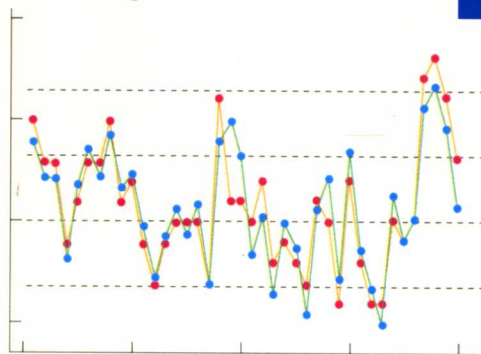


高等学校教材系列

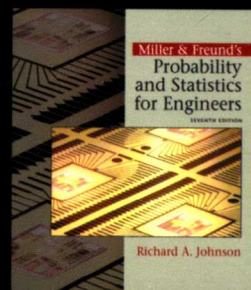
PEARSON
Prentice
Hall



概率论与数理统计

(第七版) (英文版)

Miller & Freund's
Probability and Statistics
for Engineers Seventh Edition



[美] Richard A. Johnson 著

章栋恩 改编



电子工业出版社
Publishing House of Electronics Industry
<http://www.phei.com.cn>

高等学校教材系列

概率论与数理统计

(英文版)(第七版)

Miller & Freund's
Probability and Statistics for Engineers

Seventh Edition

[英] Richard A. Johnson 著

章栋恩 改编

电子工业出版社
Publishing House of Electronics Industry
北京·BEIJING

内 容 简 介

这是一本为工程、物理等专业大学低年级学生准备的入门级教材。本书讲述了统计分布、概率密度、抽样分布、均值、方差、曲线拟合等概率论与数理统计中涉及到的基本概念。其特点是内容全面、编排合理，各章之间的衔接较为密切，附录中还提供了大量的常用统计表。全书用丰富的例子详细说明每一个概念，而省略了不必要的理论证明，例子中的数据大多来自作者的亲身经历及实际的案例。

本书适合作为大学工程、物理等专业一、二年级学生的教材，也适合作为工程技术人员的参考材料。

Authorized Adaptation from the English language edition, entitled Miller & Freund's Probability and Statistics for Engineers, 7th Edition, 0131437453 by Johnson, Richard; Miller, Irwin; Freund, John, published by Pearson Education, Inc, publishing as Prentice Hall, Copyright © 2005 by Pearson Education, Inc.

All rights reserved. No part of this book may be reproduced or transmitted in any form or by any means, electronics or mechanical, including photocopying, recording or by any information storage retrieval system, without permission from Pearson Education, Inc.

English language adaptation edition published by Pearson Education Asia Ltd. and Publishing House of Electronics Industry, Copyright © 2005.

English language adaptation edition is manufactured in the People's Republic of China, and is authorized for sale only in the Peoples's Republic of China excluding Hong Kong and Macau.

本书影印改编版由电子工业出版社和 Pearson Education 培生教育出版亚洲有限公司合作出版。未经出版者预先书面许可，不得以任何方式复制或抄袭本书的任何部分。

本书影印改编版在中国大陆地区生产，仅限于在中国大陆地区销售。

本书影印改编版贴有 Pearson Education 培生教育出版集团激光防伪标签，无标签者不得销售。

版权贸易合同登记号 图字：01-2005-5659

图书在版编目 (CIP) 数据

概率论与数理统计 = Miller & Freund's Probability and Statistics for Engineers: 第7版 / (美) 约翰逊 (Johnson, R. A.) 著; 章栋恩改编. - 北京: 电子工业出版社, 2005.12

(高等学校教材系列)

