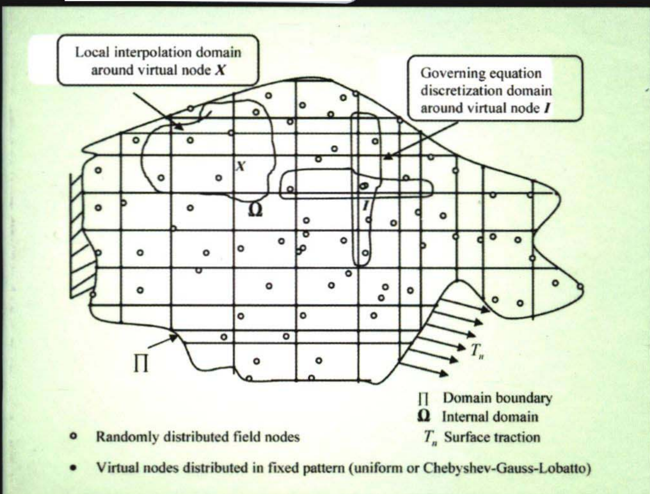
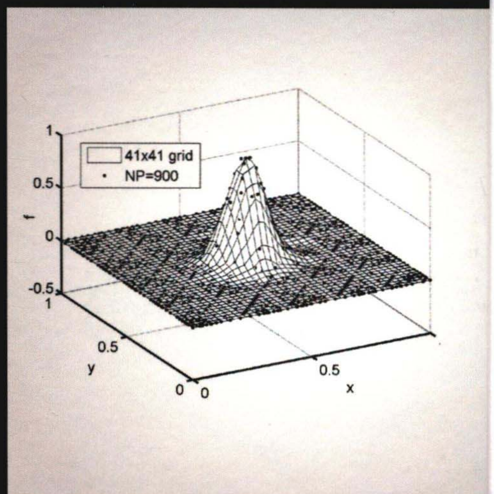
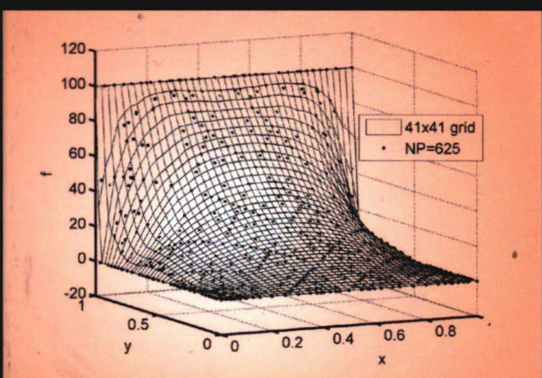
MESHLESS METHODS AND THEIR NUMERICAL PROPERTIES







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MESHLESS METHODS AND THEIR NUMERICAL PROPERTIES

Dedicated first and foremost to my motherland,
and to Duer, Anne, and my parents

Hua Li

Dedicated to my parents, and Janaki, Kaustubh,
Supriti, and little angel Aadhip

Shantanu S. Mulay

Preface

For a long time, the finite element method (FEM) has been a standard tool for numerically solving a wide range of engineering problems. Today's real-world problems are becoming so highly complex that FEM alone is inadequate to solve them. Some of the limitations are listed below.

- It is not easy for FEM to generate a good quality mesh that should be correct according to the geometry and the specific requirements of a physical phenomenon. This problem will become more serious when we solve large deformation problems such as crack propagation, astrophysics phenomena, and extrusion.
- It is difficult for FEM to treat discontinuities properly as this process depends on mesh quality. Therefore, computational results may be incorrect due to high discontinuity if mesh is distorted.
- Adaptive meshing in FEM may cause degradation of accuracy in complex programs. It is impractical to solve systems of equations based on billions of elements.
- FEM is required to handle a lot of geometry degeneracy cases to generate correct and good quality mesh for complex geometry. This makes programs highly complex and slow while running.
- Mesh generation and mesh refinement in FEM are computationally expensive.

