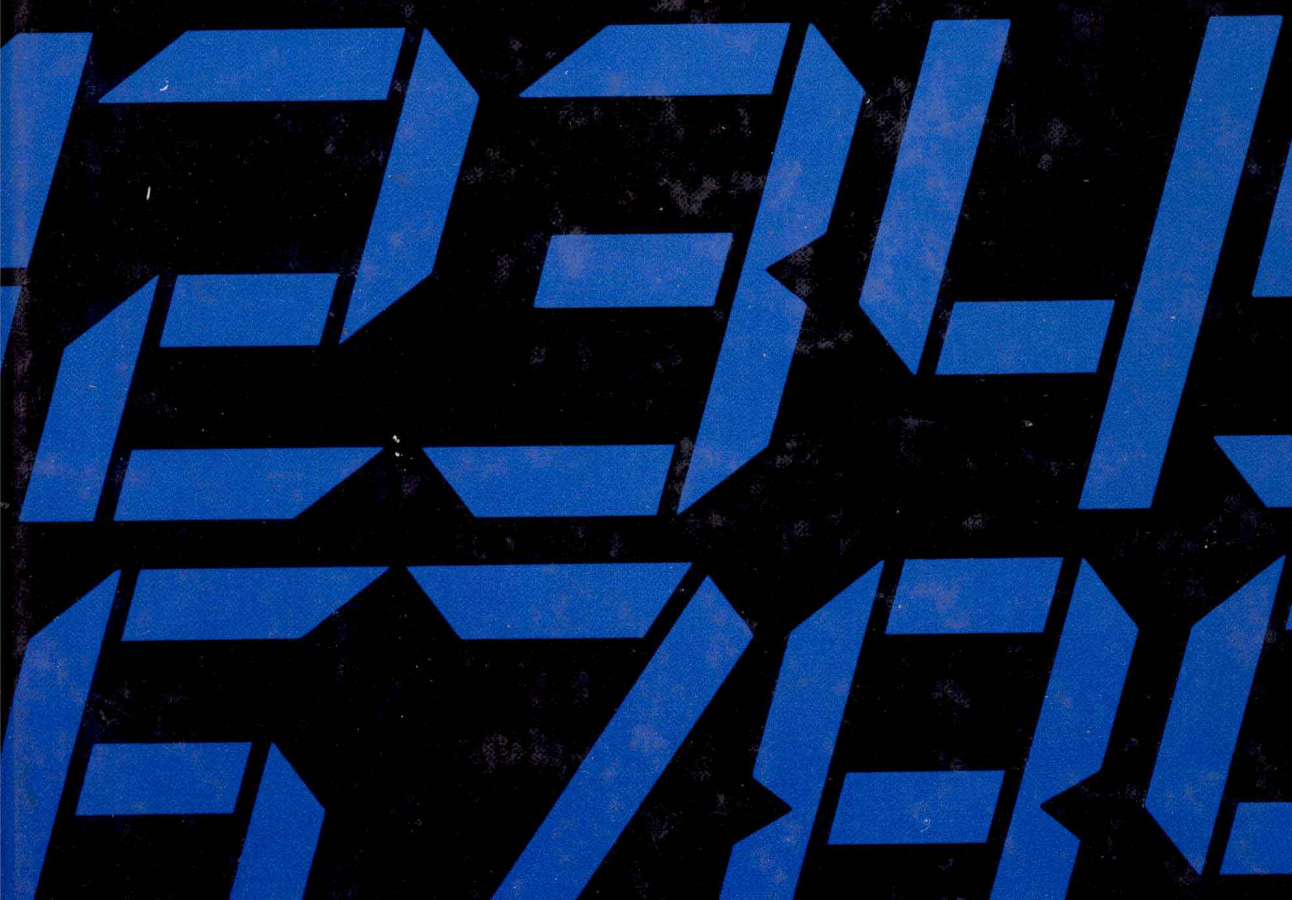
Gilbert Gordon
Israel Pressman

QUANTITATIVE DECISION MAKING FOR BUSINESS

SECOND EDITION



Quantitative Decision Making for Business

SECOND EDITION

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Preface

In the preface to the first edition of this book we noted that our intent was to "provide a clear readable presentation of the essential topics of quantitative decision making." That intent is still a prime objective of this second edition, and all additions have been designed with that goal in mind.