ISBN 7-121-01931-0

I. 概... II. ①约... ②章... III. ①概率论-高等学校-教材-英文 ②数理统计-高等学校-教材-英文 IV. O21
中国版本图书馆 CIP 数据核字 (2005) 第 131048 号

责任编辑: 许菊芳

印 刷: 北京市天竺颖华印刷厂

出版发行: 电子工业出版社

北京市海淀区万寿路 173 信箱 邮编: 100036

经 销: 各地新华书店

开 本: 787 × 980 1/16 印张: 36.25 字数: 812 千字

印 次: 2005 年 12 月第 1 次印刷

定 价: 58.00 元

凡购买电子工业出版社的图书, 如有缺损问题, 请向购买书店调换; 若书店售缺, 请与本社发行部联系。联系电话: (010) 68279077。质量投诉请发邮件至 zltts@phei.com.cn, 盗版侵权举报请发邮件至 dbqq@phei.com.cn。

Preface

This book has been written for an introductory course in probability and statistics for students of engineering and the physical sciences. Each chapter is keynoted by an introductory statement and has a checklist of key terms (with page numbers) at the end. Important formulas, theorems, and rules are set out in boxes. Confidence intervals are stressed as the major procedure for making inferences. Their properties are carefully described and their interpretation is frequently discussed. The steps for hypothesis testing are clearly and consistently delineated in each application. The examples include the interpretation and calculation of a P -value.

The text has been tested extensively in courses for university students as well as by in-plant training of engineers. The whole book can be covered in a two-semester or three-quarter course consisting of three lectures a week. The book also makes an excellent basis for a one-semester course where the lecturer can choose topics to emphasize theory or application. The author covers most of the first seven chapters, straight-line regression, and the graphic presentation of factorial designs in one semester.

In this seventh edition, we have continued to build on the strengths of the previous editions while adding a few more actual data sets and examples of the application of statistics in scientific investigations. The new data sets, like many of those already in the text, arose in the author's consulting activities or in discussions with scientists and engineers about their statistical problems. Data from some companies have been disguised, but they still retain all of the features necessary to illustrate the statistical methods and the reasoning required to make generalizations from data collected in an experiment. A set of statistical guidelines for correctly applying statistical procedures, and avoiding common pitfalls, has been added to each chapter under the title *Do's and Don'ts*.

To give students an early preview of statistics, descriptive statistics are covered in Chapter 2. Chapters 3 through 6 provide a brief, though rigorous, introduction to the theory of statistics, are suitable for an introductory semester (or quarter) course on probability and statistics. Chapters 7, 8, and 9 are the core material on the key concepts and elementary methods of statistical inference. Chapters 11, 12, and 13 comprise an introduction to some of the standard, though more advanced, topics of experimental design and regression.

The mathematical background expected of the reader is a year course in calculus; actually, calculus is required mainly for Chapter 5 dealing with basic distribution theory in the continuous case.

We wish to thank MINITAB (State College, Pennsylvania) for permission to include commands and output from their MINITAB software program, and

the SAS institute (Gary, North Carolina) for permission to include output from their SAS program.

Thanks go to Paul M. Berthouex, Michael Brajac, Smiley Cheng, Don Ermer, Jim Evans, Cheryl Hatfield, David Steinberg, and Steve Verrill for contributing data sets in earlier editions. Kuo-Tsung Wu provided valuable help for improving the presentation. Thanks also to Song Yong Sim and Erik Johnson for their help. The current edition benefited from suggestions from George Lobell, Prentice Hall, and the input of the reviewers: Bradford R. Crain, Portland State University; James G. Surles, Texas Tech University; Ulric Lund, California Polytechnic State University; Peter Macdonald, McMaster University; and Jeffrey Proehl. Thanks also go to Ting-Li Lin, who helped integrate some of new MINITAB 14 output into the manuscript and to Moo Chung who helped with some software issues. I am most grateful to Ruojia Li for her considerable help, which enabled this revision to be completed on time.

All revisions in this edition were the responsibility of R. A. Johnson.

