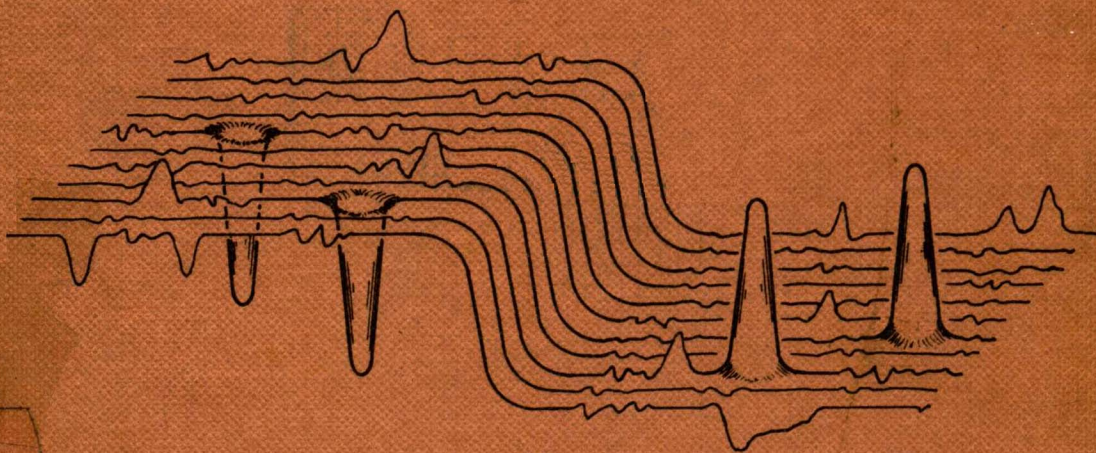


James Glimm
Arthur Jaffe

Quantum Physics

A Functional Integral
Point of View



Springer-Verlag
New York Heidelberg Berlin

James Glimm
Arthur Jaffe

Quantum Physics

A Functional Integral Point of View

With 43 Illustrations



Springer-Verlag
New York Heidelberg Berlin

James Glimm
The Rockefeller University
New York, NY 10021
USA

Arthur Jaffe
Harvard University
Cambridge, MA 02138
USA

Library of Congress Cataloging in Publication Data

Glimm, James,
Quantum physics.

Bibliography: p.
Includes index.

1. Quantum field theory. 2. Quantum theory.
3. Statistical physics. I. Jaffe, Arthur, joint
author. II. Title.

QC174.45.J33 530.1'43 81-17

© 1981 by Springer-Verlag New York Inc.
All rights reserved.

No part of this book may be translated or reproduced in any form without written
permission from Springer-Verlag, 175 Fifth Ave., New York, NY 10010, USA.

Printed in the United States of America.

9 8 7 6 5 4 3 2 1

ISBN 0-387-90551-0 Springer-Verlag New York Heidelberg Berlin (hard cover)
ISBN 3-540-90551-0 Springer-Verlag Berlin Heidelberg New York (hard cover)
ISBN 0-387-90562-6 Springer-Verlag New York Heidelberg Berlin (soft cover)
ISBN 3-540-90562-6 Springer-Verlag New York Heidelberg Berlin (soft cover)

Introduction

This book is addressed to one problem and to three audiences.

The problem is the mathematical structure of modern physics: statistical physics, quantum mechanics, and quantum fields. The unity of mathematical structure for problems of diverse origin in physics should be no surprise. For classical physics it is provided, for example, by a common mathematical formalism based on the wave equation and Laplace's equation. The unity transcends mathematical structure and encompasses basic phenomena as well. Thus particle physicists, nuclear physicists, and condensed matter physicists have considered similar scientific problems from complementary points of view.

The mathematical structure presented here can be described in various terms: partial differential equations in an infinite number of independent variables, linear operators on infinite dimensional spaces, or probability theory and analysis over function spaces. This mathematical structure of quantization is a generalization of the theory of partial differential equations, very much as the latter generalizes the theory of ordinary differential equations. Our central theme is the quantization of a nonlinear partial differential equation and the physics of systems with an infinite number of degrees of freedom.

Mathematicians, theoretical physicists, and specialists in mathematical physics are the three audiences to which the book is addressed.

Each of the three parts is written with a different scientific perspective. Part I is an introduction to modern physics. It is designed to make the treatment of physics self-contained for a mathematical audience; it covers

quantum theory, statistical mechanics, and quantum fields. Since it is addressed primarily to mathematicians, it emphasizes conceptual structure—the definition and formulation of the problem and the meaning of the answer—rather than techniques of solution. Because the emphasis differs from that of conventional physics texts, physics students may find this part a useful supplement to their normal texts. In particular, the development of quantum mechanics through the Feynman–Kac formula and the use of function space integration may appeal to physicists who want an introduction to these methods.