In response to the many readers and reviewers of the first edition and in order that the book satisfy the growing requirements of courses in quantitative decision making, two major additions have been included in the second edition. The first consists of two new chapters, one on dynamic programming and the other on Markov processes. The second is the addition of a case study problem to eight of the chapters. These cases are designed for either classroom discussion or as homework assignments. The feature of these cases is that they require the student to think about what questions are being asked and what solutions are required. Several other changes have been included. The chapter on competitive situations has been dropped and the relevant material incorporated into the chapter on simple decision problems. New material on break-even analysis, order level inventory models, and project "crashing" have also been added. Finally, we have added new problems to many chapters.

A major innovation of the first edition (i.e., the use of the computer in making decisions) has been continued in the second edition. Instructors who adopt this text can obtain from the publisher a computer tape containing programs that allow students to solve problems with practical application to real life.

The second edition is still aimed at the student with a low-to-moderate level of mathematical sophistication, and calculus is still not a prerequisitive for the use of this book.

Many people deserve our special thanks for their contributions to this edition. Once again we thank our colleagues in the Statistics and Computer Information Systems Department of Baruch College, C.U.N.Y., for

their comments and recommendations, which were based on their experience in having used the first edition for several years. We thank the many reviewers whose comments and suggestions have proven invaluable.

We thank our able typist Ruth Meyer for the many hours of typing the final manuscript. We wish to thank as well Muriel Adams of the Prentice-Hall staff for her meticulous check of the manuscript and book proofs.

Finally, as with the first edition, we again thank our wives and families for their encouragement, patience, and sacrifice. Their support was essential to the completion of this revision project.

Gilbert R. Gordon
Israel Pressman

Contents

Chapter 1

Introduction: Decisions and Models

The purpose of this chapter is to introduce you to the following concepts which are basic to obtaining a clear understanding of quantitative decision-making.

OBJECTIVES

1. The framework for decision problems.
2. The form of a mathematical model.
3. The fundamental techniques of model building.
4. Model solution and interpretation.
5. The basic elements of sensitivity analysis.

Complex organizations generate complex decision problems. Today's society is composed of a variety of such organizations—those in business, government, education, and health care, for example. They must frequently make decisions which involve a complex array of factors and which will, in turn, affect many other factors. A common example is a company's decision to introduce a new product, which will be influenced by such considerations as market conditions, labor rates and availability, and investment requirements and availability of funds. The decision will be a multidimensional response, including the production methodology, cost and quality of the product, package design, price, and marketing and advertising strategy. The results of the decision could conceivably affect every phase of the organization. Clearly, this is a complex problem.

As organizations and their decision problems become more complicated and involved, it has become increasingly more difficult for administrators (decision makers) to handle all the factors affecting the decision and to determine the results to be expected from all possible decision alternatives. What is needed is a more scientific method of investigating and analyzing these problems and thus arriving at "better" decisions.

The earliest evidence of interest in this area is the growth of the accounting discipline, which developed systematic procedures for "accounting for" the monetary operations of an organization in order that administrators could develop a clearer picture of this important business behavior. In fact, we can consider accounting statements as a "model" of the organization, a term that will be more fully discussed later in the chapter.

The logical next step after providing information for decisions is to develop decision-making aids which facilitate the investigation of these problems. The birth of this discipline, bearing such names as "quantitative decision making," "operations research," and "management science," occurred during World War II, when the general crisis situation aroused strong interest in methods of making better decisions, both military and economic. The success of the procedures developed during that period encouraged other organizations, particularly businesses and social services, to adopt these approaches to their decision problems. Since that time, the field has grown dramatically and is being applied by a wide variety of organizations to solve a myriad of decision problems.

1.2.1. Introduction. We shall develop a generalized framework for decision problems which will be useful in investigating all types of decision problems. This will facilitate the use of the techniques to be discussed later.

1.2.2. Objective of the Decision. All decision problems must have an objective—or else no problem would exist. One of the most difficult aspects of a problem is usually the choice of the specific objective to be used. For example, in the case of the introduction of a new product, some reasonable objectives might be:

1. Make the greatest profit next year.
2. Make the greatest profit over the next 10 years.
3. Make the greatest return on investment over the next 10 years.
4. Increase the net worth of the company as much as possible at the end of 10 years.
5. Obtain the greatest market share possible with a profit not lower than $10,000 per year.
6. Achieve the highest possible sales given a profit not lower than $10,000 per year.

Naturally, these are only a few of the almost infinite possibilities that could be used. Each different objective will strongly affect the amount of satisfaction to be realized by making a given decision. One decision may yield very satisfactory results when evaluated by one objective and very poor results when evaluated by another. The problem becomes even more complex when some of the factors affecting the decision are variable (i.e., are not known with certainty). For example, for a new product introduction, which includes decisions about price, quality, package, and so on, demand cannot be known with certainty. A forecast could be made, but it would be an estimate at best and thus would be subject to error.