Richard A. Johnson
rich@stat.wisc.edu

教学支持说明

本书系我社获全球最大的教育出版集团——Pearson Education Group 独家授权之英文改编版。

Pearson Education 旗下的国际知名教育图书出版公司 Prentice Hall 的高品质的基础科学类出版物是全美及全球高校采用率最高的教材, 享誉全球教育界、工商界和技术界。为秉承 Prentice Hall 出版公司对于其教材类产品的一贯教学支持, Pearson Education 将向采纳本书作为教材的教师免费提供网上教学支持课件。任何一位注册的教师都可直接下载所有在线的教学辅助资料。

为确保此资源仅为教师教学所使用, 烦请填写如下情况调查表, 并回寄或传真给电子工业出版社。

证 明

兹证明_____大学_____系 / 院_____专业_____学年 (学期)
开设的_____课程, 共_____学时, 现采用电子工业出版社出版的英文原版 / 简体中文
版_____ (书名 / 作者) 作为主要教材。任课教师为_____, 学生_____个班共_____人。

任课教师需要与本书配套的教学辅助资料。

电 话: _____

传 真: _____

E-mail: _____

联系地址: _____

邮 编: _____

建议和要求:

系 / 院主任: _____ (签字)

(系 / 院办公室章)

____年____月____日

本书还配有其他教学辅导资料, 相关事宜敬请访问 Prentice Hall 的相关网站: <http://www.prenhall.com>。

请与我们联系

Publishing House of Electronics Industry

电子工业出版社: www.phei.com.cn

www.hxedu.com.cn

北京市万寿路 173 信箱外版教材事业部

邮编: 100036

电话: 010-88254555

传真: 010-88254560

E-mail: Te_service@phei.com.cn

Pearson Education Beijing Office

培生教育出版集团北京办事处

北京市西三环北路 19 号外研大厦 2202 室

邮编: 100089

电话: 010-88819178

传真: 010-88819170

E-mail: service@pearsoned.com.cn

Contents

1 Introduction 1

- 1.1 Why Study Statistics? 1
- 1.2 Modern Statistics 2
- 1.3 Statistics and Engineering 3
- 1.4 A Case Study: Visually Inspecting Data to Improve Product Quality 3
- 1.5 Two Basic Concepts—Population and Sample 6
 - Do's and Don'ts 10
 - Review Exercises 10

2 Treatment of Data 13

- 2.1 Pareto Diagrams and Dot Diagrams 13
- 2.2 Frequency Distributions 16
- 2.3 Graphs of Frequency Distributions 19
- 2.4 Stem-and-leaf Displays 22
- 2.5 Descriptive Measures 28
- 2.6 Quartiles and Percentiles 33
- 2.7 The Calculation of \bar{x} and s 38
- 2.8 A Case Study: Problems with Aggregating Data 45
 - Do's and Don'ts 48
 - Review Exercises 48

3 Probability 53

- 3.1 Sample Spaces and Events 53
- 3.2 Counting 58
- 3.3 Probability 65
- 3.4 The Axioms of Probability 67
- 3.5 Some Elementary Theorems 70
- 3.6 Conditional Probability 78

- 3.7 Bayes' Theorem 85
- 3.8 Mathematical Expectation and Decision Making 91
 - Do's and Don'ts 96
 - Review Exercises 97

4 Probability Distributions 101

- 4.1 Random Variables 101
- 4.2 The Binomial Distribution 104
- 4.3 The Hypergeometric Distribution 110
- 4.4 The Mean and the Variance of a Probability Distribution 115
- 4.5 Chebyshev's Theorem 122
- 4.6 The Poisson Approximation to the Binomial Distribution 127
- 4.7 Poisson Processes 130
- 4.8 The Geometric Distribution 133
- 4.9 The Multinomial Distribution 136
- 4.10 Simulation 138
 - Do's and Don'ts 141
 - Review Exercises 142

5 Probability Densities 146

- 5.1 Continuous Random Variables 146
- 5.2 The Normal Distribution 153
- 5.3 The Normal Approximation to the Binomial Distribution 160
- 5.4 Other Probability Densities 165
- 5.5 The Uniform Distribution 165
- 5.6 The Log-Normal Distribution 166
- 5.7 The Gamma Distribution 169
- 5.8 The Beta Distribution 171
- 5.9 The Weibull Distribution 173
- 5.10 Joint Distributions—Discrete and Continuous 176
- 5.11 Checking if the Data Are Normal 190
- 5.12 Transforming Observations to Near Normality 193
- 5.13 Simulation 195
 - Do's and Don'ts 199
 - Review Exercises 200

