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A Reader for the Behavioral Sciences

Roger E. Kirk

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Edited by

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Brooks/Cole Publishing Company
Monterey, California
A Division of Wadsworth Publishing Company, Inc.
Belmont, California

ISBN: 0-8185-0005-0
L.C. Catalog Card No.: 73-150744
Printed in the United States of America

1 2 3 4 5 6 7 8 9 10—77 76 75 74 73 72

This book was designed by Linda Marcetti, typeset by Santype Ltd. in Salisbury, England, and printed and bound by Kingsport Press, Kingsport, Tennessee.

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Preface

This book of readings has been compiled for introductory- and intermediate-level statistics courses in the behavioral sciences; it is an outgrowth of my efforts over the years to enrich my introductory statistics course with selected articles from the statistical literature. All too often students see statistics as a series of cookbook techniques to be slavishly applied to data. Hopefully this book will dispel that notion and help the reader catch a glimpse of the excitement of statistics.

The selection of articles has been guided by a number of criteria, the most important of which is whether an article will help broaden the student's understanding of important concepts and issues in statistics. Preference has been given to articles dealing with conceptual issues as opposed to those that are technique-oriented. A number of articles have been selected because they trace the development of such controversial statistical issues as the relevance of levels of measurement for the selection of a statistic, the use of one- versus two-tailed tests, the logic of hypothesis testing, and the choice of an error rate for multiple comparisons. The articles are organized into chapters that generally parallel the contents of contemporary textbooks.

Editorial commentaries preceding the articles point up key issues to be discussed, provide background information, and in many cases summarize the conclusions of the articles. Such overviews can provide a useful conceptual framework for the integration of new ideas. An additional aid is a glossary, in the appendix, containing 160 definitions of statistical terms.

Most of the articles in this book are appropriate for students whose backgrounds include only college algebra; however, several selections require some knowledge of calculus. Chapter 10 and portions of Chapters 3 and 7 are included primarily for use in intermediate-level courses. The teacher who is familiar with the mathematical preparation of his students can best judge which articles are appropriate.

"Classic" articles in the statistical literature are always stimulating, often interestingly written, and sometimes even intelligible to college students. A number of these articles appear in the book, although many could not be included because of their length or because they assume a mathematical sophistication that the typical student does not possess. Annotated bibliographies are provided for the student who wants to pursue in depth a particular topic or statistical issue.

The preparation of a book of this type always involves a number of compromises for an editor, particularly regarding breadth of coverage and length. Nevertheless, I think this book provides a balanced presentation of contemporary thinking about statistical issues in the behavioral sciences.

Many people have contributed to the preparation of this book. Although I cannot acknowledge all of them individually, I do want to express my appreciation to the authors and publishers who gave permission to use copyrighted material and to James V. Bradley of New Mexico State University, Arthur L. and Linda W. Dudycha of Purdue University, and John C. Flynn of Baylor University, who contributed original papers. Professors J. Barnard Gilmore of the University of Toronto and William L. Sawrey of California State College at Hayward reviewed the manuscript; their comments were especially helpful. I am indebted to hundreds of undergraduate and graduate students at Baylor University, whose reactions to the proposed selections were invaluable to me in making my final choices. And I am grateful to my wife for her editorial assistance and to the staff of Brooks/Cole for their splendid assistance and cooperation.

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Behavioral Statistics: Historical Perspective and Applications

A mastery of the current knowledge in many scientific areas does not require an appreciation of, or even familiarity with, the historical backgrounds of those areas. This is especially true of statistics, which has a short but colorful history. In today's statistics courses, emphasis on theoretical and methodological topics to the exclusion of history and the men who shaped that history is a natural consequence of the ever-increasing volume of information that must be taught. After covering theorems, derivations, and formulas, the teacher has little time to detail the interesting events that led to breakthroughs in statistics. Thus it is easy for the student to lose sight of the fact that the body of knowledge in statistics is the product of people whose contributions were shaped by their personal attributes as well as the times in which they lived. This depersonalization of statistics may explain why introductory students rarely develop a real interest in their statistics course.