Another aspect which complicates matters is that within an organization there are usually several suborganizations (production, marketing, warehousing, advertising), each with its own objectives, which may be (and usually are) in conflict with other suborganizations' objectives. Salespersons may want to sell any product that any customer will buy, whereas production may wish to manufacture only a few large-volume products. Production may prefer large runs of a single product, which would create large inventories troublesome to the warehousing department. The objective or goal of the decision problem should be one that considers the overall or composite objectives of the total organization.

1.2.3. Decision Activities. The action of making a decision consists of making a choice from among a number of activities, each of which is

under the control of the decision maker. These will be referred to as *decision activities* or *controllable activities*. The terms "decision variable" and "controllable variable" will also be used since, when we discuss models, variables (such as x and y) will be used to represent these as-yet-unknown quantities. For example, in the new-product-introduction problem, some possible decision activities might be:

1. The price to be charged.
2. The package design.
3. The marketing–distribution policy to be used (e.g., direct, toward jobbers and wholesalers).
4. The number of machines to purchase for producing the product.
5. The amount of ingredient x as a percentage of the total mix of ingredients (e.g., cashews in a nut mix, gold in jewelry).

In each case the decision maker has the ability to "do" what he decides, and therefore these activities are considered controllable, that is, are subject to the human decision process of choice.

Decision variables, and variables in general, may be divided into two types: discrete and continuous. *Discrete variables* are those that can only take on one of a finite (or discrete) set of possibilities. Each of these possibilities is commonly referred to as a *strategy*.

For example, for the decision activity concerning the package design, we might have several strategies:

STRATEGY 1: White with large gold letters.

STRATEGY 2: White with small gold letters.

STRATEGY 3: White with large black letters.

⋮ ⋮

The decision activity for the number of machines to purchase might be:

STRATEGY 1: Buy one machine.

STRATEGY 2: Buy two machines.

STRATEGY 3: Buy three machines.

⋮ ⋮

Continuous variables, on the other hand, are those for which one of an infinite number of possible values must be chosen. For example, the price to be charged could be \$100.00, \$120.00, \$127.00, \$127.50, \$127.53,....

Making a decision, therefore, involves selecting a strategy for each discrete decision variable and selecting a value for each continuous decision variable.

1.2.4. Uncontrollable Activities. Decision problems frequently contain activities that are not under the control of the decision maker. These will be called *uncontrollable activities* or *uncontrollable variables*. For example, in the new-product-introduction problem, the uncontrollable activities might be the raw-material costs, economic conditions in the future, the number of other companies who will also market a similar product, or other factors of this type. These will be determined by "forces" not completely under the control of the decision maker. Uncontrollable variables can also be either discrete (number of other companies marketing a similar product) or continuous (the cost of raw materials).

We shall consider uncontrollable activities of two types:

1. *Known value*. These are usually referred to as the *parameters* of the problem. If we know that our raw-material cost is \$0.50 per pound, we may simply use the value 0.50 in considering the problem.
2. *Unknown value*. These values are not known with certainty.

The manner in which a value is selected for an uncontrollable variable with unknown value is an important part of a decision problem. The two extreme cases are called state-of-nature and competitive-strategy types of variables.

State-of-nature variables are those which are determined by a process that does not consider the decision maker at all. For example, the general economic conditions which will exist in the future will not be determined (most likely) by a process that concerns itself with the interests of the decision maker. Rather, there are an almost infinite number of other factors which will determine this outcome. The same is true for a farmer who is deciding which crops to plant. His decision problem may have as an uncontrollable variable, the weather. It is safe to assume that the rainfall during the growing season will be determined by a process (nature) that will "make its decision" without regard to the likes or dislikes of the farmer.

Competitive strategies, on the other hand, exist where an individual's decision will be influenced by a second decision maker, who is in competition with the first decision maker. The competitor's decision will be made so as to improve his own satisfaction, which, owing to the competition, will indirectly act so as to reduce the satisfaction of the first decision maker. The price that is charged by one's competitor for a similar product is an example of a competitive-strategy type of uncontrollable variable.

1.2.5. Partially Controllable Activities. In some cases an activity is neither completely controllable or uncontrollable, and these will be called *partially controllable activities* or *partially controllable variables*. In these cases some of the decision activities of the decision maker affect the