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Estimation of Rare Event Probabilities in Complex Aerospace and Other Systems

A Practical Approach

Jérôme Morio and Mathieu Balesdent

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To my wife Anne, J.M.
To my father, M.B.

Preface

This book is an opportunity to share our practical experience on rare event probability estimation. We tried to write the book that we would have appreciated having when we started working in this research domain several years ago. The book gives a broad view of current research on rare event probability estimation, and we hope that it will satisfy the readers.

We thank the contributors to this book, namely M. Brevault, Dr. De Visscher, M. Dolado-Perez, Dr. Duponcheel, Dr. Jacquemart, M. Lacaze, Prof. Le Gland, Dr. Missoum, Dr. Pastel, Dr. Vergé, and Prof. Winckelmans, for their helpful collaboration and for the time they devoted to the project. We also thank Prof. Raphael T. Haftka who has done us the great honor of writing a foreword to this book. The works and the daily interactions with the current and former PhD students whom we have supervized at ONERA have also an important part in this book.

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We could not conclude this section without thanking our families for always being there for us and being incredibly supportive.

Jérôme Morio and Mathieu Balesdent

Jérôme Morio has been working at ONERA-The French Aerospace Lab since 2007 as research engineer in the System Design and Performance Evaluation Department. He obtained a Ph.D. in physics from the University Aix-Marseille Paul Cézanne (France) in 2007 and defended his habilitation to supervise research in 2013. He is also a lecturer in probability and statistics at ISAE and ENAC. His main research interests include rare event probability estimation, sensitivity analysis, and uncertainty management.

Mathieu Balesdent has been working at ONERA-The French Aerospace Lab since 2011 as research engineer in the System Design and Performance Evaluation

Department. He obtained a Ph.D. in mechanical engineering from the "Ecole Centrale de Nantes" (France) in 2011. His main research interests include rare event probability estimation, reliability based and multidisciplinary optimization, and aerospace vehicle design. He is also a regular referee for several international conferences and journals such as the American Institute of Aeronautics and Astronautics, Springer and Elsevier.

Foreword

It is a great pleasure to be invited to write the foreword to Morio and Balesdent's book *Estimation of Rare Event Probabilities in Complex Aerospace (and other) Systems—A Practical Approach*, because it is a very timely and needed book and it is done well.

Wikipedia defines rare events as events that occur with low frequency. It says that the term is conventionally applied for those events that have potentially widespread impact and which might destabilize society. Rare events encompass natural phenomena (major earthquakes, tsunamis, hurricanes, floods, asteroid impacts, solar flares, etc.). In aerospace engineering, the term is applied also to less catastrophic events that may happen at low probabilities, typically less than 10^{-6}. A typical example in this book is the probability that two aircraft will get dangerously close to each other in a given airspace.

Calculating accurately the probability of a rare event is usually a challenge, both in terms of the required data and the computational effort required to translate the data into a probability estimate. There are two important reasons why estimating such probabilities has become a hot topic in the past decade or so. First, safety has become much more important to the public, and so the demands from automotive, civil, and aerospace designers are much tougher than they used to be. The rising level of safety means that the causes for failure have become rarer. As one Boeing engineer told me a few years ago, "It used to be that airplane accidents were due to one unlikely thing gone wrong. Now they mostly happen due to two or three unlikely things going wrong at the same time."

The second reason is our increasing ability to estimate well the probabilities of rare events. Big data developments mean that we more often have the required information. Better education of engineers in statistics means that our engineering workforce is capable of applying properly the sophisticated statistical methods required for that purpose. Faster computers allow us to estimate accurately the probabilities of rarer and rarer events.

However, because we usually strive to calculate the probabilities of rarer events than the combination of computer power and present algorithms permit, there has been a strong burst of efficient algorithm development for that purpose in the past 20 years. This book is a welcome treatise on most of the currently available methods and algorithms.

The book is certainly comprehensive. Even though I have worked in this field for more than a decade, I have come across many useful techniques that I did not know about. This appears to be partly due to the contribution of several other contributors, but the two main authors integrated the contributions very well. So the book retains a unity of notation and style.

The book has the right balance of mathematical rigor and practical implementation of the different techniques. Its "French connection" may be partly responsible for the former, and the practical experience of the authors at ONERA, the French Aerospace Lab, for the latter.

On the one hand, the book has a set of toy problems to which each technique is applied, but then it has aerospace applications that show how many of the techniques are applied to real important engineering problems.

I am thus looking forward to using the book in the near future. On the one hand, it will be a valuable resource for my research group. On the other hand, I will be using it in courses where I teach uncertainty quantification and optimization under uncertainty. I may even be tempted to offer a new course on probability estimation of rare events.

Prof. Raphael T. Haftka
Distinguished Professor of Mechanical and
Aerospace Engineering at the University of Florida

Biography of the external contributors to this book

Loïc Brevault received both his engineering degree from "Arts et Métiers" ParisTech (France) and an M.Sc. in Aerospace at Georgia Institute of Technology (USA) in 2013. He is currently preparing a Ph.D. thesis on multidisciplinary design optimization under uncertainty with application for launch vehicle design. His thesis is funded by ONERA and CNES under the supervision of Dr. Le Riche, Dr. Balesdent, M. Bérend, and M. Oswald. His fields of research are multidisciplinary optimization under uncertainty, aerospace vehicle design, and rare event probability estimation.

Ivan De Visscher holds a mechanical engineering degree and a Ph.D. degree from the "Université Catholique de Louvain" (UCL in Belgium). He is one of the developers of the DVM/PVM/WAKE4D wake vortex simulation platform and the author of various papers on wake vortex behavior simulation and modeling.