6 Sampling Distributions 203

- 6.1 Populations and Samples 203
- 6.2 The Sampling Distribution of the Mean (σ known) 207

- 6.3** The Sampling Distribution of the Mean (σ unknown) 216
- 6.4** The Sampling Distribution of the Variance 218
 - Do's and Don'ts 222
 - Review Exercises 223

7 Inferences Concerning Means 226

- 7.1** Point Estimation 226
- 7.2** Interval Estimation 231
- 7.3** Tests of Hypotheses 238
- 7.4** Null Hypotheses and Tests of Hypotheses 240
- 7.5** Hypotheses Concerning One Mean 246
- 7.6** The Relation Between Tests and Confidence Intervals 251
- 7.7** Operating Characteristic Curves 252
- 7.8** Inference Concerning Two Means 260
- 7.9** Design Issues—Randomization and Pairing 272
 - Do's and Don'ts 274
 - Review Exercises 276

8 Inferences Concerning Variances 281

- 8.1** The Estimation of Variances 281
- 8.2** Hypotheses Concerning One Variance 284
- 8.3** Hypotheses Concerning Two Variances 286
 - Do's and Don'ts 290
 - Review Exercises 290

9 Inferences Concerning Proportions 292

- 9.1** Estimation of Proportions 292
- 9.2** Hypotheses Concerning One Proportion 298
- 9.3** Hypotheses Concerning Several Proportions 300
- 9.4** Analysis of $r \times c$ Tables 308
- 9.5** Goodness of Fit 311
 - Do's and Don'ts 316
 - Review Exercises 317

10 Nonparametric Tests 320

- 10.1** Introduction 320
- 10.2** The Sign Test 321

- 10.3** Rank-Sum Tests 322
- 10.4** Tests of Randomness 328
- 10.5** The Kolmogorov-Smirnov and Anderson-Darling Tests 331
 - Do's and Don'ts 334
 - Review Exercises 335

11 Curve Fitting 337

- 11.1** The Method of Least Squares 337
- 11.2** Inferences Based on the Least Squares Estimators 344
- 11.3** Curvilinear Regression 358
- 11.4** Multiple Regression 363
- 11.5** Checking the Adequacy of the Model 367
- 11.6** Correlation 374
- 11.7** Multiple Linear Regression (Matrix Notation) 387
 - Do's and Don'ts 391
 - Review Exercises 392

12 Analysis of Variance 397

- 12.1** Some General Principles 397
- 12.2** Completely Randomized Designs 400
- 12.3** Randomized-Block Designs 417
- 12.4** Multiple Comparisons 425
- 12.5** Some Further Experimental Designs 431
- 12.6** Analysis of Covariance 439
 - Do's and Don'ts 445
 - Review Exercises 445

13 Factorial Experimentation 450

- 13.1** Two-Factor Experiments 450
- 13.2** Multifactor Experiments 459
- 13.3** 2^n Factorial Experiments 470
- 13.4** The Graphic Presentation of 2^2 and 2^3 Experiments 478
- 13.5** Confounding in a 2^n Factorial Experiment 494
- 13.6** Fractional Replication 498
 - Do's and Don'ts 504
 - Review Exercises 504

Appendix A	Bibliography	510
Appendix B	Statistical Tables	511
Appendix C	Answers to Odd-Numbered Exercises	548
Index		564

Chapter 1

Introduction

Everything dealing with the collection, processing, analysis, and interpretation of numerical data belongs to the domain of statistics. In engineering, this includes such diversified tasks as calculating the average length of the downtimes of a computer, collecting and presenting data on the numbers of persons attending seminars on solar energy, evaluating the effectiveness of commercial products, predicting the reliability of a rocket, or studying the vibrations of airplane wings.