Clearly what is needed is an interestingly written overview of the development of statistics, with particular emphasis on the personal attributes of the giants in the field. The first article in this chapter provides such an overview. The authors, Arthur L. and Linda W. Dudycha, begin with the earliest work on probability and conclude with the contributions of Ronald A. Fisher, Jerzy Neyman, and Egon Pearson to statistical theory and practice. Their paper has three sections: (I) Probability Theory and the Normal Curve, (II) Descriptive and National Statistics, and (III) Statistical Inference and Experimental Design. These sections are largely self-contained and can be assigned in the sequence in which they are taught in class. Subsequent articles by L. McMullen and William G. Cochran provide personal reminiscences of two figures prominent in the development of modern statistics, W. S. Gosset and R. A. Fisher.

1.1 Behavioral Statistics: An Historical Perspective

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Prologue

Contemporary behavioral statistics, so impersonally discussed in often forbidding textbooks, has its primary roots—both subtle and impassioned—in the post-Renaissance period. The statistics used and revered in the behavioral sciences today descended from a varied and colorful ancestry: from the greed of ancient monarchs and gamblers to the quest for knowledge by the intellectually elite, ranging from pure mathematicians to clergymen to a brewer.

Forty years ago Helen Walker noted the rapidity with which the use of statistics was advancing and argued for an understanding of its origin and development by its students. Her prophecy concerning the salience of statistics was indeed accurate—perhaps understated. Statistics has become one of the major modes of communication among behavioral scientists—sometimes to the detriment of sound deliberate thought on the behavioral phenomenon in question. For example, "statistical significance" to many has become almost sacred and itself a goal, all too often at the sacrifice of practical or meaningful significance. Nonetheless, social scientists must continue to measure behavioral phenomena and submit these data to rigorous statistical analysis, but a close relationship must be maintained cognitively between the measured phenomenon and the statistical analysis.

Students of behavior, through exposure to the historical perspectives of statistics, are provided with the opportunity to gain a deeper appreciation for the necessity of this partnership. However, prior to Walker's book *Studies in the History of Statistical Method* (1929), there was, and still is, a paucity of definitive writing in the area.[1] This is not to imply that statistical history is not well documented. Various writers do take cognizance of statistics' heritage, but the literature is not replete with chronologies of statistical development.

This article will show some of the currents in the stream of statistical development through the thoughts, contributions, and personalities of its forefathers. Ideally, to best understand their contributions one should discuss these men through the prevailing social, political, economical, and religious frameworks of their times. We cannot accomplish completely so laudable a mission in one article, but we can provide a sample (non-random) of the contributions and polemics of these great men.

[1] Koren, John, *The History of Statistics*, 1918; Westergaard, Harald, *Contributions to the History of Statistics*, 1932.

A knowledge of the backgrounds of the giants in the evolution of statistics will give the serious student a better perspective from which to understand and evaluate current statistical practices in his science. For those yet needing reassurance that the perhaps not-so-palatable subject of statistics is both important and necessary, Sir Francis Galton's words surely must suffice:

General impressions are never to be trusted. Unfortunately when they are of long standing they become fixed rules of life, and assume a prescriptive right not to be questioned. Consequently, those who are not accustomed to original inquiry entertain a hatred and a horror of statistics. They cannot endure the idea of submitting their sacred impressions to cold-blooded verification. But it is the triumph of scientific men to rise superior to such superstitions, to desire tests by which the value of beliefs may be ascertained, and to feel sufficiently masters of themselves to discard contemptuously whatever may be found untrue [1908].

I. Probability Theory and the Normal Curve

Since the origin of most statistical concepts is rooted in the mathematical theory of probability, it seems appropriate to begin with the developments in that field.

