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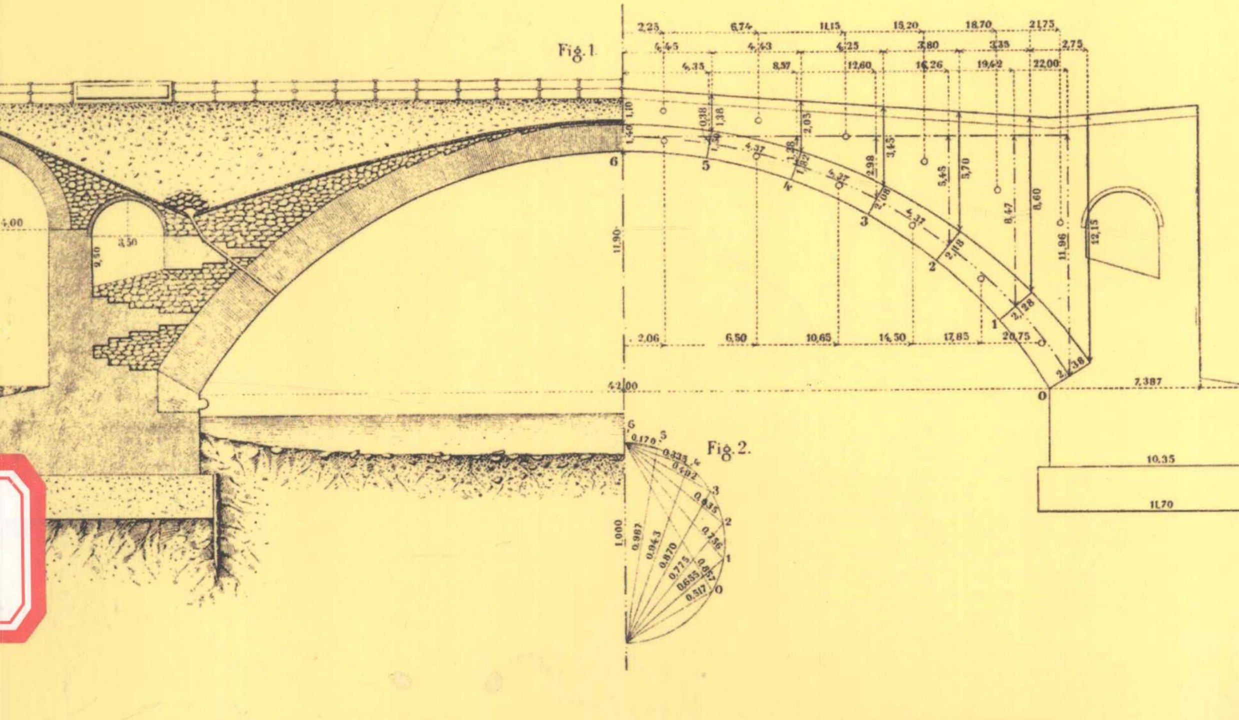
ELASTIC STRESSES IN STRUCTURES

Translated from Castigliano's *Théorem de l'equibre des systèmes élastiques et ses applications*

Alberto Castigliano

Translated by Ewart S. Andrews



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ELASTIC STRESSES IN STRUCTURES

TRANSLATED FROM CASTIGLIANO'S "THÉOREM DE L'EQUIBRE DES SYSTÈMES ÉLASTIQUES ET SES APPLICATIONS"

BY

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AUTHOR'S PREFACE.

THIS book contains the theory of elastic stresses in structures explained in accordance with a new method, based upon theorems which are either quite new or little known; incorporated in this theory will be found the mathematical theory of elastic solids, considered particularly from the standpoint of the strength of materials.

We believe that the time has arrived for introducing into our courses of instruction this scientific method of treating the strength of materials and thus abandoning the older methods which Lamé has fairly characterised as "neither scientific nor empirical, serving only to hinder the approach of true science".

