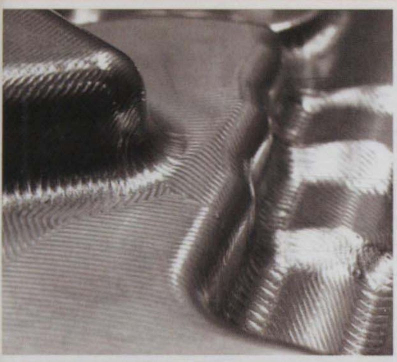
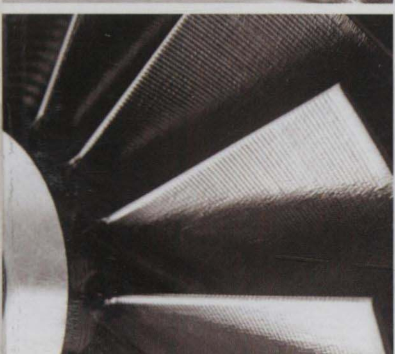
STEPHEN P. RADZEVICH

# GEOMETRY OF SURFACES

## A PRACTICAL GUIDE FOR MECHANICAL ENGINEERS





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# GEOMETRY OF SURFACES

## A PRACTICAL GUIDE FOR MECHANICAL ENGINEERS

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WILEY
A John Wiley & Sons, Ltd., Publication

This edition first published 2013


*Registered office*
John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

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*Library of Congress Cataloging-in-Publication Data*

Radzevich, S. P. (Stephen P.)
Geometry of surfaces : a practical guide for mechanical engineers / by Stephen P. Radzevich.
pages cm
Includes bibliographical references and index.
ISBN 978-1-118-52031-4 (hardback : alk. paper) – ISBN 978-1-118-52243-1 (mobi) – ISBN 978-1-118-52270-7 (ebook) – ISBN 978-1-118-52271-4 (epub) – ISBN 978-1-118-52272-1 (ebook/epdf)
1. Mechanical engineering–Mathematics. 2. Surfaces (Technology)–Mathematical models. 3. Geometry, Differential. I. Title.
TA418.7.R33 2013
516.3024′621–dc23

2012035458

A catalogue record for this book is available from the British Library.

ISBN: 978-1-118-52031-4

Typeset in 10/12pt Times by Aptara Inc., New Delhi, India
Printed and bound in Malaysia by Vivar Printing Sdn Bhd

# GEOMETRY OF SURFACES

*This book is dedicated to my wife Natasha*

# About the Author

Dr. Stephen P. Radzevich is a Professor of Mechanical Engineering and a Professor of Manufacturing Engineering. He received the M.Sc. (1976), Ph.D. (1982) and Dr.(Eng.)Sc. (1991) – all in mechanical engineering. Dr. Radzevich has extensive industrial experience in gear design and manufacture. He has developed numerous software packages dealing with CAD and CAM of precise gear finishing for a variety of industrial sponsors. His main research interest is in the kinematic geometry of surface generation, particularly focusing on (a) precision gear design, (b) high power density gear trains, (c) torque share in multi-flow gear trains, (d) design of special purpose gear cutting/finishing tools, (e) design and machining (finishing) of precision gears for low-noise/noiseless transmissions for cars, light trucks, etc. Dr. Radzevich has spent about 40 years developing software, hardware and other processes for gear design and optimization. Besides his work for industry, he trains engineering students at universities and gear engineers in companies. He has authored and co-authored over 30 monographs, handbooks and textbooks. Monographs entitled "*Generation of Surfaces*" (2001), "*Kinematic Geometry of Surface Machining*" (CRC Press, 2008), "*CAD/CAM of Sculptured Surfaces on Multi-Axis NC Machine: The DG/K-Based Approach*" (M&C Publishers, 2008), "*Gear Cutting Tools: Fundamentals of Design and Computation*" (CRC Press, 2010), "*Precision Gear Shaving*" (Nova Science Publishers, 2010), "*Dudley's Handbook of Practical Gear Design and Manufacture*" (CRC Press, 2012) and "*Theory of Gearing: Kinematics, Geometry, and Synthesis*" (CRC Press, 2012) are among recently published volumes. He has also authored and co-authored over 250 scientific papers, and holds over 200 patents on inventions in the field.

# Preface

This book is about the geometry of part surfaces, their generation and interaction with one another. Written by a mechanical engineer, this book is ***not*** on the differential geometry of surfaces. Instead, this book is devoted to the application of methods developed in the differential geometry of surfaces, for the purpose of solving problems in mechanical engineering.

A paradox exists in our present understanding of geometry of surfaces: we know everything about ideal surfaces, which do not exist in reality, and we know almost nothing about real surfaces, which exist physically. Therefore, one of the main goals of this book is to adjust our knowledge of ideal surfaces for the purpose of better understanding the geometry of real surfaces. In other words: to bridge a gap between ideal and real surfaces. One of the significant advantages of the book is that it has been written ***not*** by a mathematician, but by a mechanical engineer for mechanical engineers.

# Acknowledgments

I would like to share the credit for any research success with my numerous doctoral students, with whom I have tested the proposed ideas and applied them in industry. The many friends, colleagues and students who contributed are overwhelming in number and cannot be acknowledged individually – as much as they have contributed, their kindness and help must go unrecorded.