The earliest faint traces of probability, found in the Orient around 200 B.C., were concerned with whether an expected child would be a male or a female. However, the first real cornerstone of the calculus of probability seems to have been laid in Italy when a commentary (Venice, 1477) on Dante's *Divine Comedy* referenced the different throws which could be made with three dice[2] in the game of Hazard. The first mathematical treatment of gambling problems was *Suma* (1494) written by Luca Paccioli (1445–1509). This work gives the first version of the celebrated "problem of points," which concerns the "equitable division of the stakes between two players of unequal skill when the game is interrupted before its conclusion" (Walker, 1929, p. 5). This problem was destined to occupy the minds of probability theorists for two centuries. However, neither Paccioli nor the later Cardano (1501–1576), who wrote what is considered a "gambler's handbook" (1663), offered any general principles of probability, and they were often incorrect in solutions to the simple problems they did consider.

Though the conception of probability occurred in Italy, French mathematicians deserve credit for the first concerted efforts to master problems in probability. Blaise Pascal (1623–1662), who gained renown as a mathematician and physicist (but became a religious recluse at age 25), and Pierre de Fermat (1601–1665), a distinguished mathematician, exchanged a volley of letters during 1654 on problems suggested to Pascal by a gambler, Chevalier de Méré. Among these was the problem of points. Through the year, Pascal and Fermat gradually chiseled out of this "gambler's perplexity" an extremely important foundation stone in mathematical concepts—later to be known as the "theory of probability." Unfortunately, this correspondence was temporarily obscured by the then highly visible writings of Newton and Leibnitz; thus, the calculus of probabilities was not yet placed on a sound footing.

According to David, Christianus Huygens was "the scientist who first put forward in a systematic way the new propositions evoked by the problems set to Pascal and Fermat, who gave the rules and who first made definitive the idea of mathematical expectation" (1962, p. 110). Lord Huygens (1629–1695) was a Dutch astronomer and natural scientist who came from a family of wealth and position (as did many of the pioneers whom this article will mention). His mathematical treatise on dice games, *De Ratiociniis in Aleae Ludo* (1657), stood for a half century as the "unique" introduction to the theory of probability, and was only superceded when it

[2] The modern day die probably originated from the *astragalus*, used especially in the gaming of the Middle Ages. It is a knuckle bone having four sides on which it can rest (scored 1, 3, 4, 6); the other two sides are rounded (David, 1962).

inspired the major works of James Bernoulli, Pierre Montmort, and Abraham de Moivre.

James Bernoulli (1654–1705) was the eldest son of a family of Swiss merchant bankers in Basel, and the first of nine distinguished mathematicians of that famed name. He first took a degree in theology because his parents expected him to become a minister of the Reformed Church. What first set James on the path of astronomy and mathematics is speculative, but early in his career he became interested in the calculus of probability, as evidenced by a number of papers on this subject which were obviously inspired by Huygens. The single work for which James Bernoulli is best known is *Ars Conjectandi*, which was written during the latter part of his life but not published until eight years after his death. Nicholas Bernoulli, though only 18 years of age at the time, a nephew and pupil of James and already a probability theorist of some stature himself, was asked to edit James' all-but-complete manuscript for publication. Nicholas felt reticent and incompetent to do so (possibly because of Leibnitz's criticism of it), but finally capitulated when the pressure of public opinion allowed no further delay.

Ars Conjectandi is divided into four parts. In the preface Nicholas states:

> ... the first contains the treatise of the illustrious Huygens, "Reasoning on Games of Chance," with notes, in which one finds the first elements of the art of conjecture. The second part is comprised of the theory of permutations and combinations, theory so necessary for the calculation of probabilities and the use of which he explains in the third part for solution of games of chance. In the fourth part he undertook to apply the principles previously developed to civil, moral and economic affairs. But held back for a long time by ill-health, and at last prevented by death itself, he was obliged to leave it imperfect [from David, 1962, p. 134].

