



ADVANCED THEORY OF CONSTRAINT AND MOTION ANALYSIS FOR ROBOT MECHANISMS

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All machines are made of mechanisms. Before design, the degree of freedom of the end effector of a robot mechanism should first be determined, and then the actuation scheme can be investigated. Thus, it is the fundamental basis and prerequisite for structural analysis and synthesis to analyze the degree of freedom of the end effector of a mechanism. Structure analysis of a mechanism is focusing on proposing the structure principle of mechanisms, the classification of mechanisms, and their free motions. Meanwhile, structure analysis of mechanisms is an important aspect of kinematics, dynamics analysis and mechanism synthesis. The rapid development of science and technology calls for the creation and design of better mechanisms. Therefore, it is essential to conduct further theoretical research into the structure, type, and kinematic characteristics of mechanisms. Structure analysis presents a basis to determine the number of the actuators and carry out kinematics and dynamics research. Therefore, it is one of the most important concepts in mechanism. It offers a guarantee for innovation and structure synthesis, and lays a foundation for further analyzing the driving, and kinematics and dynamics performances of mechanisms.

This book studies the constraint and motion analysis for spatial robot mechanisms based on the unified mathematical representation of its kinematic geometry constraints. Chapter 1 roughly surveys the development and problem in the structure analysis of mechanisms in the past centuries. In order to limit the mathematical prerequisites to the knowledge of elementary screw algebra and matrix calculus, it was felt necessary to present in Chapter 2 an introduction to screw theory. The material covered is limited to that necessary for an understanding of the book, and no attempt has been made to study thoroughly such topics. It describes the primary knowledge about the screw algebra from aspects such as the Plücker column vector of a line, motion representation of rigid bodies, the screw description of the motion and force, reciprocal product of the screw and its geometry significance, the linear combination of screws and the principal screws in a screw system.

In Chapter 3, the motion screw system of kinematical chains is established through the screw representation of the free motions of kinematic pairs; the terminal constraints of the kinematic chain are derived according to the reciprocal relationship between twists and wrenches. The equivalent substitution and synthesis of kinematic chains under the conditions of the same terminal constraints are studied. The investigation into the mechanical equivalent substitution theorem lays a necessary foundation for the selection and optimization of the kinematic pairs of the spatial mechanism which has purely serial kinematic chains, the research into the idle degrees of freedom of kinematic

chains, especially the complex spatial kinematic chains, and the free motion analysis of the end effector in a mechanism and the analysis of the actuation scheme. According to reciprocal screw theory, terminal constraints of the kinematic chain depend entirely on the largest linearly independent set of the kinematic screws. Therefore, the structural optimization problem of the kinematic chain can be addressed through the algebraic analysis to the specified terminal constraints; and it can also be resolved through algebra analysis of the screws. In fact, the free motions of the end effector are the results of the combination of branch kinematic chains. Therefore, the study of spatial serial mechanisms will focus on the constraints of kinematic chains to the end effector. How to determine the types of various kinematic chains and how to complete the design with the simplest and economical structure arrangement have become key issues in the synthesis of kinematic chains. This is also one of the key issues in this chapter and subsequent chapters.

In Chapter 4, terminal constraints of robot end effectors from kinematic chains and the free motions under these constraints are discussed. The analytical theory of constraint space of the end effector of parallel robot mechanism is studied according to the terminal constraint space of its kinematic chains. The degree of freedom of the end effector of a parallel robot should have the three attributes of *quantity*, *type*, and *direction*. The degree of freedom of the end effector of a robot is different from the actuations to control the mechanism. However, the traditional methods did not make any distinction between the end effector and the entire mechanical system. This chapter is dedicated to establishing an analytical theory for the constrained space of the end effectors. On this basis, the equivalent description of the complex kinematic chain is discussed, which provides a theoretical support for the equivalent description of the complex kinematic chains to pure serial kinematic chains. It allows us to transform the parallel kinematic chains into serial chains with the equivalent substitution for the compound kinematic chains. This lays a foundation for the study of actuation of the prescribed end effector.

In Chapter 5, the search for reachable workspace of the spatial robot mechanism and its symmetric homomorphism with the mechanism itself is discussed. It is proved that the structural symmetry of the mechanism itself will inevitably lead to the symmetry of the reachable workspace and the dexterous workspace of its end effector. The corresponding relation between the reachable workspace and the dexterous workspace is homomorphism mapping. A quantitative description of such symmetry relations provides certain theoretical basis for the analysis, synthesis, and structure innovative design of robot mechanism. Based on this, the coordinate transformation method for resolving the workspace based on free motion analysis of the end effector of spatial parallel mechanism is also covered in this chapter, and the synthesis of the dexterous workspace is discussed with the Schoenflies-type spatial parallel mechanism as an example. By analyzing the end effectors' degrees of freedom, a minimum of variables can be chosen to establish the kinematic constraint equations and thus the objective

function can be built up. By searching the desired dexterous workspace, the optimal length of linkages in kinematic chains can be obtained. Application examples show that the method can be widely applied to the synthesis of the dexterous workspace of various parallel robot mechanisms.