In Sections 1.2, 1.3 and 1.4 we discuss the recent growth of statistics and, in particular, its applications to problems of engineering. Statistics plays a major role in the improvement of quality of any product or service. An engineer using the techniques described in this book can become much more effective in all phases of work relating to research, development, or production. In Section 1.5 we begin our introduction to statistical concepts by emphasizing the distinction between a population and a sample.

CHAPTER OUTLINE

- 1.1 Why Study Statistics? 1**
- 1.2 Modern Statistics 2**
- 1.3 Statistics and Engineering 3**
- 1.4 A Case Study: Visually Inspecting Data to Improve Product Quality 3**
- 1.5 Two Basic Concepts—Population and Sample 6**
- Review Exercises 10**
- Key Terms 12**

1.1 Why Study Statistics?

Answers provided by statistical analysis can provide the basis for making decisions or choosing actions. For example, city officials might want to know whether the level of lead in the water supply is within safety standards. Because not all of the water can be checked, answers must be based on the partial information from samples of water that are collected for this purpose. As another example, a civil engineer must determine the strength of supports for generators at a power plant. A number of those available must be loaded to failure, and their strengths will provide the basis for assessing the strength of other supports. The proportion of all supports available with strengths that lie below a design limit needs to be determined.

When information is sought, statistical ideas suggest a typical collection process with four crucial steps.

1. **Set clearly defined goals for the investigation.**
2. **Make a plan of what data to collect and how to collect it.**
3. **Apply appropriate statistical methods to extract information from the data.**
4. **Interpret the information and draw conclusions.**

These indispensable steps will provide a frame of reference throughout as we develop the key ideas of statistics. Statistical reasoning and methods can help you become efficient at obtaining information and making useful conclusions.

1.2 Modern Statistics

The origin of statistics can be traced to two areas of interest that, on the surface, have little in common: games of chance and what is now called political science. Mid-eighteenth-century studies in probability, motivated largely by interest in games of chance, led to the mathematical treatment of errors of measurement and the theory that now forms the foundation of statistics. In the same century, interest in the numerical description of political units (cities, provinces, countries, etc.) led to what is now called **descriptive statistics**. At first, descriptive statistics consisted merely of the presentation of data in tables and charts; nowadays, it includes the summarization of data by means of numerical descriptions and graphs.

In recent decades, the growth of statistics has made itself felt in almost every major phase of activity, and the most important feature of its growth has been the shift in emphasis from *descriptive statistics* to **statistical inference**. Statistical inference concerns generalization based on sample data; it applies to such problems as estimating an engine's average emission of pollutants from trial runs, testing a manufacturer's claim on the basis of measurements performed on samples of his product, and predicting the fidelity of an audio system on the basis of sample data pertaining to the performance of its components.

When one makes a statistical inference, namely, an inference that goes beyond the information contained in a set of data, one must always proceed with caution. One must decide carefully how far to go in generalizing from a given set of data, whether such generalizations are at all reasonable or justifiable, whether it might be wise to wait until there are more data, and so forth. Indeed, some of the most important problems of statistical inference concern the appraisal of the risks and the consequences to which one might be exposed by making generalizations from sample data. This includes an appraisal of the probabilities of making wrong decisions, the chances of making incorrect predictions, and the possibility of obtaining estimates that do not lie within permissible limits.

We shall approach the subject of statistics as a science, developing each statistical idea insofar as possible from its probabilistic foundation, and applying

each idea to problems of physical or engineering science as soon as it has been developed. The great majority of the methods we shall use in stating and solving these problems belong to the **classical approach**, because they do not formally take into account the various subjective factors mentioned above. However, we shall endeavor continually to make the reader aware that the subjective factors do exist, and to indicate whenever possible what role they might play in making a final decision. This “bread-and-butter” approach to statistics presents the subject in the form in which it has successfully contributed to engineering science, as well as to the natural and social sciences, in the last half of the twentieth century and beyond.

