

FRANK S. BUDNICK

APPLIED MATHEMATICS

FOR BUSINESS, ECONOMICS, AND THE SOCIAL SCIENCES

APPLIED MATHEMATICS

for Business,
Economics,
and the
Social Sciences

Second Edition

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PREFACE

INTRODUCTION

Mathematics is an integral part of the education of students in business, economics, and the social sciences. There is increasingly a desire to improve the level of quantitative sophistication possessed by graduates in these types of programs. The objective is not to make mathematicians of these students, but to make them as comfortable as possible in an environment which increasingly makes use of quantitative analysis and the computer. Students are discovering that they must integrate mathematics, statistical analysis, and the computer in both required and elective courses within their programs. Furthermore, organizations are becoming more effective users of quantitative tools and the computer. Decision makers will be better equipped to operate within this type of environment if they are familiar with the more commonly used types of quantitative analyses and the technology of the computer. Such familiarity can assist them in being better "critics" and "users" of these tools, and hopefully, better decision makers.

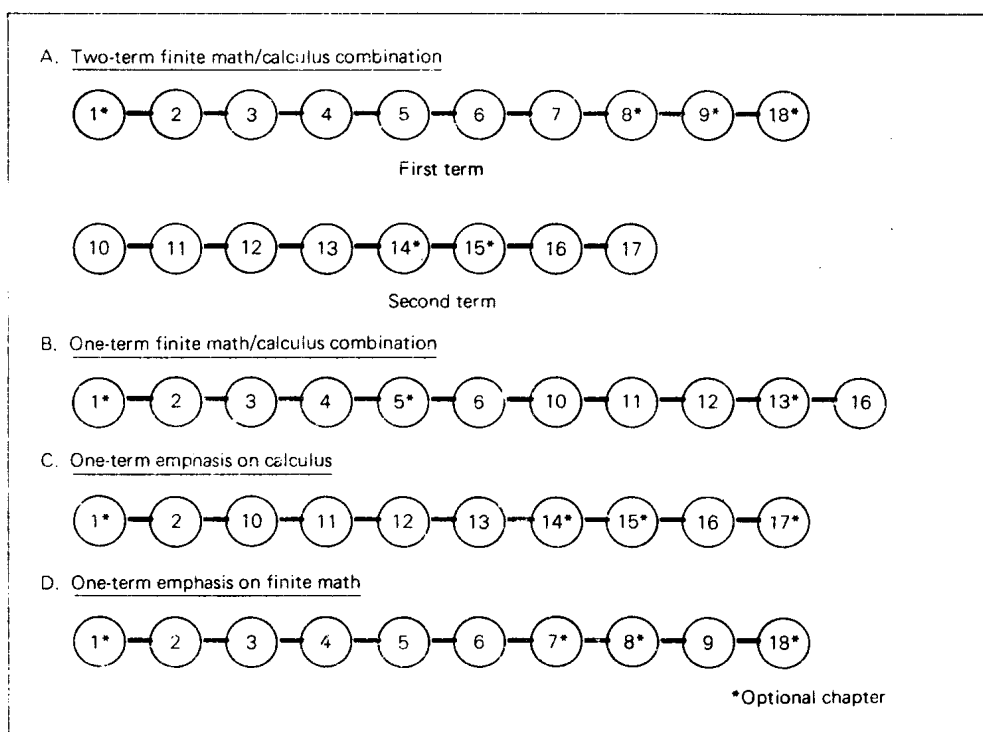
DESIGN OF BOOK

This book is an applied mathematics book for students in business, economics, and the social sciences. It provides a comprehensive treatment of selected topics in both finite mathematics and calculus. Although intended principally for students in business and economics, the book is appropriate for students in the social sciences. Designed primarily for a two-term course, the book can be adapted easily for a one-term course. It is appropriate for use in both two-year schools and four-year schools, as well as at the "foundation" level for graduate programs which require some mathematics background. M.B.A. and M.P.A. programs are typical graduate programs having this type of requirement.

The figure on the following page illustrates some *suggested* ways in which this text might be used.

Specific features of this book include:

- 1 A level of presentation which carefully develops and reinforces topics.
- 2 A style which appeals to the intuition of students and provides a great deal of visual reinforcement (over 300 figures).
- 3 An applied orientation which motivates students and provides a sense of purpose for studying mathematics.
- 4 An approach which first develops the mathematical concept and then reinforces with applications.
- 5 An approach which minimizes the use of rigorous mathematical proofs. Proofs are included at the end of selected chapters for interested persons.
- 6 Special aids which address the most universal shortcoming of students entering this type of course: weak algebra skills. These aids include a review of key algebra principles in Chapter 1. A chapter pretest allows the student and



instructor to identify areas requiring special attention. In addition, “Algebra Flashbacks” are used throughout the book to assist the student in the recall of key rules or concepts. The flashback usually consists of a restatement of a rule or concept with a reference to the appropriate section in Chapter 1.