In Chapter 6, the singularity of the spatial robot mechanism in the workspace is studied and it is proved that the structural symmetry of the mechanisms themselves will inevitably lead, not only to the symmetry of reachable workspace and dexterous workspace of its end effector, but also to that of singular workspace. A quantitative description of such a symmetric relation provides some theoretical basis for the analysis and synthesis of workspace of symmetrical mechanisms. With screw theory, a group of balance equations which can be used to analyze the singularity of mechanisms have been obtained and the conditions for singular mechanism are derived. The order of the coefficient matrix of final equations is identical to the number of degrees of freedom of the end effector, which constitutes great advantages for the singularity analysis of the end effector of the mechanism with less than six degrees of freedom. The fewer degrees of freedom the end effector has, the lower the order of the finally converted coefficient matrix will be.

In Chapter 7, kinematics modeling of the spatial parallel robot mechanism is discussed. On the basis of the natural coordinate method, the sufficiency and necessity have been proved that a type of kinematics model spatial parallel mechanism can be completely established with only four non-collinear points with Cartesian coordinates. This method has a significant advantage in dealing with kinematics issues of spatial parallel mechanisms with more than three degrees of freedom. And the established kinematics model shares the same advantages of general natural coordinate method and can also minimize the number of independent variables. As this model only introduces the Cartesian coordinates, and the elements of the derivative matrix merely contain the linear items or even the constant items, while the elements of the derivative matrix obtained through the modeling method based on the rotation matrix transformation are often non-linear, and even contain transcendental functions. Therefore, these advantages of the four-point coordinate method have brought great convenience to kinematics and dynamics analysis.

In Chapter 8, the kinematics and static forces of robot mechanisms are discussed. Based on the analysis of terminal constraints of the kinematic chain in Chapter 4, detailed analysis is given to the motion between the end effector and actuation and the rigid body statics. Through the analysis of the velocity of mechanism end effector, the basic conditions for the feasible motion of robot end effector have been discussed and the forward and inverse solutions of velocity are also investigated. On this basis, robot static equilibrium conditions and the forward and inverse solutions of statics are derived according to virtual power principle.

In Chapter 9, the basic way to analyze the degree of freedom is studied from the theoretical basis for the analysis model. Through investigating the

theoretical basis of the degree of freedom of the end effector and the configuration degree of freedom of a mechanism, the key for analyzing the mechanism is revealed: it is a necessary condition to distinguish the degree of freedom of the end effector and the independent actuators required to control the end effector. Application examples of spatial mechanisms demonstrate that it is the necessary requirement for correctly analyzing a mechanism, and it satisfies the current needs of engineering as well.

In Chapter 10, by analyzing the calculation of configuration degrees of freedom of spatial robot mechanism under a single Cartesian coordinate system, the basic motion characteristics of the robot mechanism in the workspace are further investigated. Based on the analysis of free motion and terminal constraint of the kinematic chain in Chapter 4, specific studies have been performed to analyze the degree of freedom of the end effectors within a single Cartesian coordinate system; the configuration degrees of freedom describing the dynamic drive control of mechanisms are investigated by studying the dynamic degree of freedom of the end effector under different actuation schemes. On the basis of the unified mathematical description of kinematic geometry constraints, the analysis theory for the degree of freedom of spatial robot mechanisms has also been discussed. The theory consists of two main parts: the first is the analysis theory targeted at the end effector; the second is the analysis theory for the configuration degree of freedom mechanism with the designated end effectors. The concept of configuration degrees of freedom means that singularity analysis of complex parallel mechanisms are naturally included in the analysis of the configuration degrees of freedom; at the same time, the approach for analyzing the structural stability of mechanisms is also provided: only when the mechanism's motion inputs remain consistent with its configuration degree of freedom in the space at any time, can the entire mechanism system be stably controlled.

In Chapter 11, the mechanism theory of spatial deployable structure based on the scissor-like-element (SLE) is dealt with. To begin with, the motion analysis of the simple planar linkage, the characteristics of the deployable structure and its elements are studied step-by-step. Through in-depth analysis of the double rotating hinge connected by two SLEs, the mechanism theory in the constitution of elements of the deployable structure has been revealed. Through the analysis of elements of the planar deployable structure, the cylindrical deployable structure, and the spherical deployable structure, the constitution mechanism of spatial deployable structures based on the SLE has been investigated. This has important practical value for the promotion of deployable structures in mechanical engineering, construction engineering, and robot engineering.

In Chapter 12, application of the analytical theory in structure synthesis and innovation of mechanisms is studied. Through presenting general steps in mechanism synthesis, the basic method for the innovative design of mechanism with the universal significance is also discussed. With a comprehensive case study for the innovative design of the lower limb of a biped humanoid robot

and the independent suspension of the vehicle, applications of the theory in the innovative design of mechanisms are investigated.