Part II presents quantum fields. Boson fields with polynomial self-interaction in two space-time dimensions— $P(\phi)_2$ fields—are constructed. This treatment is mathematically complete and self-contained, assuming some knowledge of Hilbert space operators and of function space integrals. The original construction of the authors has been replaced by successive improvements and simplifications accumulated for more than a decade. This development is due to the efforts of a small and dedicated group of some thirty constructive field theorists including Fröhlich, Guerra, Nelson, Osterwalder, Rosen, Schrader, Simon, Spencer, and Symanzik, as well as the authors. Physicists may find Part II useful as a supplement to a conventional quantum field text, since the mathematical structure (normally omitted from such texts) is developed here.

Part II contains the resolution of a scientific controversy. For years physicists and mathematicians questioned whether nonlinear field theory is compatible with relativistic quantum mechanics. Could quantization defined by renormalized perturbation theory be implemented mathematically? The mathematically complete construction of $P(\phi)_2$ fields presented here and the construction of Yukawa_{2,3}, ϕ_3^4 , sine-Gordon₂, Higgs₂, etc., fields in the literature provide the proof. Central among the issues resolved by this work is the meaning of renormalization outside perturbation theory. The mathematical framework for this analysis includes the theory of renormalization of function space integrals. From the viewpoint of mathematics the implementation of these ideas has involved essentially the creation of a new branch of mathematics.

Whether the equations are mathematically consistent in four space-time dimensions has not been resolved. There is speculation, for example, that the equations for coupled photons and electrons (in isolation from other particles) may be inconsistent, but that the inclusion of coupling to the quark field may give a consistent set of equations. A proper discussion of this issue is beyond the scope of this book, but is alluded to in Chapters 6 and 17.

Particle interaction, scattering, bound states, phase transitions, and critical point theory form the subject of Part III. Here we develop the consequences of the Part II existence theory and make contact with issues of broad concern to physics. This part of the book is written at a more advanced level, and is addressed mainly to theoretical and mathematical physicists. It is neither self-contained nor complete, but is intended to

develop central ideas, explain main results of a mathematical nature, and provide an introduction to the literature.

Condensed matter physicists may find interesting the discussion of phase transitions and critical phenomena. The central matters are series expansions and correlation bounds. These methods find application in diverse areas. We give detailed justification of the connection (by analytic continuation) between quantum fields and classical statistical mechanics. Professional physicists could well start directly in Part III, returning to earlier material only as necessary.

Readers interested in the historical development of constructive quantum field theory are referred to the various survey articles of the authors and others. In this book the specific, detailed references are minimized, especially in the self-contained Parts I and II. A large bibliography has been included; we apologize for the inevitable omissions.

Numerous colleagues, students, and friends helped make this book possible. Of particular importance were R. D'Arcangelo, R. Brandenberger, B. Drauschke, J.-P. Eckmann, J. Gonzalez, W. Minty, K. Peterson, P. Petti, the staff at Springer Verlag, and especially our wives Adele and Nora. We are also grateful to the ETH, the IHES, the University of Marseilles and the CEN Saclay for hospitality as well as to the Guggenheim Foundation and the NSF for support.

Conventions and Formulas

Fourier transforms

$$f(x) = (2\pi)^{-d/2} \int e^{ipx} f^\sim(p) dp,$$

$$f^\sim(p) = (2\pi)^{-d/2} \int e^{-ipx} f(x) dx,$$

$$f(\theta) = (2\pi)^{-d/2} \sum e^{in\theta} f^\sim(n),$$

$$f^\sim(n) = (2\pi)^{-d/2} \int_0^{2\pi} e^{-in\theta} f(\theta) d\theta.$$

Minkowski vectors

$$x = (x_0, \mathbf{x}) = (x_0, \dots, x_{d-1}),$$

$$x^2 = x \cdot x = -x_0^2 + \mathbf{x}^2, \quad p^2 = p \cdot p = -p_0^2 + \mathbf{p}^2,$$

$$x \cdot p = \sum x_i p^i = -x_0 p_0 + \mathbf{x} \cdot \mathbf{p},$$

$$\square = -\partial_i^2 + \Delta = -\partial x_0^2 + \sum_{i=1}^{d-1} \partial x_i^2.$$

