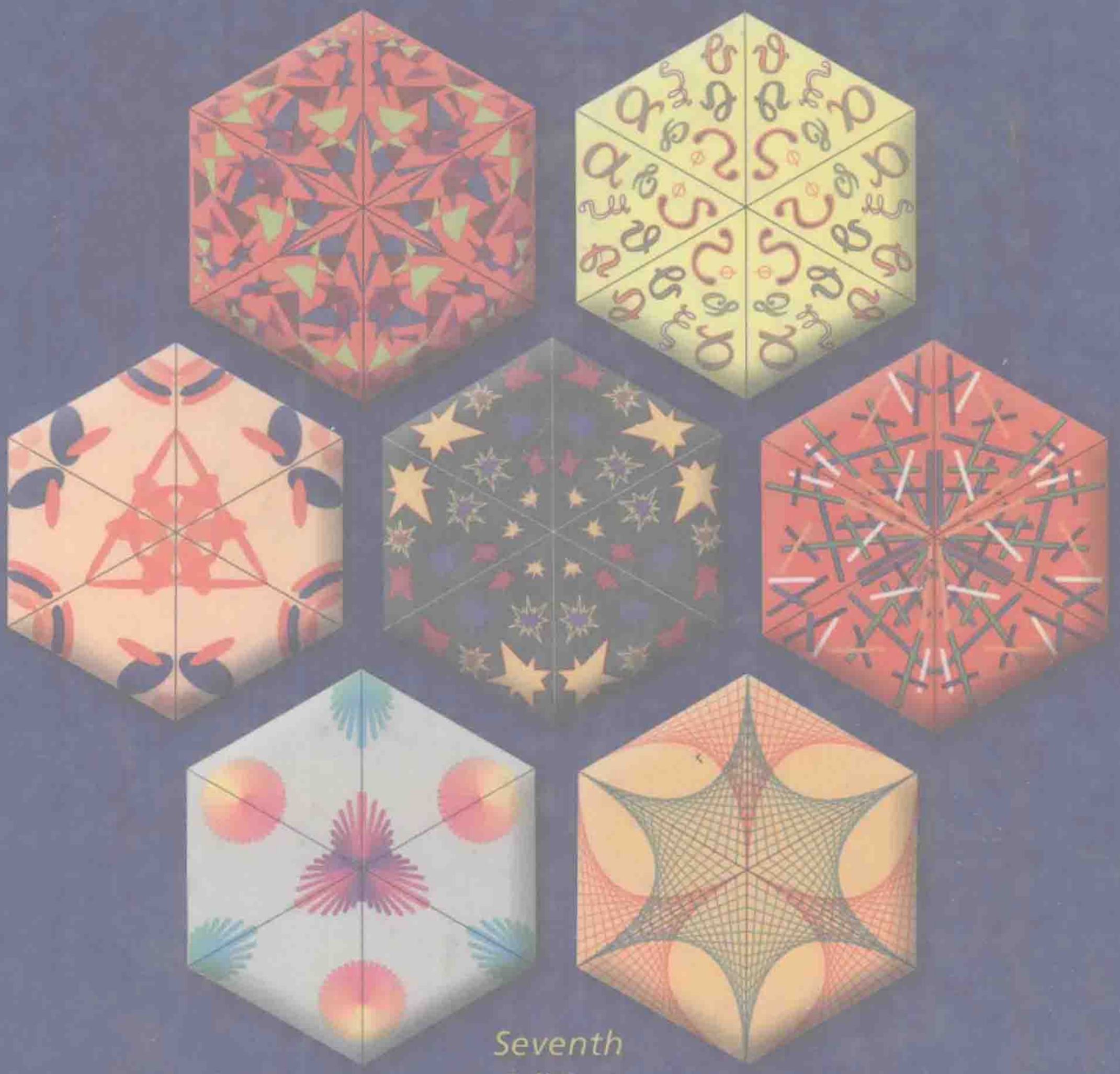
A Problem Solving Approach to

MATHEMATICS

for Elementary School Teachers



Seventh Edition

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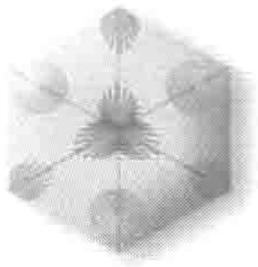