1.3 Statistics and Engineering

There are few areas where the impact of the recent growth of statistics has been felt more strongly than in engineering and industrial management. Indeed, it would be difficult to overestimate the contributions statistics has made to solving production problems, to the effective use of materials and labor, to basic research, and to the development of new products. As in other sciences, statistics has become a vital tool to engineers. It enables them to understand phenomena subject to variation and to effectively predict or control them.

In this text, our attention will be directed largely toward engineering applications, but we shall not hesitate to refer also to other areas to impress upon the reader the great generality of most statistical techniques: The statistical method used to estimate the average coefficient of thermal expansion of a metal serves also to estimate the average time it takes a secretary to perform a given task, the average thickness of a pelican eggshell, or the average IQ of first-year college students. Similarly, the statistical method used to compare the strength of two alloys serves also to compare the effectiveness of two teaching methods, or the merits of two insect sprays.

1.4 A Case Study: Visually Inspecting Data to Improve Product Quality

This study¹ dramatically illustrates the important advantages gained by appropriately plotting and then monitoring manufacturing data. It concerns a ceramic

¹Courtesy of Don Ermer

part used in popular coffee makers. This ceramic part is made by filling the cavity between two dies of a pressing machine with a mixture of clay, water, and oil. After pressing, but before the part is dried to a hardened state, critical dimensions are measured. The depth of the slot is of interest here.

Because of natural uncontrolled variation in the clay-water-oil mixture, the condition of the press, differences in operators, and so on, we cannot expect all of the slot measurements to be exactly the same. Some variation in the depth of slots is inevitable, but the depth needs to be controlled within certain limits for the part to fit when assembled.

Slot depth was measured on three ceramic parts selected from production every half hour during the first shift from 6 A.M. to 3 P.M. The data in Table 1.1 were obtained on a Friday. The sample mean, or average, for the first sample of 214, 211, and 218 (thousandths of an inch) is

$$\frac{214 + 211 + 218}{3} = \frac{643}{3} = 214.3.$$

The graphical procedure, called an \bar{X} -bar chart, consists of plotting the sample averages versus time order. This plot will indicate when changes have occurred and actions need to be taken to correct the process.

From a prior statistical study, it was known that the process was stable about a value of 217.5 thousandths of an inch. This value will be taken as the central

Table 1.1 Slot depth (thousandths of an inch)

Time	6:30	7:00	7:30	8:00	8:30	9:00	9:30	10:00
1	214	218	218	216	217	218	218	219
2	211	217	218	218	220	219	217	219
3	218	219	217	219	221	216	217	218
Sum	643	654	653	653	658	653	652	656
\bar{x}	214.3	218.0	217.7	217.7	219.3	217.7	217.3	218.7

Time	10:30	11:00	11:30	12:30	1:00	1:30	2:00	2:30
1	216	216	218	219	217	219	217	215
2	219	218	219	220	220	219	220	215
3	218	217	220	221	216	220	218	214
Sum	653	651	657	660	653	658	655	644
\bar{x}	217.7	217.0	219.0	220.0	217.7	219.3	218.3	214.7

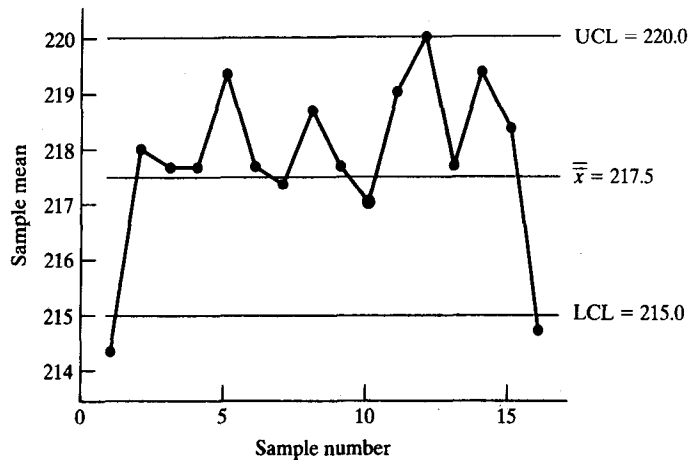


Figure 1.1
X-bar chart for depth

line of the X-bar chart in Figure 1.1.

$$\text{central line : } \bar{\bar{x}} = 217.5$$

It was further established that the process was capable of making mostly good ceramic parts if the average slot dimension for a sample remained between certain control limits.