Euclidean vectors

$$x_d = ix_0,$$

$$x^2 = x \cdot x = \sum_{i=1}^d x_i^2,$$

$$\Delta = \sum_{i=1}^d \partial x_i^2.$$

Schrödinger's equation

$$\begin{aligned} \hbar &= h/2\pi, \\ i\hbar \dot{\theta} &= H\theta, \quad \theta(t) = e^{-itH/\hbar}\theta(0), \\ p &= -i\hbar \frac{\partial}{\partial q}, \quad [p(x), q(y)] = -i\hbar \delta(x - y) \end{aligned}$$

Covariance operators $C_m \in \mathcal{C}_m$ satisfy

$$(-\Delta + m^2)C_m = \delta.$$

σ and γ matrices

$$\begin{aligned} \sigma_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ \gamma_i &= \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, & i &= 1, 2, 3, \\ \gamma_0 &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, & \gamma_5 &= \gamma_0 \gamma_1 \gamma_2 \gamma_3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \\ \not{a} &= \sum a_\mu \gamma_\mu, \\ \not{a}^2 &= \sum a_\mu^2 = a^2. \end{aligned}$$

Dirac equation (zero field)

$$(h\not{\partial} - mc)\psi = 0.$$

Dirac equation in external field A

$$\left(h\not{\partial} + i \frac{e}{c} \not{A} - mc \right) \psi = 0.$$

List of Symbols

a, a^*, A, A^*	annihilation and creation operators
a, A	free energy
A	antisymmetrization operator
\mathcal{A}	action
$\mathfrak{A}, \mathcal{A}$	algebra of operators
b	bond
B	observable; region in space-time
\mathcal{B}	set of bonds
c	diagonal values of C , $c(x) = C(x, x)$; critical (as a subscript); constant
cr	critical
cl	classical
C	covariance; (Chap. 7) complex numbers
$\mathcal{C}, \mathcal{C}_m$	a class of covariance operators (Sec. 7.9)
d	dimension of space-time
D	Dirichlet boundary conditions
\mathcal{D}	domain of an operator; C_0^∞ test function space
\mathcal{D}'	Schwartz distribution space
$\mathcal{D}^{(j)}$	domain for irreducible spin j representation of $SU(2, C)$
E	energy level; eigenvalue for H ; Euclidean transformation; Euclidean group

\mathcal{E}	Euclidean group; Euclidean Hilbert space; time strip (Section 10.5)
f	test function; free energy
\mathcal{F}	Fock space
g	test function
\mathcal{G}	group
h, \hbar	Planck's constant
h	external field
H	Hamiltonian
HS	Hilbert–Schmidt
$\mathcal{H}(\mathbf{x})$	Hamiltonian density
\mathcal{H}	Hilbert space of quantum states
I	identity operator
J	interaction strength for Ising ferromagnet
j, \mathbf{J}	angular momentum
k	Boltzmann's constant
\mathcal{K}	kernel of semigroup
K	kernel of Bethe–Salpeter equation
L	angular momentum (Section 15.1)
L_s, L_i, L	lines in Feynman graphs (self-interacting, interacting)
\mathcal{L}	Lagrangian; lattice; multiple reflection norm (Section 10.5); Lorentz group
m, M	mass; magnetization; multiple reflection norm (Section 10.5)
n	number of field components; degree of polynomial P
nn	nearest neighbor
N	Neumann boundary condition; $N(f)$ = norm of f .
\mathcal{N}	null space for inner product
p	period boundary conditions; pressure; Lebesgue index; degree of polynomial P
p, P	momenta; momentum operator; momentum space
P	polynomial interaction; projection operator
P_n	Hermite polynomial
q, Q	configuration; configuration space; Lebesgue index
R	real numbers; multiple reflection norm (Section 10.5)
R^d	Euclidean d -space
s	time

s, S	entropy
S	generating function; Schwinger function; sphere; symmetrization operator
$ S^n $	volume of n -sphere
\mathcal{S}	Schwartz space of rapidly decreasing test functions
\mathcal{S}'	Schwartz space of tempered distributions
\mathfrak{S}_n	symmetric group on n elements (permutation group)
t	Euclidean time ($=x_d$); Minkowski time ($=x_0$)
T	time ordering; truncation
U, V	unitary operator on Hilbert space
V	potential
W	Wightman function
dW	Wiener measure
\mathcal{W}	Wiener path space
\mathcal{X}	phase space
x	point in space time
\mathbf{x}	point in space
z	fugacity; activity
Z	partition function; field strength renormalization constant; integers
Z_+	nonnegative integers; partition function
β	$(1/kT)$ inverse temperature
γ	critical exponent
γ, Γ	boundary; phase boundary
Γ	Dirichlet boundary conditions on Γ ; inverse to propagator or two point function
$ \Gamma $	length or area of Γ
δ	Dirac δ function; Kronecker δ function; lattice spacing; critical exponent
Δ	Laplacian; special solution of wave or Laplace equation (propagator) also unit square
ε	$2\theta - 1$ (a type of Heaviside function); lattice spacing; reduced temperature $(T - T_c)/T_c$
ζ, η	critical exponents
θ	reflection operator; Heaviside function; state in \mathcal{H}
κ	momentum cutoff
λ	coupling constant
Λ	bounded region of space