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Preface

The seventh edition of *A Problem Solving Approach to Mathematics for Elementary School Teachers* continues to strive to meet the expectations for the mathematics education of elementary teachers in the new century.

Standards of the NCTM

In the seventh edition we focus on the 2000 National Council of Teachers of Mathematics (NCTM) publication, *Principles and Standards for School Mathematics* (hereafter referred to as *Principles and Standards*). We also continue to emphasize the need for the teaching of mathematics to include

- logic and mathematical evidence as verification;
- mathematical reasoning;
- conjecturing, inventing, and problem solving; and
- connecting mathematics, its ideas, and its applications.

The seventh edition

- allows instructors a variety of approaches to teaching,
- encourages discussion and collaboration among students and with their instructors,
- allows for the integration of projects into the curriculum, and
- promotes discovery and active learning.

Goals

In the seventh edition our goals are

- to present appropriate mathematics in an intellectually honest and mathematically correct manner.
- to use problem solving as an integral part of mathematics.
- to approach mathematics in a sequence that instills confidence, then challenges students.
- to provide opportunities for alternative forms of teaching and learning.
- to offer communication problems to develop writing skills and allow students to practice explanation.
- to encourage integration of technology tools.
- to provide core mathematics for prospective elementary school teachers that challenges them to determine why mathematics is done the way it is.
- to provide core mathematics that allows instructors to use methods integrated with content.

Problem Solving in the Seventh Edition

We showcase problem-solving skills by

- devoting Chapter 1 to problem-solving skills, and emphasizing deductive versus inductive reasoning.
- using a four-step problem-solving process to solve problems throughout the text.
- beginning each chapter with a preliminary problem that poses a question students can answer with the skills mastered from that chapter.

We encourage teachers to point out and discuss the preliminary problem at the beginning of each chapter to show how the techniques therein are necessary to solve the problem.

Features Retained in this Edition

We continue to incorporate various study aids and features that facilitate learning:

- **Historical Notes** add context and humanize the mathematics.
- **Brainteasers** provide a different avenue for problem solving. They are solved in the instructor's guide, and may be assigned or used by the teacher to challenge students.
- **Laboratory Activities** are integrated throughout the book to provide hands-on learning exercises. A separate activities book is also available.
- **Cartoons** teach or emphasize important material, and add levity.
- **Key terms** are presented in the margins for quick review.
- **Definitions** are either set off in text or presented as key terms in the margin.
- Optional sections as well as problems based on these sections are marked with an asterisk (*). **More difficult problems** are marked with a star (★). **Problems numbered in color** have answers at the back of the book.
- **Questions from the Classroom** allow instructors to use questions posed by students when building a course syllabus. Instructors may require students to write two answers to the questions—one mathematical and one pedagogical—using student texts and professional journals for research.
- **Chapter Outlines** at the end of each chapter help students review the chapter.
- **Chapter Reviews** at the end of each chapter allow students to test themselves.
- **Selected Bibliographies** have been updated and revised, and appear at the end of each chapter.
- **Problem-solving strategies** are often highlighted in italics, and indicated by ◪.
- **Relevant quotes** from the *Principles and Standards* are incorporated throughout the text, and are marked by the standard icon ▲.
- **Student pages** are included to show how the mathematics is actually introduced to the K–8 student.
- **Communication Problems** require students to explain or justify their answers.
- **Full color** has been used for pedagogical reasons and to help students visualize concepts better. Figures are more modern, attractive, and easy to follow. All of the pages taken from elementary mathematics texts are presented in full color.
- **Problem sets** contain six types of problems: (1) ongoing assessment, (2) communication, (3) open-ended, (4) cooperative learning, (5) technology, and (6) review. **Communication, Open-Ended,** and **Cooperative Learning** sections are included to conform with the major *Standards'* processes.
- **Relevant and realistic problems** are more accessible and appealing to students of diverse backgrounds.
- **Technology Corners** include use of Logo, spreadsheets, both graphing and scientific calculators, Geometer's Sketchpad, and computer activities.
- **Now Try This** activities appear throughout each chapter, and are intended to help students become more involved in their learning, to facilitate the development and improvement of their critical thinking and problem-solving skills, and to stimulate both in-class and out-of-class discussion.

