FEEDBACK CONTROL OF DYNAMIC SYSTEMS

FIFTH EDITION

GENE F. FRANKLIN
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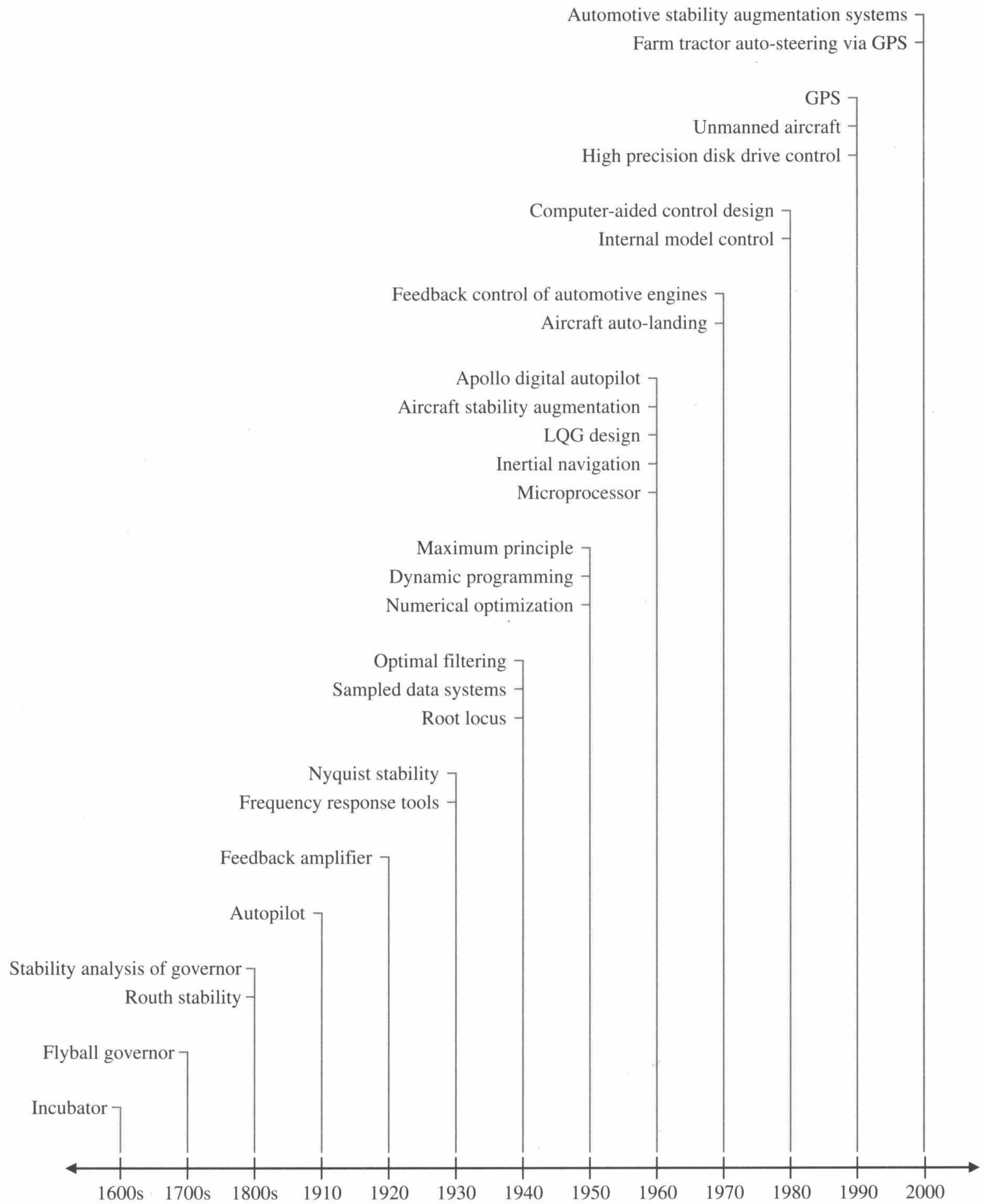
Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-at}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

Chronological History of Feedback Control



To Gertrude, David, Carole,
Valerie, Daisy, Annika, Davenport,
Malahat, Sheila, and Nima

Preface

In this fifth edition we again present a text in support of a first course in control and have retained the best features of our earlier editions. For this edition, we have substantially rewritten Chapter 4 on the basic properties of feedback, placing the material in a more logical order and presenting it in a much more effective manner. We have also included a new Chapter 9 on nonlinear systems. In this chapter we have collected the bits and pieces of nonlinear problems into one place, organized them into a coherent story, and added new material in order to give a careful first introduction to this most important area. With this organization, the distractions caused by having the optional material on nonlinear systems scattered throughout the book are removed. Those instructors who wish to introduce the material earlier can easily do so, of course.