The first three parts alone would have established James Bernoulli as a probability theorist. It was, however, in the fourth part, *Pars Quarta*, that he introduced his celebrated but controversial "golden theorem"—his solution of the problem of "assigning the limits within which, by the repetition of experiments, the probability of an event may approach indefinitely to a given probability"—about which he wrote:

> This is therefore the problem that I now wish to publish here, having considered it closely for a period of twenty years, and it is a problem of which the novelty, as well as the high utility, together with its grave difficulty, exceed in value all the remaining chapters of my doctrine. Before I treat of this "Golden Theorem" I will show that a few objections, which certain learned men [e.g., Leibnitz] have raised against my propositions, are not valid [*Ars Conjectandi*, 1713, p. 327; from K. Pearson, 1925, p. 206].

Bernoulli's Golden Theorem, which has since become the well-known "Bernoulli Theorem," was usually incorrectly phrased by textbook writers until the early 1900's as: "Accuracy increases with the square root of the number of observations." Further, early writers repeatedly stated that an illustration of Bernoulli's principle was "the fact that the constants [statistics] of frequency distributions in the case of large samples have standard deviations varying inversely as the square root of the size of the sample ...." This principle—currently often called the Law of Large Numbers—although admittedly closely allied to both Bernoulli's Theorem and the Tchebycheff Inequality, should be rightfully attributed to de Moivre (K. Pearson, 1925, p. 201). Pearson, taking note of the manner in which history often manages to misrepresent authorships, set the record straight as to what Bernoulli "really did achieve," which was "... to prove that by increasing sufficiently the number of observations he can cause the probability—i.e., that the ratio of observed successful to unsuccessful occurrences will differ from the true ratio within certain small limits—to diverge from certainty by an assignable limit" (K. Pearson, 1925, pp. 201–202). In modern terminology, the Bernoulli Theorem states that the "probability that a frequency v/n

differs from its mean value p by a quantity of modulus at least equal to ε tends to zero as $n \to \infty$, however small $\varepsilon > 0$ is chosen" (Cramér, 1946, p. 196; see also Hays, 1963).

Also in *Pars Quarta* Bernoulli proceeded to turn his argument for the Golden Theorem around, which resulted in the first definite suggestion of inverse and fiducial probability that is so essential to modern statistical theory in the now familiar form of confidence intervals. It is to this reverse principle, which Bernoulli simply stated without offering proof, that Leibnitz raised strong objections. The Golden Theorem thus sparked the beginning of the controversy on inverse probability and foreshadowed further developments of it in the writings of de Moivre, Bayes, Laplace, and Gauss.

Bernoulli, through *Ars Conjectandi*, is responsible for many other of our "modern" ideas. He developed the binomial theorem which is the basis for many distribution-free tests. Binomial trials often go by his name—Bernoulli trials. He also can be accounted responsible for inspiring de Moivre's derivation of the "normal curve" limit to the sum of a number of binomial probabilities when he (de Moivre) refined the Golden Theorem. If it were not for James Bernoulli, who had the mathematical prowess to digest the then modern analysis of Leibnitz (1646–1716) and Newton (1642–1727) and apply it to the analysis of games of chance, it is doubtful whether Montmort or de Moivre would have contributed what they did to the development of probability theory—to which we shall now turn.

Pierre Rémond de Montmort (1678–1719) was born in Paris of nobility. Contemptuous of parental control, young Pierre left home to avoid having to study law as his father had intended and instead traveled throughout Europe. In 1699, however, Pierre returned home and made peace with his father, who died shortly afterwards, leaving him a large fortune. Though having received this large inheritance, he did not plunge into the dissolute life of wine, women, and song which was thought natural for a young nobleman of his time. Instead, he purchased the estate of Montmort, resigned his stall as a canon of Notre Dame in order to marry, and settled down in relative seclusion on his country estate to work on problems of probability—though he was no gambler.