Content

The streamlined seventh edition retains the core of the sixth edition and includes some new content.

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Chapter 1 An Introduction to Problem Solving

Chapter 1 has more on inductive versus deductive reasoning, and has a new section on algebraic thinking.

Chapter 2 Sets, Whole Numbers, and Functions

In Section 2-3 we introduce whole numbers and basic operations on whole numbers using set concepts.

We have rewritten Section 2-5 on functions from a more concrete and application-oriented perspective. Relations are introduced in the problem set as generalizations of functions.

Chapter 3 Whole-Number Computation

Chapter 3 introduces numeration systems and other number bases. We explore algorithms for addition, subtraction, multiplication, and addition, along with mental mathematics and estimation for whole-number computation.

Chapter 4 Integers and Number Theory

In Chapter 4 we introduce the system of integers and develop an understanding of basic number theory. In several historical notes we tell about recent developments in number theory.

Chapter 5 Rational Numbers as Fractions

We reorganized Chapter 5. The topic of comparing rational numbers is now placed at the beginning of the chapter. In addition, this chapter contains more pictures to help describe the concepts and to assist students with visualization. There is a new section on proportional reasoning.

Chapter 6 Decimals, Percents, and Real Numbers

Chapter 6 now provides a full treatment of decimals, percents, and real numbers. Computing interest is covered as an application of percents and decimals.

Chapter 7 Probability

Chapter 7 introduces elementary probability and methods of counting.

Chapter 8 Statistics: An Introduction

Chapter 8 includes an introduction to different types of graphs. There is a section on measures of central tendency and variation, as well as a section on abuses of statistics.

Chapter 9 Introductory Geometry

This revised chapter has less emphasis on definitions.

Chapter 10 Constructions, Congruence, and Similarity

Chapter 10 includes a reorganized discussion of constructions and congruence. Some material on coordinate geometry has been moved into a section on similarity to show how slope and equations of lines is developed.

Chapter 11 Concepts of Measurement

Chapter 11 now covers linear measure along with areas, surface areas, the Pythagorean Theorem, volume, mass, and temperature.

Chapter 12 Motion Geometry and Tessellations

This chapter has been revised slightly from the sixth edition.

Appendices

Appendices in this edition include

- graphing calculators,
- Geometry Utility (based on Geometer's Sketchpad),
- spreadsheets (based on Microsoft Excel), and
- Logo Turtle Graphics.

Use of Calculators

As the *Principles and Standards* state, coverage of calculators is necessary and timely. The use of the graphing calculator is presented, where relevant, in the Technology Corners. In addition, problems involving the use of both scientific/fraction and graphing calculators appear in the problem sets.

Supplements for the Student

Student's Solutions Manual, ISBN 0-201-61141-4, by Louis Levy, contains detailed solutions to all odd-numbered exercises.

Activities Manual—Mathematics Activities for Elementary School Teachers: A Problem Solving Approach, Fourth Edition, ISBN 0-201-61321-2, by Daniel Dolan, Jim Williamson, and Mari Muri. This revised edition features activities that can be used to develop, reinforce, and apply mathematical concepts. The activities for each concept are ordered by developmental level in each chapter.