Due to these limitations of FEM, it is necessary to develop other numerical techniques that should be able to solve complex problems without generating mesh. A technique should also correctly handle governing equations by conserving essential parameters. As a result, the meshless or meshfree method is proposed as one such numerical technique to overcome these limitations.

Meshless methods have been developed in the past decade, and significant progress has been achieved recently for numerical computations of wide ranging engineering problems. These meshless methods do not require mesh for discretisation of problem domains, and they construct the approximate functions only via a set of nodes where no element is required for approximation of functions. They overcome the limitations of FEM. Several examples of the advantages of the meshless methods include

Computational cost is reduced significantly since no mesh is required.
Higher computational accuracy is achieved easily by simply adding nodes, especially for cases where more refinement is required.
High-order shape functions are constructed easily.
Compared with FEM, the meshless methods can easily handle large deformation and strongly nonlinear problems, since the connectivity among the nodes is generated as a portion of computation and it can change with time.

To date, about 15 books have been published on meshless methods. However, the authors focused primarily on methods that they developed. It is thus really necessary to publish a handbook type volume that provides the complete mathematical formulations for each of the most important and classic meshless methods that are well known and widely accepted and cover recent developments. It is also necessary to demonstrate a rigorous mathematical treatment of the numerical properties of meshless methods that will give sufficient confidence to users about the capabilities of particular meshless methods. This is especially important for readers who are interested in the individual meshless methods and seek full background information about all the most important and classic methods. This information will also be useful to an individual researcher who wants to embark on a journey of meshless method development.

A comprehensive introduction of the most important and classic meshless methods through complete mathematical formulations is thus warranted to provide overall insight into the meshless methods, theoretical understanding of the difference between FEM and meshless methods, and explanations of the detailed numerical computational characteristics of the methods. However, as noted, there is a lack of comprehensive publications on the formulations of the most important and classic methods. This monograph is thus written to systematically document the most important and classic meshless methods and the analyses of numerical properties.

In this book, the introduction for each of the most important and classic meshless methods is provided along with the complete mathematical formulations. In total, it presents 19 meshless methods, including the authors' recent contribution, in detail with full mathematical formulations and performance studies for the methods developed by the authors showing numerical properties such as convergence, consistency, stability, and adaptive analyses systematically.

Several engineering applications of the meshless methods are also included, for example, the CAD designing of MEMS devices, the nonlinear fluid structure analysis of near-bed submarine pipelines, and two-dimensional multiphysics simulations of pH-sensitive hydrogels.

This is the first monograph of its kind in which a comprehensive and systematic introduction of the most important and classic meshless methods is provided by complete mathematical formulations with full development information, although this is not the first book about meshless methods. It also covers the recent development of the meshless methods, mainly contributed by the authors. The methods are fully formulated mathematically and their numerical properties, such as convergence, consistency, stability, and adaptivity are studied in detail. Further, the benchmark results for engineering applications of the methods are also documented. Finally, this monograph is written in as simple a manner as possible so that it is informative and easy reading for researchers and can also serve as a rich reference source, for example, as a handbook for a graduate student who intends to work in the area of numerical computational techniques.

This monograph is intended to meet the needs of scientists and engineers in the broad areas of computational science and engineering. It will be especially useful for them as a reference book, and also if they wish to conduct further studies to extend their work to modeling and simulation of practical engineering problems. Another important primary audience is postgraduate students in the areas of computational theory, numerical methods, and discrete mathematics, especially those involved in developing new high-performance numerical methods. Possible secondary audiences include undergraduate students taking advanced numerical analysis courses covering discrete numerical analysis and methods. The chapters on the formulation of the selected classic meshless methods will be especially useful to these students. Correspondingly, course lecturers will also find this book a good reference source.