We will now give some historical notes of the discovery of the theorems of which we make repeated use in this book.

These theorems are the following three:—

1. That of the differential coefficients of internal work, 1st part.

2. That of the differential coefficients of internal work, 2nd part.

3. That of the least internal work.

The first has already been employed by the celebrated English astronomer Green, but only for a particular problem; it has not been enunciated and proved in a general way such as we give here.

The second is the converse of the first, and we believe that it was first enunciated and proved in our thesis for the diploma of engineering at Turin; we have given it at greater length in our memoir entitled "Nuova teoria intorno all' equibrio dei sistemi elastici," published in the Transactions of the Academy of Science of Turin in 1875.

The third theorem can be regarded as a corollary of the second; but as in some other problems of maxima and minima, it has been partly known several years before the discovery of the principal theorem.

In the year 1818 Captain Vène, of the French Engineer Corps, enunciated a principle which was absolutely incorrect under the conditions to which he wished to apply it, but which, by one of those peculiar combination of circumstances of which science presents several examples, was destined to lead later to the discovery of the theorem of least work.

After this first step, the distinguished scientists, MM. A. Cournot, Pagani, Mossotti, A. Dorna, and General L. F. Ménabréa investigated the question. The last-mentioned gave the name "principle of elasticity" to the theorem of least work, and made it the subject of his researches, in a first memoir presented in 1857 to the Academy of Science of Turin, later in a second presented in 1858 to the Academy of Science of Paris, and again in a third submitted in 1868 to the Turin Academy. Since, however, the proofs given by M. Ménabréa were not exact, the "principle of elasticity" was not accepted by the greater number of the authorities, and some of them published memoranda to show the fallacy of it. It was not until 1873 that we gave, in our above-mentioned thesis, the first rigorous proof, in a form which appeared to us clear and exact, of the theorem of least work. Afterwards, in our thesis of 1875, we demonstrated that the theorem of least work is only a corollary of that of the differential coefficients of internal work.

We can thus state that the present book, comprising the complete theory of elastic stresses in structures, of which the mathematical theory of the elasticity of solid bodies comprises only a chapter, is wholly based on the theorems of the differential coefficients of internal work. As our object is not only to expound a theory, but further to show its advantages of brevity and simplicity in practical applications, we have solved, according to the new method, the greater number of the general problems dealt with in courses on the strength of materials, and have added several numerical examples for the calculation of the stresses in the more important types of structure. Each of these examples is, so to speak, a particular application of the theory to one of these structures, but in order to give our book a more practical value we have put each example in the form which would arise in an actual design, in order to justify the dimensions of the principal members.

Moreover, since in our view these examples ought to serve as a model for similar calculations, we have always examined several cases of loading and taken account of the effects due to temperature variations; that is to say, that for each assumed load we have determined the stresses occurring at different temperatures. By taking account of these circumstances and by following the new method of calculation, which permits of the solution of all questions on the stresses in elastic structures without the introduction of any arbitrary assumption, we have the advantage of being able to adopt higher working stresses; for one of the causes which often compels, in practice, the adoption of small values for these stresses, is the imperfection of the principles upon which the calculation of the stresses is based; and another reason for the adoption of low working stresses is that we cannot take account of all the circumstances liable to cause these stresses to increase. With regard to the calculations, we may say that they are but a little longer than in methods ordinarily followed, and moreover, they can nearly always be materially shortened by neglecting some terms having small influence on the result.

In conclusion, we have heartily to thank M. Louis Reeb, Permanent Way Superintendent of the Northern Italian Railway, who has kindly undertaken the proof revision.

TRANSLATOR'S PREFACE.

CASTIGLIANO's work is referred to as a classic in the leading text-books dealing with the advanced Theory of Structures, but few students in modern times have had an opportunity of studying it, since, as far as we have been able to ascertain, there is no previous translation in English, and the copy in French is out of print and very scarce.

