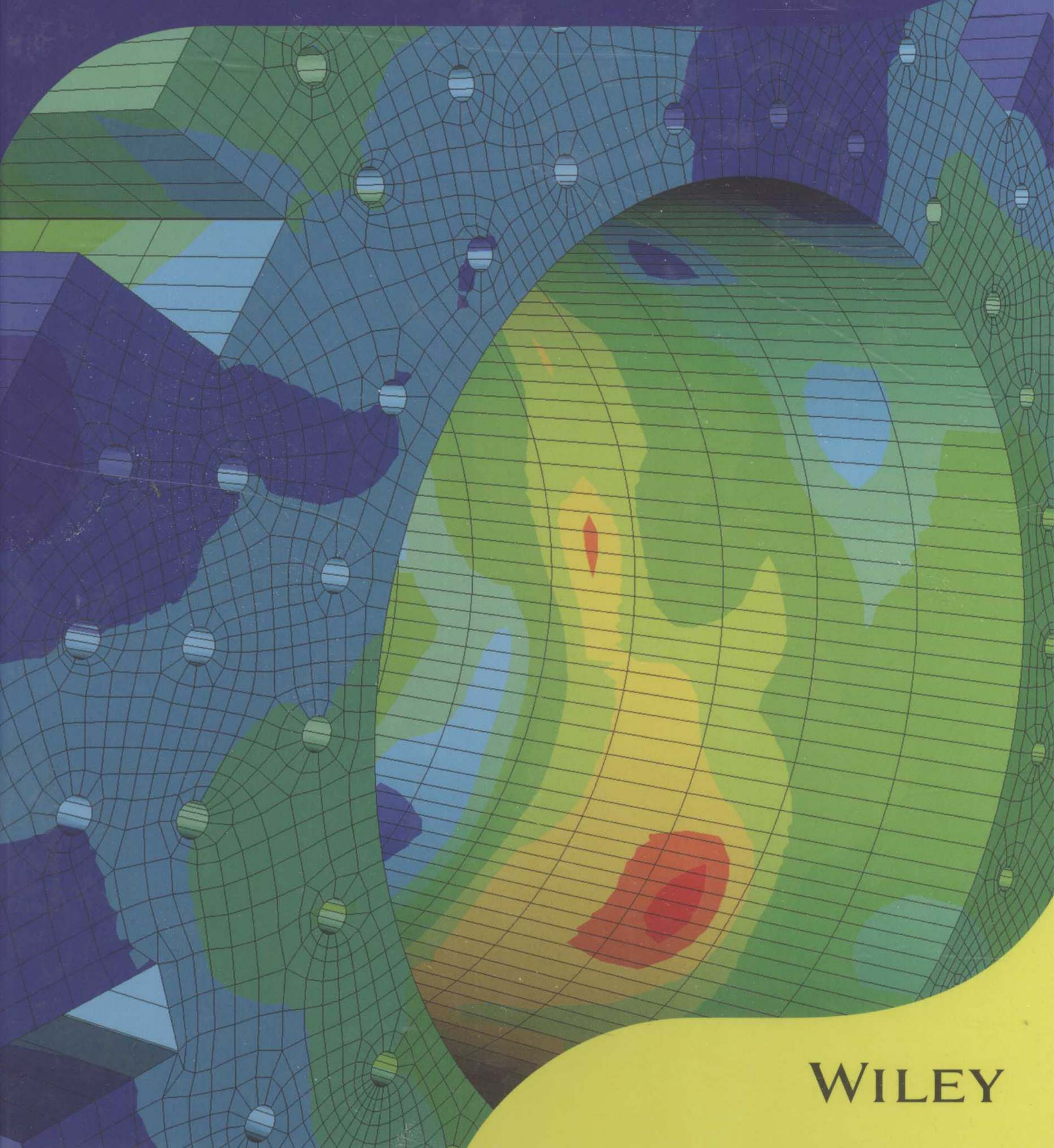
Fifth Edition

Programming the Finite Element Method

I. M. Smith, D. V. Griffiths and L. Margetts



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PROGRAMMING THE FINITE ELEMENT METHOD

Preface to Fifth Edition

This edition maintains the successful theme of previous editions, namely a modular programming style which leads to concise, easy to read computer programs for the solution of a wide range of problems in engineering and science governed by partial differential equations.