The basic structure of the book is unchanged, and we continue to combine analysis with design using the three approaches of the root locus, frequency response, and state variable equations. The text continues to include many carefully worked out examples to illustrate the material. As before, we provide a set of review questions at the end of each chapter with answers in the back of the book to assist the students in verifying that they have learned the material.

In the three central chapters on design methods we continue to expect the students to learn how to perform the basic calculations by hand and make a quick sketch of a root locus or Bode plot as a guide to design. However, we introduce the use of MATLAB® early on in recognition of the universal use of software tools in control analysis and design. Furthermore, in recognition of the fact that increasingly controllers are implemented in embedded computers, we again introduce digital control in Chapter 4 and in a number of cases compare the responses of feedback systems using analog controllers with those having a digital "equivalent" controller. As before, we have prepared a collection of all the MATLAB "m" files used to produce the figures in the book and these are available under "publications" at the following website:

http://www.scsolutions.com/feedbackcontrol.html

The case studies are now in Chapter 10, including control of the read–write head assembly of a computer hard disk and temperature control of a silicon wafer in a Rapid Thermal Processor used in the fabrication of integrated circuits.

We feel that this fifth edition presents the material with good pedagogical support, provides strong motivation for the study of control, and represents a solid foundation for meeting the educational challenges of introducing study of feedback control.

Addressing the Educational Challenges

Some of the educational challenges facing students of feedback control are long-standing; others have emerged in recent years. Some of the challenges remain for students across their entire engineering education; others are unique to this relatively sophisticated course. Whether they are old or new, general or particular, the educational challenges we perceived were critical to the evolution of this text. Here we will state several educational challenges and describe our approaches to each of them.

- CHALLENGE *Students must master design as well as analysis techniques.*

Design is central to all of engineering and especially so to control systems. Students find that design issues, with their corresponding opportunities to tackle practical applications, particularly motivating. But students also find design problems difficult because design problem statements are usually poorly posed and lack unique solutions. Because of both its inherent importance for and its motivational effect on students, design is emphasized throughout this text so that confidence in solving design problems is developed from the start.

The emphasis on design begins in Chapter 4 following the development of modeling and dynamic response. The basic idea of feedback is introduced first, showing its influence on disturbance rejection, tracking accuracy, and robustness to parameter changes. The design orientation continues with uniform treatments of the root locus, frequency response, and state variable feedback techniques. All the treatments are aimed at providing the knowledge necessary to find a good feedback control design with no more complex mathematical development than is essential to clear understanding.

Throughout the text, examples are used to compare and contrast the design techniques afforded by the different design methods and, in the capstone case studies of Chapter 10, complex real-world design problems are tackled using all the methods in a unified way.

- CHALLENGE *New ideas continue to be introduced into control.*

Control is an active field of research and hence there is a steady influx of new concepts, ideas, and techniques. In time, some of these elements develop to the point where they join the list of things every control engineer must know. This text is devoted to supporting students equally in their need to grasp both traditional and more modern topics.

In each of our editions we have tried to give equal time to root locus, frequency response, and state variable methods for design. In this edition we continue to emphasize solid mastery of the underlying techniques. coupled with computer-based methods for detailed calculation. We also provide an early introduction to data sampling and discrete controllers in recognition of the major role played by digital controllers in our field. While this material can be skipped to save time without harm to the flow of the text, we feel that it is very important for students to understand that computer control is widely used and that the most basic techniques of computer control are easily mastered.

- CHALLENGE *Students need to manage a great deal of information.*

The vast array of systems to which feedback control is applied and the growing variety of techniques available for the solution of control problems means that today's student of feedback control must learn many new ideas. How do students keep their perspective as they plow through lengthy and complex textual passages? How to do they identify highlights and draw appropriate conclusions? How do they review for exams? Helping students with these tasks was a criterion for the fourth edition and continues be addressed in this fifth edition. We outline these features below.