The results of his efforts were first published in *Essai d'Analyse sur les Jeux de Hasard* (1708). A much more comprehensive second edition, which also included the extensive Montmort-Bernoulli (John and Nicholas) correspondence, was published in 1714. In the first edition Montmort began by finding the chances involved in various card games. He exhibited great agility and insight in his use of the principle of conditional probability, often attributed to de Moivre but probably dating back to Huygens or James Bernoulli. He also decided that while the rules of probability could be applied to the game of life, the chances in this game were too difficult to compute! The second edition, which contained much new material, reflected Montmort's maturity of thought on the subject and the influence of Nicholas Bernoulli. Here he presented generalized solutions for many games of chance discussed in the first edition, made the first but rather maladroit attempts toward questions of annuities, and solved the problem of points in full generality with two players of unequal skill. Montmort's contributions to probability theory probably lie not in the novel ideas he introduced but in his algebraic methods of attack (David, 1962).

Montmort, however, felt a strong compunction for having spent most of his life working on gambling problems and so apologized:

> It is particularly in games of chance that the weakness of the human mind appears and its leaning towards superstition. ... There are those who will play only with packs of cards with which they have won, with the thought that good luck is attached to them. Others on the contrary prefer packs with which they have lost, with the idea that having lost a few times with them it is less likely that they will go on losing, as if the past can decide something for the future.... Others refuse to shuffle the cards and believe they must infallibly lose if they deviate from their

rules. Finally there are those who look for advantage where there is none, or at least so small as to be negligible. Nearly the *same thing can be said of the conduct of men in all situations of life where chance plays a part.* It is the *same* superstitions which govern them, the *same* imagination which rules their method of procedure and which blinds their fears and hopes. ... The general principle of these superstitions and errors is that most men attribute the distribution of good and evil and generally all the happenings in this world to a fatal power which works without order or rule.... I think therefore it would be useful, not only to gamesters *but to all men* in general, to know that *chance has rules which can be known,* and that through not knowing these rules they make faults every day, the results of which with more reason may be imputed to themselves than to the destiny which they accuse.... It is certain that men do not work honestly as hard to obtain what they want as they do in the pursuit of Fortune or Destiny.... The conduct of men usually makes their good fortune, and wise men leave as little to chance as possible [from David, 1962, pp. 143–144; present authors' italics].

His prophecy concerning the applicability of "chance rules" to all walks of life seems to have been accurate, and his advice seems as necessary in the twentieth century as in the eighteenth.

Abraham de Moivre (1667–1754), an extremely influential but often underrated probabilitist of this period, was born in Champagne—then a province of eastern France (and still famous for its dry white wine). He, like Bernoulli, was a Huguenot, but unlike many other early probability theorists was neither wealthy nor of noble birth. At the age of eleven de Moivre began studying the humanities at a Protestant college. It was not until his family moved to Paris and he began attending classes at the Sorbonne, where he came into contact with the great teacher of mathematics Ozanam, that de Moivre became interested in mathematics. His education, however, was interrupted when the Edit of Nantes[3] was revoked by Louis XIV in 1685 and de Moivre, then eighteen, was imprisoned. He was released in 1688 and fled from France immediately, never to return and never to publish in his native tongue. The embittered young de Moivre landed in England devoid of money, friends, and influence and soon realized he had been disillusioned even about his "profound" knowledge of mathematics.

De Moivre spent his early years in London as a visiting tutor to the sons of noblemen. Finally he broke into the charmed circle of English mathematicians and was elected a Fellow of the Royal Society in 1697. Even so, de Moivre was destined to tramp about the London streets from pupil to pupil. In an effort to escape these humble circumstances, he begged John Bernoulli (James' brother) to intercede with Leibnitz and use his influence to get him a university post somewhere, but to no avail. Nevertheless, while augmenting his income by calculating odds for gamblers at a coffee-house that he frequented after his long days of tutoring, de Moivre unexpectedly found and developed a lasting friendship with Newton.

The early part of the eighteenth century saw de Moivre grow rapidly in mathematical stature. In 1711 he published *De Mensura Sortis, seu de Probabilitate Eventuum in Ludis a Casu Fortuito Pendentibus*, which led to a flurry of charges and counter-charges between Montmort and de Moivre. De Moivre insinuated that Montmort had done no more than slightly improve on Huygens, while Montmort insinuated that de Moivre had taken his ideas solely from *Essai d'Analyse.* De Moivre wrote in the preface of his memoir:

Huygens, first, as I know, set down rules for the solution of the same kind of problem as those which the new French author[4] illustrates freely with diverse examples. But these famous men do not seem to have been accustomed to that simplicity and generality which the nature of the thing demands [from David, 1962, p. 152].