This book provides both casual and interested readers with insight into the special features and intricacies of meshless methods. It will also be invaluable to design engineers using CAD software for modeling and simulation of a wide range of engineering problems, serving as a useful reference containing benchmark formulations to compare and verify other numerical methods.

The authors would like to thank Professor Tom Hou of Caltech for his guidance in the stability analysis of the RDQ method, and Professors Khin-Yong Lam, Gui-Rong Liu, and J. N. Reddy for their constant support and

encouragement. The authors would also like to thank J.Q. Cheng and Q.X. Wang for their invaluable contributions to this research.

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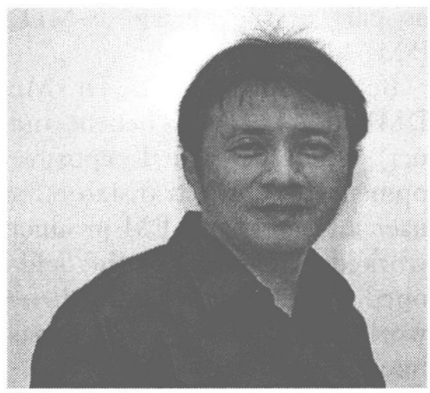
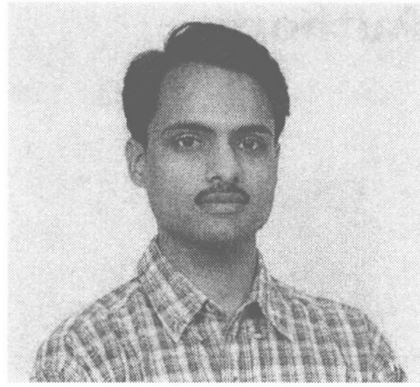


Dr. Hua Li earned BSc and MEng degrees in engineering mechanics from the Wuhan University of Technology, in the Peoples Republic of China in 1982 and 1987, respectively. He obtained his PhD in mechanical engineering from the National University of Singapore in 1999. From 2000 to 2001, Dr. Li was a postdoctoral associate at the Beckman Institute for Advanced Science and Technology, University of Illinois at Urbana-Champaign in the United States. In late 2005, he served as an invited visiting scientist in the Department of Chemical and Biomolecular Engineering of Johns Hopkins University. From 2001 to 2006, he was a research scientist at the A*STAR Institute of High Performance Computing.

Dr. Li is currently an assistant professor at the School of Mechanical and Aerospace Engineering at Nanyang Technological University. His research interests include the modeling and simulation of MEMS, focusing on the use of smart hydrogels in BioMEMS applications; the development of advanced numerical methodologies; and the dynamics of high-speed rotating shell structures.

He is the sole author of a monograph entitled *Smart Hydrogel Modeling* published by Springer. He has co-authored a book entitled *Rotating Shell Dynamics* published by Elsevier and book chapters on MEMS simulation and hydrogel drug delivery system modeling. He has authored or co-authored over 110 articles published in top international peer-reviewed journals. His research has been extensively funded by agencies and industries and he acted as the principal investigator of a computational BioMEMS project awarded under A*STAR's Strategic Research Programme in MEMS.



Dr. Shantanu S. Mulay earned a BEng in mechanical engineering from the Maharastra Institute of Technology of the University of Pune, India in 2001 and a PhD in mechanical engineering from Nanyang Technological University (NTU) in Singapore in 2011.

Since August 2010, Dr. Mulay has worked as a postdoctoral associate with Professor Rohan Abeyaratne of the Massachusetts Institute of Technology as part of the Singapore–MIT Alliance for Research and Technology (SMART).

Before joining NTU, Dr. Mulay worked in product enhancement of DMU (CATIA workbench) and the development of NISA (FEM product), where he gained exposure to a variety of areas such as the development of CAD translators, computational geometry, and handling user interfaces of FEM products. During his PhD program, Dr. Mulay worked extensively in the field of computational mechanics and developed a meshless random differential quadrature (RDQ) method. His work on the RDQ method has culminated in several chapters of this book.

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