FEATURE	REFERENCE EXAMPLE
Chapter openers offer perspective and overview. They place the specific chapter topic in the context of the discipline as a whole and they briefly overview the chapter sections.	Chapter 3 opener, pp. 94–95
Margin notes help students scan for chapter highlights. They point to important definitions, equations, and concepts.	pp. 49–50
Boxed highlights identify key concepts within the running text. They also function to summarize important design procedures.	Advantage of feedback, p. 206; compensation design, p. 440
Bulleted chapter summaries help with student review and prioritization. These summaries briefly reiterate the key concepts and conclusions of the chapter.	Chapter 2 summary, pp. 77–78
Synopsis of design aids. Relationships used in design and throughout the book are collected in one place for easy reference.	Inside back cover
The *color blue* is used (1) to highlight useful; pedagogical features; (2) to highlight components under particular scrutiny within block diagrams; (3) to distinguish curves on graphs; and (4) to lend a more realistic look to figures of physical systems.	Fig. 5.43, p. 330 Fig. 2.9, p. 32
Review questions at the end of each chapter with solutions in the back guide the student in self-study.	Chapter 2, p. 78

- CHALLENGE *Students of feedback control come from a wide range of disciplines.*

Feedback control is an interdisciplinary field in that control is applied to systems in every conceivable area of engineering. Consequently, some schools have separate introductory courses for control within the standard disciplines and some, like Stanford, have a single set of courses taken by students from many disciplines. However, to restrict the examples to one field is to miss much of the range and power of feedback but to cover the whole range of applications is overwhelming. In this book we develop the interdisciplinary nature of the field and provide review material for several of the most common technologies so that students from many disciplines will be comfortable with the presentation. For electrical engineering students who typically have a good background in transform analysis, we include in Chapter 2 an introduction to writing equations of motion for mechanical mechanisms. For mechanical engineers, we include in Chapter 3 a review of the Laplace Transform and dynamic response as needed in control. In addition, we introduce other technologies briefly and, from time to time, we present the equations of motion of a physical system without derivation but with enough physical description to be understood from a response point of view. Examples of some of the physical systems represented in the text include the read–write head for a computer disk drive, a satellite tracking system, the fuel–air ratio in an automobile engine, and an airplane automatic pilot system.

Outline of the Book

The contents of the book are organized into ten chapters and seven appendixes. Optional sections of advanced or enrichment material marked with a triangle (▲) are included at the end of some chapters. Examples and problems based on this material are also marked with a triangle (▲). The appendixes include background and reference material such as Laplace transform tables, a review of complex variables, a review of matrix theory, and answers to the end-of-chapter review questions.

In Chapter 1, the essential ideas of feedback and some of the key design issues are introduced. This chapter also contains a brief history of control, from the ancient beginnings of process control to flight control and electronic feedback amplifiers. It is hoped that this brief history will give a context for the field, introduce some of the key figures who contributed to its development, and provide motivation to the student for the studies to come.

Chapter 2 is a short presentation of dynamic modeling and includes mechanical, electrical, electromechanical, fluid, and thermodynamic devices. This material can be omitted, used as the basis of review homework to smooth out the usual nonuniform preparation of students, or covered in depth depending on the needs of the students.

Chapter 3 covers dynamic response as used in control. Again, much of this material may have been covered previously, especially by electrical engineering students. For many students, the correlation between pole locations and transient response and the effects of extra zeros and poles on dynamic response is new material. Stability of dynamic systems is also introduced in this chapter. This material needs to be covered carefully.

Chapter 4 presents the basic equations and transfer functions of feedback along with the definitions of the sensitivity and complementary sensitivity functions. With these tools, open-loop and closed-loop control are compared with respect to disturbance rejection, tracking accuracy, and sensitivity to model errors. Classification of systems according to their ability to track polynomial reference signals or to reject polynomial disturbances is described with the concept of system type. Finally, the classical proportional, integral, and derivative (PID) control structure is introduced and the influence of the controller parameters on a system's characteristic equation is explored. The end-of-chapter optional sections treat digital control, PID tuning, and time response sensitivity.

Following the overview of feedback in Chapter 4, the core of the book presents the design methods based on root locus, frequency response, and state variable feedback in Chapters 5, 6, and 7, respectively.

Chapter 8 develops in more detail the tools needed to design feedback control for implementation in a digital computer. However, for a complete treatment of feedback control using digital computers, the reader is referred to the companion text, *Digital Control of Dynamic Systems*, by Franklin, Powell, and Workman.

In Chapter 9 the nonlinear material includes techniques for the linearization of equations of motion, analysis of zero memory nonlinearity as a variable gain, frequency response as a describing function, the phase plane, Lyapunov stability theory, and the circle stability criterion.

In Chapter 10 the three primary approaches are integrated in several case studies and a framework for design is described that includes a touch of the real-world context of practical control design.