Lower control limit: LCL = 215.0

Upper control limit: UCL = 220.0

What does the chart tell us? The mean of 214.3 for the first sample, taken at approximately 6:30 A.M., is outside the lower control limit. Further, a measure of the variation in this sample

$$\text{range} = \text{largest} - \text{smallest} = 218 - 211 = 7$$

is large compared to the others. This evidence suggests that the pressing machine had not yet reached a steady state. The control chart suggests that it is necessary to warm up the pressing machine before the first shift begins at 6 A.M. Management and engineering implemented an early start-up and thereby improved the process. The operator and foreman did not have the authority to make this change. Deming claimed that 85% or more of our quality problems are in the system and that the operator and others responsible for the day-to-day operation are responsible for 15% or less of our quality problems.

The X-bar chart further shows that, throughout the day, the process was stable but a little on the high side, although no points were out of control until the last sample of the day. Here an unfortunate oversight occurred. The operator did not report the out-of-control value to either the set-up person or the foreman because it was near the end of her shift and the start of her weekend. She also knew the set-up person was already cleaning up for the end of the shift and that the foreman was likely thinking about going across the street to the Legion Bar for some refreshments as soon as the shift ended. She did not want to ruin anyone's plans so she kept quiet.

On Monday morning when the operator started up the pressing machine, one of the dies broke. The cost of the die was over a thousand dollars. But this was not the biggest cost. When a customer was called and told there would be a delay in delivering the ceramic parts, he canceled the order. Certainly the loss of a customer is an expensive item. Deming referred to this type of cost as the unknown and unknowable, but at the same time it is probably the most important cost of poor quality.

On Friday the chart had predicted a problem. Afterward it was determined that the most likely difficulty was that the clay had dried and stuck to the die, leading to the break. The chart indicated the problem but someone had to act; for a statistical charting procedure to be truly effective action must be taken.

1.5 Two Basic Concepts—Population and Sample

The examples above, where the evaluation of actual information is essential for acquiring new knowledge, motivate the development of statistical reasoning and tools taught in this text. Most experiments and investigations conducted by engineers in the course of investigating, be it a physical phenomenon, production process, or manufactured unit, share some common characteristics.

A first step in any study is to develop a clear, well-defined **statement of purpose**. For example, a mechanical engineer wants to determine whether a new additive will increase the tensile strength of plastic parts produced on an injection molding machine. Not only must the additive increase the tensile strength, it needs to increase it by enough to be of engineering importance. He therefore created the following statement.

Purpose: Determine whether a particular amount of an additive can be found that will increase the tensile strength of the plastic parts by at least 10 pounds per square inch.

In any statement of purpose, try to avoid words like soft, hard, large enough, and so on, which are difficult to quantify. The statement of purpose can help us to decide on what data to collect. For example, the mechanical engineer tried two different amounts of additive and produced 25 specimens of the plastic part with each mixture. The tensile strength was obtained for each of 50 specimens.

Relevant data must be collected. But it is often physically impossible or infeasible from a practical standpoint to obtain a complete set of data. When data are obtained from laboratory experiments, no matter how much experimentation has been performed, more could always be done. To collect an exhaustive set of data related to the damage sustained by all cars of a particular model under collision at a specified speed, every car of that model coming off the production lines would have to be subjected to a collision! In most situations, we must work with only partial information. The distinction between the data actually acquired and the vast collection of all potential observations is a key to understanding statistics.

The source of each measurement is called a **unit**. It is usually an object or a person. To emphasize the term "population" for the entire collection of units, we call the entire collection the **population of units**.