NEW: *A Problem Solving Approach to Mathematics for Elementary School Teachers* Videotapes A complete set of videotapes for use by students is available to departments that adopt the seventh edition. All of the basic concepts from the text are reinforced in these videotapes. An instructor experienced in mathematics for elementary school teachers works through detailed examples taken from the text. Videos are available to departments through your Addison Wesley Longman representative.

NEW: InterAct Math CD Tutorial Software ISBN 0-201-61319-0 Available in Windows and Macintosh versions, InterAct Math Tutorial Software includes exercises that are linked with every objective in the textbook and require the same computational and problem-solving skills as their companion exercises in the text. Each exercise has an example and an interactive guided solution that are designed to involve students in the solution process and to help them identify precisely where they are having trouble. In addition, the software recognizes common student errors and provides students appropriate customized feedback. With its sophisticated answer-recognition capabilities, InterAct Math Tutorial Software recognizes appropriate forms of the same answer for any kind of input.

It also tracks student activity and scores. Contact your Addison Wesley Longman representative.

Supplements for the Instructor

NEW: Instructor's Edition includes sequential answers to all text exercises in a special section at the back of the book.

Instructor's Solutions Manual, ISBN 0-201-61142-2, by Louis Levy, contains detailed solutions to all exercises.

Instructor's Resource Guide, ISBN 0-201-61143-0, includes two forms of chapter assessments with answers for each chapter, suggested answers to Questions from the Classroom, Solutions to the Brainteasers, and suggested answers to the Now Try This activities.

Instructor's Guide to *Mathematics Activities for Elementary School Teachers: A Problem Solving Approach, Fourth Edition,* ISBN 0-201-61322-0, by Daniel Dolan, Jim Williamson, and Mari Muri, contains answers for all activities, as well as additional teaching suggestions for some activities.

NEW: TestGen-EQ CD with QuizMaster-EQ ISBN 0-201-61317-4 TestGen-EQ is a computerized test generator with algorithmically defined problems organized specifically for this textbook. Its user-friendly graphical interface enables instructors to select, view, edit, and add test items, then print tests in a variety of fonts and forms. Seven types of questions are available, and search and sort features let the instructor quickly locate questions and arrange them in a preferred order. A built-in question editor gives the user the power to create graphs, import graphics, insert mathematical symbols and templates, and insert variable numbers or text. An "Export to HTML" feature lets instructors create practice tests that can be posted to a Web site. Tests created with TestGen-EQ can be used with QuizMaster-EQ, which enables students to take exams on a computer network. QuizMaster-EQ automatically grades the exams, stores results on disk, and allows the instructor to view or print a variety of reports for individual students, classes, or courses. This program is available in Windows and Macintosh formats. Contact your Addison Wesley Longman representative.

NEW: Web site http://www.awl.com/Billstein
The Web site contains additional resources for instructors and students.

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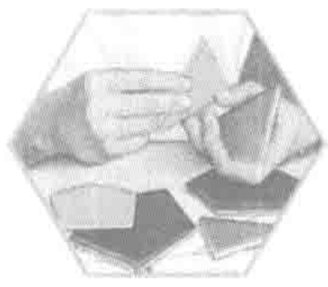