7 Notes to students which provide them with special insights.

8 “Points for Thought and Discussion” which allow students to pause for a moment and reconsider a concept or example from a different perspective. Their purpose is to reinforce and extend the student’s understanding.

9 “Minicases” at the end of many chapters which provide challenging applications.

10 A multitude of other learning aids, including almost 450 solved examples, a wealth of exercises (over 2,200, most of which are new) chapter tests, chapter objectives, lists of key terms and concepts, and summary lists of important formulas.

11 An instructors manual which contains answers for all exercises and tests, suggestions for different course structures, prototype examples for new applications, transparency masters for selected figures, and a bank of questions for constructing quizzes and tests.

Although applications are presented throughout the book, Chapters 5 and 13 are devoted entirely to applications. The intent is that instructors cover as many applications in these chapters as they feel appropriate for their students.

Some exercises in the book are considered to be of a higher level of difficulty than most others. These are preceded by an asterisk (*).

SUMMARY OF NEW FEATURES AND CHANGES FOR THIS EDITION

In preparing this revision I surveyed those who taught from the first edition as well as those who decided not to adopt the first edition. The comments of those surveyed proved extremely valuable. The redesign of the book has resulted in the elimination or reduced coverage of some material and the expansion of other material, as well as some rearrangement of subject matter. I hope that those who continue to use the book find it more satisfactory for their needs; to those who are adopting for the first time, I hope that you and your students will be pleased.

Highlights of the significant changes follow:

- In this edition, the treatment of *algebra* and *set theory* has been consolidated in one chapter. The algebra treatment is a bit more abbreviated but continues to be as inclusive as in the first edition. The set theory portion of this chapter is also shortened from the chapter status it enjoyed in the first edition.
- In Chapter 2, the discussion of cartesian products has been eliminated. A new section provides a brief overview of different classes of mathematical functions.
- In Chapter 5, the material on break-even models has been expanded to include examples of *three-alternative analysis* and *multiproduct break-even analysis*.
- Chapter 6 represents a repositioning of the material on matrix algebra. Aside from a new subsection on the properties of determinants, the most significant change is an *expanded section on applications*.
- Chapter 7 represents a major expansion of the material on linear programming. More attention is given to problem formulation, and a new section discusses *computer solution methods*. A sample LP package is illustrated. *Shadow price* and *sensitivity analysis* concepts are introduced and illustrated using the LP package.
- Chapter 8 is an expanded treatment of the original simplex chapter. A new section discusses *alternative optimal solutions*, *no feasible solution*, and *unbounded solutions*. Another new section introduces the *dual problem*.
- Chapter 9 represents a repositioning of the material on probability theory.
- The material on exponential and logarithmic functions has been separated from other functional forms. Chapter 10 introduces *nonlinear functions* by discussing quadratic and other polynomial functions and their application. Chapter 14 consolidates the presentation of exponential and logarithmic functions and their application. New sections discuss the *conversion to base-e-functions* and *solving logarithmic and exponential equations*. This chapter also contains an *expanded set of applications*.
- Chapter 15 represents a significant expansion in the treatment of the calculus of functions of several variables. New (optional) sections examine *n-variable optimization* and *optimization subject to constraints*. This chapter also illustrates *three-dimensional graphics capabilities of computers*.

- The material on integral calculus has been expanded slightly and repackaged in two chapters. Chapter 16 introduces integral calculus and its methodology. The focus is upon the techniques of integration. A new section discusses *differential equations*. Chapter 17 focuses upon applications of integral calculus.
- Chapter 18 updates the material on the mathematics of finance by expanding the interest table ranges. A new extension of this material is a section which discusses *cost-benefit analysis*.
- Another new feature in this edition is the inclusion of *minicases* at the ends of 11 chapters. These provide a challenge to students which goes beyond the regular exercises in the text.

ACKNOWLEDGMENTS

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I also want to thank Nancy Nakamoto, Sue Rose, and Joe Daly for their assistance in developing problems and solution sets; and Ede Williams and Charlotte Manni for their superb work in typing the manuscript and its revisions. Special thanks also go to my students who served as "guinea pigs" for debugging new exercise sets.

I also wish to thank my parents, Mr. and Mrs. Willard L. Budnick, for their continued support, encouragement, and love during this and all other endeavors.