It is particularly surprising that Montmort, who usually avoided getting embroiled in

[3] A law promulgated by Henry IV of France in 1598, granting considerable religious and civil liberty to the Huguenots.

[4] Referring to *L'Analyse des Jeux de Hasard* by Montmort.

arguments, replied so vigorously to the *De Mensura Sortis* in the "Avertissement" of the second edition of *Essai d'Analyse* (1714), writing:

> The author did me the honour of sending me a copy.... M. Moivre[5] was right to think I would need his book to reply to the criticism he made of mine in his introduction. His praise-worthy intention of boosting and increasing the value of his work has led him to disparage mine and to deny my methods the merit of novelty. As he imagined he could attack me without giving me reason for complaint against him, I think I can reply to him without giving him cause to complain against me.... [from David, 1962, p. 153].

And indeed he did reply—with both length and uncharacteristic adamance—expounding on the history of probability theory from the Pascal-Fermat debate up to and including his own contributions. A partial precipitating factor to this entire controversy may also have been that Montmort felt impelled to react against the émigré Frenchman in London, especially since a few years earlier English troops had been knocking at the gates of France. Apparently the quarrel had resolved itself by 1718, because in that year de Moivre acted as Montmort's guide and interpreter during a visit he made to London (David, 1962).

In 1718, de Moivre published and dedicated to Newton *The Doctrine of Chances*, which was an expanded version, in English, of *De Mensura Sortis*. In the preface of this first edition de Moivre was obviously trying to gain favor with Montmort and the Bernoullis when he wrote apologetically for his remarks about Montmort's memoir:

> As for the French book, I had run it over but cursorily, by reason I had observed that the Author chiefly insisted on the Method of Huygens, which I was absolutely resolved to reject.... However, had I allowed myself a little more time to consider it, I had certainly done the Justice to its Author, to have owned that he had not only illustrated Huygen's Method by a great variety of well chosen examples, but that he had added to it several curious things of his own Invention.... [He] published a Second Edition of that Book, in which he has particularly given many proofs of his singular Genius and extraordinary Capacity; which Testimony I give both to Truth, and to the Friendship with which he is pleased to Honour me.... [from David, 1962, p. 166].

As David notes, it is revealing that the words after "reject" were omitted from the second edition (1738) and the posthumous third edition (1756)—both of which were published well after the death of Montmort in 1719!

The Preface of de Moivre's *Doctrine* contains a lengthy summary of the contents of the book, discusses the problem of points for two gamblers of unequal skill, and relates the author's general ideas about chance. He begins by giving the definitions of probability, the addition of probabilities, expectation, independence of events, and joint and conditional probabilities. The remainder of the book is divided into discussions of specific problems. It is of note that in Problem V, de Moivre reaches what has been commonly called Poisson's (1781–1840) binomial exponential limit, and in Problem VII he gives us the multinomial distribution, which was also arrived at independently by Montmort and Nicholas Bernoulli in their correspondence. There is no doubt that the first edition was written by "... a man who was already superior to Montmort and the Bernoullis in his mathematical powers, and who, when he came to maturity, was to produce in this third edition the first modern book on probability theory" (David, 1962, p. 171).

De Moivre also had a life-long interest in the theory of annuities, an interest that undoubtedly grew from his early and lasting acquaintance with Edmund Halley,[6] who had

[5] Note that Montmort did not use the noble prefix "de," which de Moivre himself is believed to have added while crossing the English Channel.

[6] It was Halley (1656–1742) who in 1692, while Secretary to the Royal Society, met de Moivre and introduced him to the mathematical elite of Britain and tried to interest him in astronomy. Halley, an astronomer, was the first to predict the return of the comet which now bears his name.