With the development in practical design of methods of calculation involving considerations of internal work arising in "Higher Structures," the work of Castigliano becomes of fresh interest to engineers. Although the book is now practically forty years old, it is surprising that it is by no means out of date; the reason for this is that though practice may vary, principles are almost invariable. Castigliano's work gives us the most complete analysis of the theory of elasticity applied to the determination of stresses in structures that we have yet met, and we believe that it deserves to receive a close study by all engineers and students who wish to follow the logical development of structural theory and its application to practical design.

In view of the very large number of tabulated numerical values and of the resulting labour involved in translating these values from metric into British units, it has been decided to preserve the metric units; it is thought that this does not detract from the value of the book in demonstrating methods of design for British designers. In order, however, to bring the notation into more modern and standardised form, the mnemonic notation prepared by the Concrete Institute has been followed as far as possible.

The author's thanks are due to Mr. C. Paice of the Examining Staff of H.M. Patent Office for assistance in the troublesome task of reading the proofs, and to Mr. C. Wyndham Hulme, Librarian of H.M. Patent Office, for courtesy in giving special access to the copy of M. Castigliano's book in the Patent Office Library.

EWART S. ANDREWS.

ROLLS CHAMBERS, 89 CHANCERY LANE,
LONDON, W.C. 2, *July*, 1918.

FOLDING PLATES

(Following Index).

NOTATION.

The following is the general notation adopted throughout the book; wherever additional notation is necessitated, its explanation is given in the text:—

A	= Area of cross section.
B	= Bending moment.
C	= Constant.
E	= Elastic modulus (Young's modulus).
F	= Force (general).
F	= Coefficient of shear stress in beam formulæ.
G	= Glide, shear or rigidity modulus.
H	= Horizontal thrust in arches.
I	= Moment of inertia (about a line).
I_p	= Polar moment of inertia.
M	= Moment (general).
M	= Modulus of section.
M_I	= First moment of a force.
	= First moment about N.A. of area above N.A. in shear in beams.
N.A.	= Neutral axis.
N	= Node.
P	= Pressure or thrust.
R	= Radius of an arch.
R	= Resultant force.
R	= Reaction.
S	= Shearing force.
T	= Tensile force.
T_M	= Twisting moment.
V	= Volume.
W	= Weight.
W_i	= Internal work.
X, Y, Z	= Components of a force parallel to the corresponding axes.
a	= An arm or distance (general).
a	= Length of prismatic body of cylinder.
a	= Breadth of a rectangle.
a	= Semi-axis of an ellipse.
b	= Semi-axis of an ellipse.
b	= Depth of a rectangle.
c	= Compressive stress.
d	= Diagonal length.
d	= Depth of a beam (general).
d_n	= Deflection.
e	= Eccentricity of load.

f	= Force per unit area.
g	= Gyration radius.
l	= Length (general).
	= Span of a beam or arch.
n	= A number.
	= Neutral axis depth from compressed edge of beam.
r	= Rise of an arch.
r	= Radius or radial length.
s	= Shear stress.
t	= Tensile stress.
$t°$	= Temperature.
u, v, w	= Displacements (p. 36).
w	= Weight or load per unit length.
x	= Extension.
x, y, z	= co-ordinates of a point.
α, β, γ	= Angles.
ϵ	$= \frac{EA}{l}$.
ϵ	= A coefficient (p. 40).
ξ	= Increment of x.
η	= Increment of y.
ζ	= Increment of z.
λ	= Increment of l.
ρ	= Increment of r.
κ	= Compressive strain.
σ	= Shear strain.
τ	= Tensile strain.
ν	= Dilatation or volume strain.
Π	= Poisson's ratio. [Lateral contraction to longitudinal extension.]
π	= Circle function 3·14159
θ	= Angle of torsion.
Θ, ϕ	= Angles generally.

CONTENTS.