Stability, Oscillations and Optimization of Systems

STABILITY ANALYSIS OF NONLINEAR SYSTEMS UNDER STRUCTURAL PERTURBATIONS

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INTRODUCTION TO THE SERIES

Modern stability theory, oscillations and optimization of nonlinear systems have developed in response to the practical problems of celestial mechanics and applied engineering has become an integral part of human activity and development at the end of the 20th century.

For a process or a phenomenon, such as atom oscillations or supernova explosion, if a mathematical model is constructed in the form of a system of differential equations, then the investigation is possible either by a direct (numerical as a rule) or integration of the equations or by analysis by qualitative methods.

In the 20th century, the fundamental works by Euler (1707 — 1783), Lagrange (1736 — 1813), Poincare (1854 — 1912), Liapunov (1857 — 1918) and others have been thoroughly developed and applied in the investigation of stability and oscillations of natural phenomena and the solution of many problems of technology.

In particular, the problems of piloted space flights and those of astrodynamics were solved due to the modern achievements of stability theory and motion control. The Poincare and Liapunov methods of qualitative investigation of solutions to nonlinear systems of differential equations in macroworld study have been refined to a great extent though not completed. Also modelling and establishing stability conditions for microprocesses are still at the stage of accumulating ideas and facts and forming the principles; for examples, the stability problem of thermonuclear synthesis.

Obviously, this is one of the areas for the application of stability and control theory in the 21th century. The development of efficient methods and algorithms in this area requires the interaction and publication of ideas and results of various mathematical theories as well as the co-operation of scientists specializing in different areas of mathematics and engineering.

The mathematical theory of optimal control (of moving objects, water resources, global processes in world economy, etc.) is being developed in terms of basic ideas and results obtained in 1956 — 1961 and formulated in Pontryagin's principle of optimality and Bellman's principle of dynamical programming. The efforts of many scholars and engineers in the framework of these ideas resulted in the efficient methods of control for many concrete systems and technological processes.

Thus, the development of classical ideas and results of stability and control theory remains the principle direction for scholars and experts modern stage of the mathematical theories. The aim of the international book series; **Stability, Oscillations and Optimization of Systems** is to provide a medium for the rapid publications of high quality original monographs in the following areas:

Development of the theory and methods of stability analysis:

a. Nonlinear systems (ordinary differential equations, partial differential equations, stochastic differential equations, functional differential equations, integral equations, difference equations, etc.)

b. Nonlinear operators (bifurcations and singularity, critical point theory, polystability, etc.)

Development of up-to-date methods of the theory of nonlinear oscillations:

a. Analytical methods.

b. Qualitative methods.

c. Topological methods.

d. Numerical and computational methods, etc.

Development of the theory and up-to-date methods of optimization of systems:

a. Optimal control of systems involving ODE, PDE, integral equations, equations with retarded argument, etc.

b. Nonsmooth analysis.

c. Necessary and sufficient conditions for optimality.

d. Hamilton-Jacobi theories.

e. Methods of successive approximations, etc.

Applications:

a. Physical sciences (classical mechanics, including fluid and solid mechanics, quantum and statistical mechanics, plasma physics, astrophysics, etc.).

b. Engineering (mechanical engineering, aeronautical engineering, electrical engineering, chemical engineering).

c. Mathematical biology and life sciences (molecular biology, population dynamics, theoretical ecology).

d. Social sciences (economics, philosophy, sociology).

In the forthcoming publications of the series the readers will find fundamental results and survey papers by international experts presenting the results of investigations in many directions of stability and control theory including uncertain systems and systems with chaotic behavior of trajectories.

It is in this spirit that we see the importance of the "Stability, Oscillations and Optimization of Systems" series, and we are would like to thank Cambridge Scientific Publishers, Ltd. for their interest and cooperation in publishing this series.

*Everything should be made as simple as possible,
but not simpler.*

ALBERT EINSTEIN

PREFACE

This monograph deals with some topical problems of stability theory of nonlinear large-scale systems. The purpose of this book is to describe some new applications of Liapunov matrix-valued functions method to the theory of stability of evolution problems governed by nonlinear equations with structural perturbations.