Course Configurations

The material in this text can be covered flexibly. Most first-course students in controls will have some dynamics and Laplace transforms. Therefore, Chapter 2 and most of Chapter 3 would be a review for those students. In a 10-week quarter, it is possible to review Chapter 3, and all of Chapters 1, 4, 5, and 6. Most optional sections should be omitted. In the second quarter, Chapters 7, and 9 can be covered comfortably including the optional sections. Alternatively, some optional sections could be omitted and selected portions of Chapter 8 included. A semester course should comfortably accommodate Chapters 1–7, including the review material of Chapters 2 and 3, if needed. If time remains after this core coverage, some introduction of digital control from Chapter 8, selected nonlinear issues from Chapter 9, and some of the case studies from Chapter 10 may be added.

The entire book can also be used for a three-quarter sequence of courses consisting of modeling and dynamic response (Chapters 2 and 3), classical control (Chapters 4, 5, and 6), and modern control (Chapters 7, 8, 9, and 10).

Two basic 10-week courses are offered at Stanford and are taken by seniors and first-year graduate students who have not had a course in control, mostly in the departments of Aeronautics and Astronautics, Mechanical Engineering, and Electrical Engineering. The first course reviews Chapters 2 and 3 and covers Chapters 4, 5, and 6. The more advanced course is intended for graduate students and reviews Chapters 4, 5, and 6 and covers Chapters 7, 8, and 10. This sequence complements a graduate course in linear systems and is the prerequisite to courses in digital control, nonlinear control, optimal control, flight control, and smart product design. Several of the subsequent courses include extensive laboratory experiments. Prerequisites for the course sequence include dynamics or circuit analysis and Laplace transforms.

Prerequisites to This Feedback Control Course

This book is for a first course at the senior level for all engineering majors. For the core topics in Chapters 4–7, prerequisite understanding of modeling and dynamic response is necessary. Many students will come into the course with sufficient background in those concepts from previous courses in physics, circuits, and dynamic response. For those needing review, Chapters 2 and 3 should fill in the gaps.

An elementary understanding of matrix algebra is necessary to understand the state-space material. While all students will have much of this in prerequisite math courses, a review of the basic relations is given in Appendix C and a brief treatment of particular material needed in control is given at the start of Chapter 7. The emphasis is on the relations between linear dynamic systems and linear algebra.

Supplements

The website mentioned earlier includes the m files used to generate all the MATLAB figures in the book and these may be copied and distributed to the students as desired. An instructor's manual with complete solutions to all homework problems is available to adopters of the fifth edition.

Acknowledgments

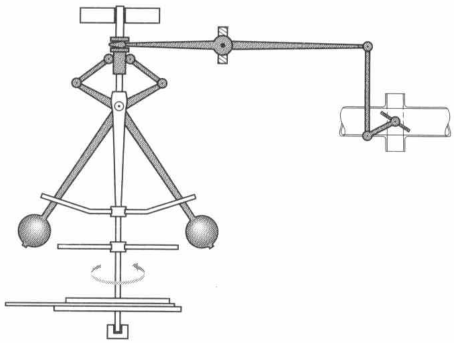
Finally, we wish to acknowledge our great debt to all those who have contributed to the development of feedback control into the exciting field it is today and specifically to the considerable help and education we have received

from our students and our colleagues. In particular, we have benefited in this effort by many discussions with the following who taught introductory control at Stanford: A. E. Bryson, Jr., R. H. Cannon, Jr., D. B. DeBra, S. Rock, S. Boyd, C. Tomlin and P. Enge.

Special thanks go to the many students who have provided almost all the solutions to the problems in the book.

G.F.F.
J.D.P.
A.E.-N.
Stanford, California

1 An Overview and Brief History of Feedback Control



A Perspective on Feedback Control

Control of dynamic systems is a very common concept with many characteristics. A system that involves a person controlling a machine, as in driving an automobile, is called **manual control**. A system that involves machines only, as when room temperature can be set by a thermostat, is called **automatic control**. Systems designed to hold an output steady against unknown disturbances are called **regulators** while systems designed to track a reference signal are called **tracking** or **servo** systems. Control systems are also classified according to the information used to compute the controlling action. If the controller does *not* use a measure of the system output being controlled in computing the control action to take, the system is called **open-loop control**. If the controlled output signal *is* measured and fed back for use in the control computation, the system is called closed-loop or **feedback control**. There are many other important properties of control systems in addition to these most basic characteristics. For example, in this book we will mainly be concerned

open-loop control

feedback control

Contents