Juan Carlos Dolado Pérez is an expert on space surveillance and tracking topics at CNES (French Space Agency). Since 2008 he has worked at the orbit determination office where his main research topics concern long- and middle-term re-entry prediction, long-term evolution of the space debris population, orbital collision risk assessment, and orbit determination from radar and optical measurements. He holds a B.Sc. in Aerospace Engineering from Madrid's Polytechnic University and an M.Sc. in Aerospace Engineering from the "Institut supérieur de l'aéronautique et de l'espace" (France).

Matthieu Duponcheel holds a mechanical engineer degree and a Ph.D. degree from UCL (Belgium) and is a research engineer at UCL. He has worked on several research projects devoted to aircraft wake vortices and participated in the development of the DVM/PVM/WAKE4D software.

Damien Jacquemart passed the agregation of mathematics in 2010 and graduated from Aix-Marseille University (France) with an M.Sc. in probability and statistics. In 2010, he joined ONERA to work on a Ph.D. in rare event probability on time-variant systems modeled with Markov processes. He defended his Ph.D. thesis the University of Rennes 1 (France) in 2014.

Sylvain Lacaze defended his Ph.D. in mechanical engineering at the University of Arizona (USA) in 2015. His current research focuses on fields such as reliability assessment, reliability-based design optimization, meta-modeling, and adaptive sampling strategies.

François Le Gland has been a research director at INRIA Rennes (France) and the current head of the research unit applications of interacting particle systems to statistics (ASPI). His main research interests include particle filtering and rare event estimation.

Raphael T. Haftka has been a distinguished Professor of Mechanical and Aerospace Engineering at the University of Florida. Before coming to the University of Florida in 1995, he taught at the Israel Institute of Technology, the Illinois institute of Technology, and Virginia Tech. His areas of research include structural and multidisciplinary optimization, design under uncertainty, and surrogate-based global optimization.

Samy Missoum has been an Associate Professor in the Aerospace and Mechanical Engineering Department at the University of Arizona (USA) since 2005. He obtained his Ph.D. in Mechanical Engineering from the Institute of Applied Sciences in Toulouse (France) and was a postdoctoral researcher in the Aerospace and Ocean Engineering Department at Virginia Polytechnic Institute and State University (USA). His current research focuses on the development of optimization and probabilistic design methods for complex systems. He is a member of the Multidisciplinary Design Optimization and Non-Deterministic Approach technical committees of the American Institute of Aeronautics and Astronautics.

Rudy Pastel was born in Martinique where he studied science until he joined ENSTA-ParisTech (France) specializing in applied mathematics. In 2008, he joined ONERA to work on a Ph.D. dealing with rare event probability and extreme quantile estimation. He defended his Ph.D. thesis in 2012 at the University of Rennes 1 (France). He is currently a data scientist in the Volkswagen data lab in Munich (Germany).

Christelle Vergé passed the agregation of mathematics in 2011 and graduated from the Bordeaux University (France) with an M.Sc. in statistics and stochastic models. She defended in 2015 a Ph.D. in applied mathematics in applied mathematics at ONERA and the "École Polytechnique," under the supervision of Prof. Del Moral, Prof. Moulines, and Dr. Morio. She is interested in parallelization of sequential Monte Carlo methods.

Grégoire Winckelmans holds a mechanical engineering degree from UCL (Belgium), a postgraduate degree in aeronautics and aerospace from the von Karman Institute (Belgium), and M.Sc. and a Ph.D. degrees in aeronautics from California Institute of Technology (USA). His expertise lies in fluid mechanics, advanced numerical simulation, and the modeling of turbulent flows. He and his team have participated on numerous research projects devoted to aircraft wake vortices, have authored papers, and have developed the DVM/PVM/WAKE4D software.

Abbreviations

^{2}SMART	support vector margin algorithm for reliability estimation
ADS	adaptive directional sampling
AMISE	asymptotic integrated square error
ANOVA	analysis of variance
as	almost surely
AST	adaptive splitting technique
AV	antithetic variates
cdf	cumulative distribution function
CE	cross entropy
CFD	computational fluid dynamics
CMC	crude Monte Carlo
CC	correlation coefficient
CV	control variate
DoE	design of experiment
DS	directional sampling
DVM	deterministic wake vortex model
DSS	directional stratified sampling
EGRA	efficient global reliability analysis
EVT	extreme value theory
FAST	Fourier amplitude sensitivity test
FORM	first-order reliability method
FOSPA	first-order saddle point approximation
GEV	generalized extreme value
GAISA	general adaptive importance splitting algorithm
GPD	generalized Pareto distribution
iid	independent and identically distributed
IS	importance sampling
kde	kernel density estimator
LDT	large deviation theory
LHS	Latin hypercube sampling
LS	line sampling
MCMC	Monte Carlo Markov chain
MLW	maximum landing weight
MISE	mean integrated square error
MSE	mean squared error
MT	mean translation
n/a	not applicable

NAIS	nonparametric adaptive importance sampling
NORAD	North American Aerospace Defense Command
OAT	one variable at a time
PCC	partial correlation coefficient
POT	peak over threshold
pdf	probability density function
PSO	particle swarm optimization
QMC	quasi-Monte Carlo
QP	quadratic programming
rv	random variable
SA	sensitivity analysis
SRC	standardized regression coefficient
SS	stratified sampling
SC	scaling
SORM	second-order reliability method
SUR	stepwise uncertainty reduction
SVM	support vector machine
SVR	support vector regression
TLE	two-line elements
WIR	weighted importance resampling

Contents