The concept of structural perturbations has extended the possibilities of engineering simulation of the classes of real world phenomena. We have written this book for the broadest audience of potentially interested readers, students and researchers: applied mathematicians, applied physicists, control and electrical engineers, communication network specialists, performance analysts, operations researchers, etc., who deal with qualitative analysis of ordinary differential equations, difference equations, impulsive equations, and singular perturbed equations.

To achieve our aims we made the analysis:

(i) general enough to apply to the wide range of of applications which arise in science and engineering, and

(ii) simple enough so that it can be understood by persons whose mathematical training does not extend beyond the classical methods of stability theories which were popular at the end of the twentieth century.

Of course, we understood that it is not possible to achieve generality and simplicity in a perfect union but, in fact, the new generalization of direct Liapunov's method give us new possibilities in the direction.

In this monograph the concept of structural perturbations is developed in the framework of four classes of systems of nonlinear equations mentioned above (ordinary differential equations, difference equations, impulsive equations and singular perturbed equations). The direct Liapunov method being one of the main methods of qualitative analysis of solutions to nonlinear systems is used in this monograph in the direction of its generalization in terms of matrix-valued auxiliary functions.

Thus, the concept of structural perturbations combined with the method of Liapunov matrix-valued functions is a methodological base for the new direction of investigations in nonlinear systems dynamics.

The monograph is arranged as follows.

Chapter 1 provides an overview of recent results for four classes of systems of equations (continuous, discrete-time, impulsive, and singular perturbed systems), which are a necessary introduction to the qualitative theory of the same classes of systems of equations but under structural perturbations.

Chapter 2 – 5 expose the mathematical stability theory of equations under structural perturbations. The sufficient existence conditions for various dynamical properties of solutions to the classes of systems of equations under consideration are obtained in terms of the matrix-valued Liapunov functions and are easily available for practical applications. All main results are illustrated by many examples from mechanics, power engineering and automatical control theory.

The final Sections of Chapters 2 – 5 deal with the discussion of some directions of further generalization of obtained results and their applications. To this end new problems of nonlinear dynamics and system theory are involved.

The idea of writing this monograph came from Professor A.A.Martynyuk with the intension to describes work done during 1989–2012.

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Also, we are exceedingly thankful to Professor V.Lakshmikantham for his kind attention to our work up until the day of June 7, 2012 when he passed away.

Great assistance in preparing the manuscript for publication has been rendered by collaborators of the Department of Processes Stability of the S.P.Timoshenko Institute of Mechanics of National Academy of Sciences of Ukraine L.N.Chernetzkaya, and S.N.Rasshyvalova. The authors express their sincere gratitude to all of these persons. We are pleased, indeed, to offer our heartfelt thanks to Mrs. Janie Wardle for her interest and cooperation in this project.

A. A. Martynyuk
V. G. Miladzhanov
Kiev-Andizhan, December 2012

Professor V.G.Miladzhanov died unexpectedly on May 13, 2013 at the age of 60. Scientific results of Professor Miladzhanov are associated with the development of the method of matrix Lyapunov functions for stability investigation of systems with quick and slow motions; stability analysis of large-scale systems under structural perturbations; stability analysis of large-scale discrete systems under structural perturbations; stability of impulsive system under structural perturbations. He proposed a method of construct- ing hierarchical matrix Lyapunov function for non-autonomous systems which is used in stability investigation of nonlinear mechanical systems. Besides, he wrote and published more than 150 scientific and methodological papers.

Cherished memory of the well-known scientist and a remarkable man Miladzhanov Vakhobzhon Ganizhonovich will linger on in the hearts of those who knew him and worked together with him.

A.A.Martynyuk
Kiev, August, 2014

NOTATION

R — the set of all real numbers

$R_+ = [0, +\infty) \subset R$ — the set of all nonnegative numbers

R^k — k-th dimensional real vector space

$R \times R^n$ — the Cartesian product of R and R^n

$G_1 \times G_1$ — topological product

(a, b) — open interval $a < t < b$

$[a, b]$ — closed interval $a \le t \le b$

$A \cup B$ — union of sets A and B

$A \cap B$ — intersection of sets A and B

$\overline{D}$ — closure of set D

∂D — boundary of set D

$N_\tau^+ \triangleq \{\tau_0, \ldots, \tau_0 + k, \ldots\}, \quad \tau_0 \ge 0, \quad k = 1, 2, \ldots$