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*indicates optional section

1

An Introduction to Problem Solving

Preliminary Problem

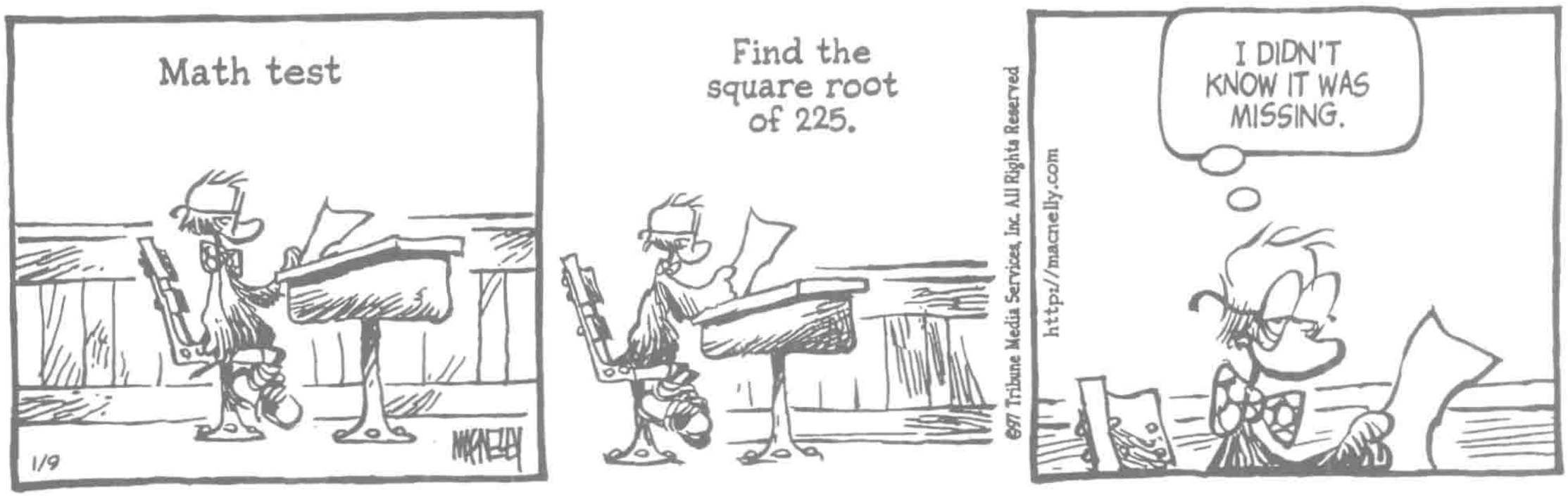
In a fourth-grade class election, there were five candidates for class president. To try to guarantee a fair election, Ms. Pendergast, the teacher, decided to have a series of head-to-head elections in which each candidate ran against each other candidate. The class president would be the person who won the most head-to-head elections. How many elections did there have to be?

Problem solving has long been recognized as one of the hallmarks of mathematics. One of the greatest goals of mathematics education is to have students become good problem solvers. We do not mean doing exercises that are routine practice for skill building. What does *problem solving* mean?

George Polya (1887–1985), one of the great mathematicians and teachers of the twentieth century, pointed out that "Solving a problem means finding a way out of difficulty, a way around an obstacle, attaining an aim which was not immediately attainable." (Polya 1981, p. ix) In *Principles and Standards for School Mathematics* (hereafter referred to as the *Principles and Standards*), we find the following:

▲ *Problem solving is the cornerstone of school mathematics. Without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills are severely limited. . . . The goal of school mathematics should be for all students to become increasingly able and willing to engage with and solve problems* (p. 182).

Exercises or practice problems serve a purpose in learning mathematics, but problem solving must be a focus of school mathematics. Your mathematical experience often determines whether situations are *problems* or *exercises.* In the "Shoe" cartoon, the math test contains what might be a problem for some, but is an exercise for others. Would you expect to find this test item in a middle-school mathematics text?



In this text, you will have many opportunities to solve problems. Each chapter opens with a problem that can be solved by using the concepts developed in the chapter.

A hint for the solution to the problem is given at the end of each chapter. Throughout the text, there are numerous other problems solved using a four-step process and others solved using other formats.

Working with other students to solve problems can enhance your problem-solving ability and communication skills. In this text, we encourage *cooperative learning* and encourage students to work in groups whenever possible. To encourage group work and help identify when cooperative learning might be useful, we identify activities that might involve tasks where it would be helpful to have several people gathering data, or the problems might be such that group discussions might lead to strategies for solving the problem.

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