$ \Lambda $	area or volume of Λ
μ	$(-\Delta + m^2)^{1/2} = (p^2 + m^2)^{1/2}$; chemical potential; external field
$d\mu$	statistical weight or ensemble
ν	frequency; critical exponent
dv	statistical weight or ensemble
ξ	random variable
Ξ	partition function
π	3.14159; momentum conjugate to field ϕ
Π	projection operator; hyperplane
Π_{\pm}	half spaces of $R^d \setminus \Pi$
ρ	density
σ	mass ² ; Ising spin variable; time
Σ	proper self-energy
ϕ, Φ	quantum field; configuration of classical field
$d\Phi_C$	Gaussian measure, covariance C
χ	susceptibility; random variable; state in \mathcal{H} ; characteristic function
ψ	quantum field; state in \mathcal{H}
ω	frequency; Wiener path; angular integration variable
Ω	vacuum state; ground state; equilibrium state
∂	derivative; boundary operator
∇	gradient; divergence
$ \cdot $	absolute value; area, volume or number of \cdot ; norm
\wedge	projection operator from the Euclidean path space to the Hilbert space of quantum states
\sim	Fourier transform
$\langle \cdot, \cdot \rangle$	inner product
$\langle \cdot \rangle$	expectation; integral with respect to $d\mu$
$[\cdot, \cdot]$	commutator: $[a, b] = ab - ba$
$\{ \cdot, \cdot \}$	anticommutator: $\{a, b\} = ab + ba$
\emptyset	free boundary conditions; empty set
\times	vector product
\cdot	time derivative; position for missing variable as in $f(\cdot) = f$ for a function f .
\setminus	set theoretic difference: $A \setminus B = \{x: x \in A, x \notin B\}$
$-$	complex conjugation; closure

Contents

Introduction
Conventions and Formulas
List of Symbols

PART I An Introduction to Modern Physics

1	Quantum Theory	3
1.1	Overview	3
1.2	Classical Mechanics	3
1.3	Quantum Mechanics	7
1.4	Interpretation	11
1.5	The Simple Harmonic Oscillator	12
1.6	Coulomb Potentials	20
1.7	The Hydrogen Atom	24
1.8	The Need for Quantum Fields	27
2	Classical Statistical Physics	28
2.1	Introduction	28
2.2	The Classical Ensembles	30
2.3	The Ising Model and Lattice Fields	36
2.4	Series Expansion Methods	37

3	The Feynman–Kac Formula	42
3.1	Wiener Measure	42
3.2	The Feynman–Kac Formula	46
3.3	Uniqueness of the Ground State	49
3.4	The Renormalized Feynman–Kac Formula	51
4	Correlation Inequalities and the Lee–Yang Theorem	55
4.1	Griffiths Inequalities	55
4.2	The Infinite Volume Limit	58
4.3	ξ^4 Inequalities	59
4.4	The FKG Inequality	63
4.5	The Lee–Yang Theorem	64
4.6	Analyticity of the Free Energy	67
4.7	Two Component Spins	70
5	Phase Transitions and Critical Points	72
5.1	Pure and Mixed Phases	72
5.2	The Mean Field Picture	74
5.3	Symmetry Breaking	77
5.4	The Droplet Model and Peierls’ Argument	80
5.5	An Example	85
6	Field Theory	88
6.1	Axioms	88
	(i) Euclidean Axioms	88
	(ii) Minkowski Space Axioms	95
6.2	The Free Field	98
6.3	Fock Space and Wick Ordering	104
6.4	Canonical Quantization	109
6.5	Fermions	113
6.6	Interacting Fields	116

PART II Function Space Integrals

7	Covariance Operator = Green’s Function = Resolvent Kernel = Euclidean Propagator = Fundamental Solution	123
7.1	Introduction	123
7.2	The Free Covariance	125
7.3	Periodic Boundary Conditions	127
7.4	Neumann Boundary Conditions	128
7.5	Dirichlet Boundary Conditions	129