$\{x\colon \Phi(x)\}$ — set of x's for which the proposition Φ is true

$\mathcal{T} = [-\infty, +\infty] = \{t\colon -\infty \le t \le +\infty\}$ — the largest time interval

$\mathcal{T}_\tau = [\tau, +\infty) = \{t\colon \tau \le t < +\infty\}$ — the right semi-open unbounded interval associated with τ

$\mathcal{T}_i \subseteq R$ — a time interval of all initial moments to under consideration (or, all admissible t_0)

$\mathcal{T}_0 = [t_0, +\infty) = \{t\colon t_0 \le t < +\infty\}$ — the right semi-open unbounded interval associated with t_0

$\|x\|$ — the Euclidean norm of vector x in R^n

$\chi(t; t_0, x_0)$ — a motion of a system at $t \in R$ iff $x(t_0) = x_0$, $\chi(t_0; t_0, x_0) \equiv x_0$

$B_\varepsilon = \{x \in R^n\colon \|x\| < \varepsilon\}$ — open ball with center at the origin and radius $\varepsilon > 0$

$\delta_M(t_0, \varepsilon) = \max\{\delta\colon \delta = \delta(t_0, \varepsilon) \ni x_0 \in B_\delta(t_0, \varepsilon) \Rightarrow \chi(t; t_0, x_0) \in B_\varepsilon,$

$\forall t \in \mathcal{T}_0\}$ — the maximal δ obeying the definition of stability

$\Delta_M(t_0) = \max\{\Delta\colon \Delta = \Delta(t_0),\ \forall \rho > 0,\ \forall x_0 \in B_\Delta,\ \exists \tau(t_0, x_0, \rho) \in (0, +\infty) \ni \chi(t; t_0, x_0) \in B_\rho,\ \forall t \in \mathcal{T}_\tau\}$ — the maximal Δ obeying the definition of attractivity

$\tau_m(t_0, x_0, \rho) = \min\{\tau\colon \tau = \tau(t_0, x_0, \rho) \ni \chi(t; t_0, x_0) \in B_\rho,\ \forall t \in \mathcal{T}_\tau\}$ — the minimal τ satisfying the definition of attractivity

$\mathcal{N}$ — a time-invariant neighborhood of original of R^n

$f\colon R \times \mathcal{N} \to R^n$ — a vector function mapping $R \times \mathcal{N}$ into R^n

$C(\mathcal{T}_\tau \times \mathcal{N})$ — the family of all functions continuous on $\mathcal{T}_\tau \times \mathcal{N}$

$C^{(i,j)}(\mathcal{T}_\tau \times \mathcal{N})$ — the family of all functions i-times differentiable on $\mathcal{T}_\tau$ and j-times differentiable on $\mathcal{N}$

$\mathcal{C} = C([-\tau, 0], R^n)$ — the space of continuous functions which map $[-\tau, 0]$ into R^n

$U(t, x)$, $U\colon \mathcal{T}_\tau \times R^n \to R^{s \times s}$ — matrix-valued Liapunov function, $s = 2, 3, \dots, m$

$V(t, x)$, $V\colon \mathcal{T}_\tau \times R^n \to R^s$ — vector Liapunov function

$v(t, x)$, $v\colon \mathcal{T}_\tau \times R^n \to R_+$ — scalar Liapunov function

$D^+v(t, x)\,(D^-v(t, x))$ — the upper right (left) Dini derivative of v along $\chi(t; t_0, x_0)$ at (t, x)

$D_+v(t, x)\,(D_-v(t, x))$ — the lower right (left) Dini derivative of v along $\chi(t; t_0, x_0)$ at (t, x)

$D^*v(t, x)$ — denotes that both $D^+v(t, x)$ and $D_+v(t, x)$ can be used

$Dv(t, x)$ — the Eulerian derivative of v along $\chi(t; t_0, x_0)$ at (t, x)

$\lambda_i(\cdot)$ — the i-th eigenvalue of a matrix $(\cdot)$

$\lambda_M(\cdot)$ — the maximal eigenvalue of a matrix $(\cdot)$

$\lambda_m(\cdot)$ — the minimal eigenvalue of a matrix $(\cdot)$

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