In Chapter 13, a new algorithm is proposed to optimize the design of a spatial parallel manipulator for the purpose of obtaining a desired dexterous workspace rather than the whole reachable workspace. With the analysis of the degree of freedom of a manipulator, we can select the least number of variables to depict the kinematic constraints of each leg of a manipulator. The optimum parameters can be obtained by searching the extreme values of the objective functions with the specified adroit workspace. An example is utilized to demonstrate the significant advantages of this method in the dexterous workspace synthesis. In applications, this method can be widely used to synthesize, optimize, and create all kinds of new spatial parallel manipulators with the desired dexterous workspace.

Regarding kinematics, many works have addressed this problem in the past. In Chapter 14, the natural coordinates are adopted to deal with the kinematics model for different spatial mechanisms. With these constraint equations in natural coordinates, we can conduct the kinematics analysis of the mechanism and then build the dynamics model with the Newton–Euler formulation.

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Introduction

Mechanism is a branch of mechanical engineering science focusing on the study of the motion and the transmission law of mechanisms, which consists of structure analysis and synthesis [1–3]. Structure analysis of mechanism mainly investigates the mobility of the structure, and explores the trajectory, displacement, velocity, and acceleration of the mechanism [2,3]. Structure synthesis of mechanism focuses on the design of mechanisms so that they meet the kinematic requirements according to the prescribed motions. Research results of assembly principles of structure, kinematics, dynamics, and precision analysis of mechanisms provide the theories and methods for various movement generations, the kinematics and dynamics design of mechanical systems [4,5].

1.1 REVIEW OF MECHANISM

With the invention and improvement of steam engines and textile machines at the beginning of the 18th century, calculus, theoretical mechanics, and other disciplines were set up and becoming more and more sophisticated, which provided a theoretical basis and means for the design and invention of various mechanisms. At that time, mechanism was still included in the field of applied mechanics. The rapid development of mechanical industry at the second half of the 19th century proposed specific requirements for the application of mechanisms and the corresponding analysis and synthesis [1]. In the United Kingdom, scholars of mechanism, represented by R. Willis, classified mechanisms according to the characteristics of motion transformation and the speed ratio relationship, and mainly studied the gear and planetary transmission [1,2]. In Germany, scholars of mechanism, represented by F. Reuleaux and L. Burmester, introduced concepts such as kinematic pair, kinematic chain, and schematic diagram of mechanism, discussed the possible types of mechanisms from the structure principle of mechanism, and further extracted the common characteristics of mechanisms, and with graphical methods, conducted the analysis and synthesis of mechanisms according to the motion geometry principle [2,3]. In Russia, scholars, represented by П. Л. Чебышев, addressed the approximate synthesis of dimension by using algebraic methods like approximation theory of functions [4,5]. Based on these results, mechanism was developed into an independent discipline [1,2,6].

Mechanism is one of the most important fundamental disciplines on which machine design relies. The invention, innovation, and improvement of mechanisms are important tasks in machine design. The basic problems of research in mechanism can be roughly divided into two major categories: structure analysis and structure synthesis. Structure analysis focuses on investigating the mobility and constraint, kinematics and dynamics of mechanisms, so as to reveal the structure constitution, the kinematics and dynamics laws, and their inter-relationships in mechanisms, and it can provide a theoretical basis for understanding the performance and mechanical synthesis of existing mechanical systems. Mechanism synthesis is defined as the theory and method of designing new mechanisms satisfying the engineering requirements in aspects such as structure, motion, kinematics, and dynamics. It includes structure synthesis, kinematics synthesis, and dynamics synthesis. Structure synthesis of mechanism is one of the most important areas in the field of mechanism. The entire mechanism is often proposed as a multi-rigid-body system, in order to reveal the assembly laws of the structure, characteristics of the structure topology, and their intrinsic relationships with kinematics, dynamics in mechanisms, so as to provide a basis for the establishment of structure synthesis of mechanism, kinematics, and dynamics theory, and to provide a theory to design new mechanisms [7].

1.1.1 Development of Mechanism

Germany was most active in the early study of mechanism. After the 1940s, and especially after the 1960s, the United States achieved rapid development in the research of mechanism. A lot of research work has been done in areas such as structure analysis and synthesis of planar and spatial linkages, cam dynamics, dynamics of elastomeric mechanical systems, as well as mechanical design optimization. The United States leads the world in the research of many aspects of mechanism. After the 1960s, the United Kingdom accelerated its pace of research in mechanism, and achieved substantial results in cam mechanism, mechanical system dynamics, as well as areas such as spatial linkages and industry robots. Ever since the establishment of the Institute of Mechanics of the Academy of Sciences in Soviet Union during the 1930s, mechanism became one of the main directions of development in the former Soviet Union. After World War II, the Soviet Union's research in mechanism developed rapidly. The Soviet Union made important contributions in aspects like mechanical structure theory, precision analysis, lower flat pair mechanical analysis, higher pair envelope theory, spatial mechanism, and system dynamics of multiple degrees of freedom (DOFs) mechanisms. Especially from the 1950s to the 1970s, the Soviet Union remained the leader in the main areas of study in mechanism in the world. Similarly, Japan studied the cam mechanism dynamics and its computer-aided design and manufacturing and the analysis and synthesis of planar linkages and so on. After the 1970s, its research in robots and manipulators developed especially fast. Significant progress had been made in the classification and