The programming style has remained essentially the same despite huge advances in computer hardware. Readers will include beginners, making acquaintance with the finite element method for the first time, and specialists solving very large problems using the latest generation of parallel supercomputers.

In this edition special attention is paid to interfacing with other open access software, for example ParaView for results visualisation, ABAQUS user subroutines for a range of material constitutive models, ARPACK for large eigenvalue analyses, and METIS for mesh partitioning.

Chapter 1 has been extensively rewritten to take account of rapid developments in computer hardware, for example the availability of GPUs and cloud computing environments. In Chapters 2 to 11 numerous additions have been made to enhance analytical options, for example new return algorithms for elastoplastic analyses, more general boundary condition specification and a complex response option for dynamic analyses.

Chapter 12 has been updated to illustrate the rapidly advancing possibilities for finite element analyses in parallel computing environments. In the fourth edition the maximum number of parallel 'processes' used was 64 whereas in this edition the number has increased to 64,000. The use of GPUs to accelerate computations is illustrated.

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There were many contributions to Chapter 12. Llion Evans, Paul Mummery, Philip Manning, Graham Hall and Dimitris Christias (all University of Manchester) provided scientific case studies. Florent Lebeau and Francois Bodin (CAPS Entreprise) evaluated the use of GPUs and Philippe Young (Simpleware Ltd) provided support in image-based modelling.

Benchmarking of the programs in Chapter 12 was carried out using supercomputers hosted by the UK National High Performance Computing Service "HECToR" (e107, e254) and the UK Regional Service "N8 HPC" (EP/K000225/1). The EU FP7 project "Venus-C" and Barcelona Supercomputing Center (Spain) provided access, resources and training to use Microsoft Azure.

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1

Preliminaries: Computer Strategies

1.1 Introduction

Many textbooks exist which describe the principles of the finite element method of analysis and the wide scope of its applications to the solution of practical engineering and scientific problems. Usually, little attention is devoted to the construction of the computer programs by which the numerical results are actually produced. It is presumed that readers have access to pre-written programs (perhaps to rather complicated 'packages') or can write their own. However, the gulf between understanding in principle what to do, and actually doing it, can still be large for those without years of experience in this field.

The present book bridges this gulf. Its intention is to help readers assemble their own computer programs to solve particular engineering and scientific problems by using a 'building block' strategy specifically designed for computations via the finite element technique. At the heart of what will be described is not a 'program' or a set of programs but rather a collection (library) of procedures or subroutines which perform certain functions analogous to the standard functions (`SIN`, `SQRT`, `ABS`, etc.) provided in permanent library form in all useful scientific computer languages. Because of the matrix structure of finite element formulations, most of the building block routines are concerned with manipulation of matrices.

The building blocks are then assembled in different patterns to make test programs for solving a variety of problems in engineering and science. The intention is that one of these test programs then serves as a platform from which new applications programs are developed by interested users.

The aim of the present book is to teach the reader to write intelligible programs and to use them. Both serial and parallel computing environments are addressed and the building block routines (numbering over 100) and all test programs (numbering over 70) have been verified on a wide range of computers. Efficiency is considered.

The chosen programming language is FORTRAN which remains, overwhelmingly, the most popular language for writing large engineering and scientific programs. Later in this chapter a brief description of the features of FORTRAN which influence the programming of the finite element method will be given. The most recent update of the language was in 2008 (ISO/IEC 1539-1:2010). For parallel environments, MPI has been used, although the programming strategy has also been tested with OpenMP, or a combination of the two.

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