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Frank S. Budnick

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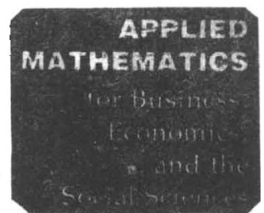
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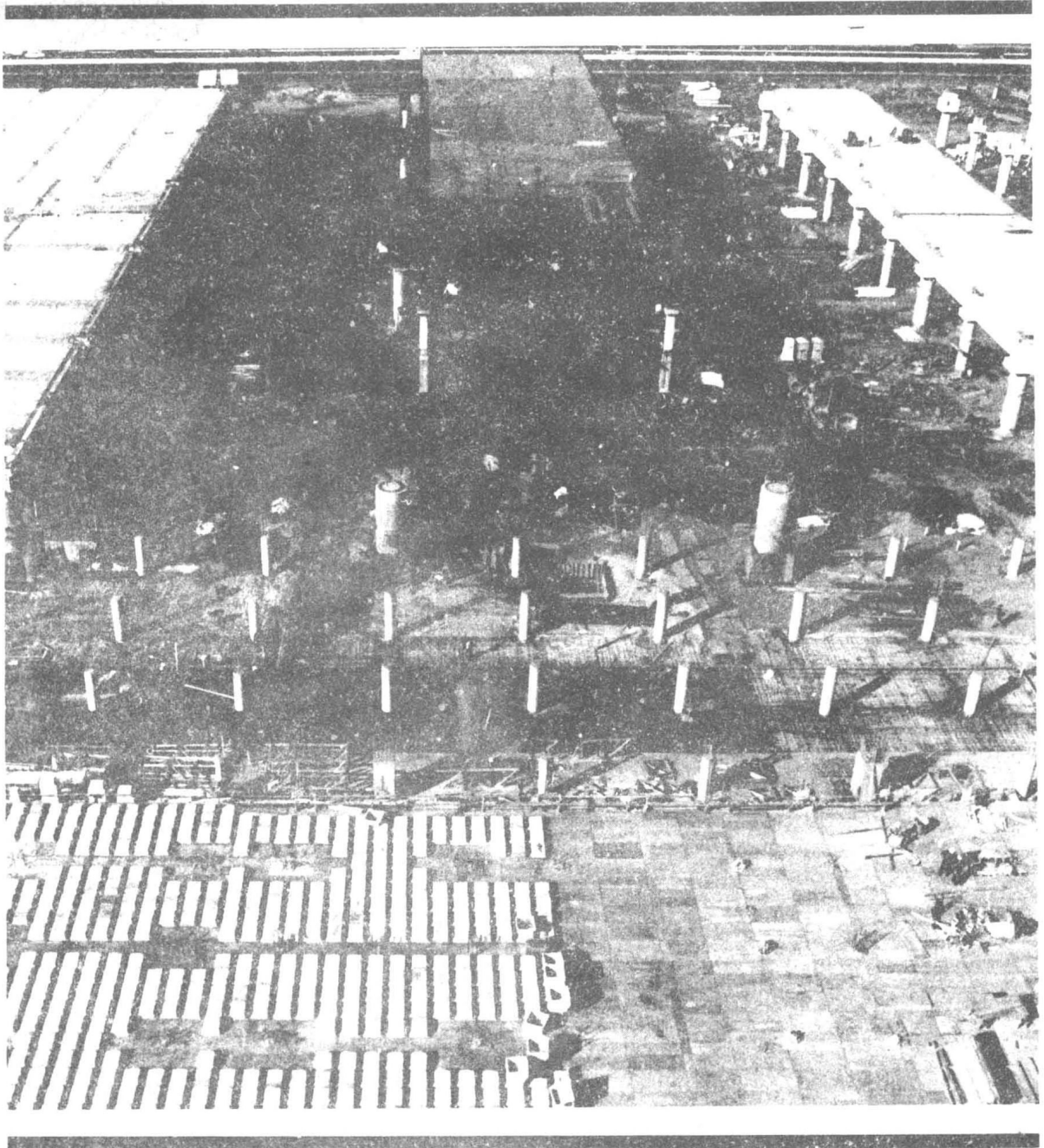
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**APPLIED
MATHEMATICS**

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and the
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1



A REVIEW OF ALGEBRA AND SET THEORY (OPTIONAL)

1.1 THE REAL NUMBER SYSTEM	1.7 SETS DEFINED
1.2 POLYNOMIALS	1.8 SET OPERATIONS
1.3 FACTORING	1.9 SAMPLE APPLICATIONS
1.4 FRACTIONS	KEY TERMS AND CONCEPTS
1.5 EXPONENTS AND RADICALS	ADDITIONAL EXERCISES
1.6 EQUATIONS	CHAPTER TEST

CHAPTER OBJECTIVES

- Review the fundamentals of algebra which are necessary for the study of the material in the remainder of this text
 - Provide an overview of the nature of sets and their representation, logic, and algebra
 - Illustrate the application of set theory
-

Algebra is the only prerequisite for using this text. The first part of this chapter provides a brief review of the elements of algebra which the author believes are important in studying the material in the remainder of the chapter. To guide you in your review of algebra it is suggested that you take the following self-correcting algebra test. Its purpose is to help you diagnose those areas in which you need more review. The results of the test can guide you in your review of Secs. 1.1 to 1.6.

Algebra Pretest

	CORRESPONDING SECTION IN CHAPTER
1 $ -10 =$	1.1
2 $x^3 \cdot x^4 =$	1.2
3 $[(x^3)^2]^3 =$	1.2
4 $x^5/x^3 =$	1.2
5 $(4x - 2y + z) - (-3x + 4y - 2z) =$	1.2
6 $\frac{2x^2(3x^3)}{(-2x^2)^2} =$	1.2
7 Factor $2a^3b^2c + 4a^2bc^2$.	1.3