7.6	Change of Boundary Conditions	130
7.7	Covariance Operator Inequalities	130
7.8	More General Dirichlet Data	132
7.9	Regularity of C_B	137
7.10	Reflection Positivity	141
8	Quantization = Integration over Function Space	144
8.1	Introduction	144
8.2	Feynman Graphs	145
8.3	Wick Products	147
8.4	Formal Perturbation Theory	150
8.5	Estimates on Gaussian Integrals	152
8.6	Non-Gaussian Integrals, $d = 2$	157
8.7	Finite Dimensional Approximations	163
9	Calculus and Renormalization on Function Space	166
9.1	A Compilation of Useful Formulas	166
	(i) Wick Product Identities	166
	(ii) Gaussian Integrals	169
	(iii) Integration by Parts	171
	(iv) Limits of Measures	172
9.2	Infinitesimal Change of Covariance	174
9.3	Quadratic Perturbations	175
9.4	Perturbative Renormalization	179
9.5	Lattice Laplace and Covariance Operators	183
9.6	Lattice Approximation of $P(\phi)_2$ Measures	189
10	Estimates Independent of Dimension	193
10.1	Introduction	193
10.2	Correlation Inequalities for $P(\phi)_2$ Fields	193
10.3	Dirichlet or Neumann Monotonicity and Decoupling	195
10.4	Reflection Positivity	198
10.5	Multiple Reflections	200
10.6	Nonsymmetric Reflections	206
11	Fields Without Cutoffs	213
11.1	Introduction	213
11.2	Monotone Convergence	213
11.3	Upper Bounds	215
12	Regularity and Axioms	219
12.1	Introduction	219
12.2	Integration by Parts	220

12.3	Nonlocal ϕ^j Bounds	223
12.4	Uniformity in the Volume	225
12.5	Regularity of the $P(\phi)_2$ Field	229

PART III The Physics of Quantum Fields

13	Scattering Theory: Time-Dependent Methods	237
13.1	Introduction	237
13.2	Multiparticle Potential Scattering	240
13.3	The Wave Operator for Quantum Fields	244
13.4	Wave Packets for Free Particles	247
13.5	The Haag–Ruelle Theory	250
14	Scattering Theory: Time-Independent Methods	256
14.1	Time-Ordered Correlation Functions	256
14.2	The S Matrix	259
14.3	Renormalization	260
14.4	The Bethe–Salpeter Kernel	264
15	The Magnetic Moment of the Electron	270
15.1	Classical Magnetic Moments	270
15.2	The Fine Structure of the Hydrogen Atom and the Dirac Equation	272
15.3	The Dirac Theory	274
15.4	The Anomalous Moment	276
15.5	The Hyperfine Structure and the Lamb Shift of the Hydrogen Atom	279
16	Phase Transitions	280
16.1	Introduction	280
16.2	The Two Phase Region	284
16.3	Symmetry Unbroken, $d = 2$	294
16.4	Symmetry Broken, $3 \leq d$	297
17	The ϕ^4 Critical Point	304
17.1	Elementary Considerations	304
17.2	The Absence of Even Bound States	305
17.3	A Bound on the Coupling Constant λ_{phys}	307
17.4	Existence of Particles and a Bound on $dm^2/d\sigma$	309
17.5	Existence of the ϕ^4 Critical Point	310

17.6	Continuity of $d\mu$ at the Critical Point	313
17.7	Critical Exponents	314
17.8	$\eta \leq 1$	316
17.9	The Scaling Limit	318
17.10	The Conjecture $\Gamma^{(6)} \leq 0$	319
18	The Cluster Expansion	321
18.1	Introduction	321
18.2	The Cluster Expansion	324
18.3	Clustering and Analyticity	330
18.4	Convergence: The Main Ideas	332
18.5	An Equation of Kirkwood–Salsburg Type	335
18.6	Covariance Operators	336
18.7	Convergence: The Proof Completed	341
19	From Path Integrals to Quantum Mechanics	344
19.1	Reconstruction of Quantum Fields	344
19.2	The Feynman–Kac Formula	347
19.3	Self-Adjoint Fields	349
19.4	Commutators	350
19.5	Lorentz Covariance	354
19.6	Locality	357
19.7	Uniqueness of the Vacuum	358
20	Further Directions	363
20.1	The ϕ_3^4 Model	363
20.2	Borel Summability	364
20.3	Euclidean Fermi Fields	365
20.4	Yukawa Interactions	366
20.5	Low Temperature Expansions and Phase Transitions	367
20.6	Debye Screening and the Sine–Gordon Transformation	368
20.7	Dipoles Don’t Screen	371
20.8	Solitons	373
20.9	Gauge Theories	374
20.10	The Higgs Model and Superconductivity	376
	Bibliography	377
	Index	411