My thanks also go to those at John Wiley who took over the final stages and will have to manage the marketing and sale of the fruit of my efforts.

# Glossary

We list, alphabetically, the most commonly used terms in engineering geometry of surfaces. In addition, most of the newly introduced terms are listed below as well.

**Auxiliary generating surface R** a smooth regular surface that is used as an intermediate (auxiliary) surface when an envelope for successive positions of a moving surface is determined.

***CA*-gearing** crossed-axis gearing, or a gearing having axes of rotation of the gear and of the pinion that are skewed in relation to one another.

**Cartesian coordinate system** a reference system comprised of three mutually perpendicular straight axes through the common origin. Determination of the location of a point in a Cartesian coordinate system is based on the distances along the coordinate axes. Commonly, the axes are labeled $X$, $Y$ and $Z$. Often, either a subscript or a superscript is added to the designation of the reference system $XYZ$.

**Center-distance** this is the closest distance of approach between the two axes of rotation. In the particular case of hypoid gearing, the center-distance is often referred to as the offset.

**Characteristic line** this is a limit configuration of the line of intersection of a moving surface that occupies two distinct positions when the distance between the surfaces in these positions is approaching zero. In the limit case, a characteristic line aligns with the line of tangency of the moving surface and with the envelope for successive positions of the moving surface.

**Darboux frame** in the differential geometry of surfaces, this is a local moving Cartesian reference system constructed on a surface. The origin of a Darboux frame is at a current point of interest on the surface. The axes of the Darboux frame are along three unit vectors, namely along the unit normal vector to the surface, and two unit tangent vectors along principal directions of the gear tooth flank. The Darboux frame is analogous to the *Frenet–Serret* frame applied to surface geometry. A Darboux frame exists at any non-umbilic point of a surface. It is named after the French mathematician *Jean Gaston Darboux*.

**Degree of conformity** this is a qualitative parameter to evaluate how close the tooth flank of one member of a gear pair is to the tooth flank of another member of the gear pair at a point of their contact (or at a point within the line of contact of the teeth flanks).

**Dynamic surface** this is a part surface that is interacting with the environment.

**Engineering surface** this is a part surface that can be reproduced on a solid using for these purposes any production method.

**Free-form surface** this is a kind of part surface (see: *sculptured part surface* for more details).

**Indicatrix of conformity** this is a planar centro-symmetrical characteristic curve of fourth order that is used for the purpose of analytical description of geometry of contact of the gear tooth flank and of the pinion tooth flank. In particular cases, the indicatrix of conformity also possesses the property of mirror symmetry.

**Natural kind of surface representation** specification of a surface in terms of the first and second fundamental forms, commonly referred to as a *natural kind of surface representation.*

**Part surface** this is one of numerous surfaces that bound a solid.

**Point of contact** this is any point at which two tooth profiles touch each other.

***R*-gearing** a kind of crossed-axis gearing that features line contact between tooth flanks of the gear and pinion.

**$\mathbb{R}$-mapping of the interacting part surfaces** a kind of mapping of one smooth regular part surface onto another smooth regular part surface, under which normal curvatures at every point of the mapped surface correspond to normal curvatures at a corresponding point of a given surface.

**Reversibly enveloping surfaces (or just $R_e$-surfaces)** a pair of smooth regular part surfaces that are enveloping one another regardless of which one of the surfaces is traveling and which one of them is enveloping.

**Rotation vector** a vector along an axis of rotation having magnitude equal to the rotation of the axis. The direction of the rotation vector depends upon the direction of the rotation. Commonly, the rotation vector is designated $\boldsymbol{\omega}$. The magnitude of the rotation vector is commonly denoted $\omega$. Therefore, the equality $\omega = |\boldsymbol{\omega}|$ is valid. The rotation vector of a gear is designated $\boldsymbol{\omega}_g$, the rotation vector of the mating pinion is designated $\boldsymbol{\omega}_p$, and the rotation vector of the plane of action is designated $\boldsymbol{\omega}_{pa}$.

**Sculptured part surface** this is a kind of part surface parameter of local geometry where every two neighboring infinitesimally small patches differ from one another. *Free-form surface* is another terminology that is used for part surfaces of this particular kind.

**$S_{pr}$-gearing** a kind of crossed-axis gearing that features base pitch of the gear, base pitch of the pinion, and operating base pitch, which are equal to one another under various values of the axis misalignment. Gearing of this kind is noiseless and capable of transmitting the highest possible power density.

**Surface that allows for *sliding over itself*** a smooth regular surface for which there exists a motion that results in the envelope for successive positions of the moving surface being congruent to the surface itself.

**Vector of instant rotation** a vector along the axis of instant rotation, either of the pinion in relation to the gear or of the gear in relation to the pinion. The direction of the vector of instant rotation depends upon the direction of rotation of the gear and of the pinion. Commonly, the rotation vector is designated $\boldsymbol{\omega}_{pl}$.

# Notation

| | |
|---|---|
| $A_{P1}$ | apex of the base cone of the part surface $P_1$ |
| $A_{P2}$ | apex of the base cone of the part surface $P_2$ |
| $A_{pa}$ | apex of the plane of action, *PA* |
| $C$ | center-distance |
| $C_{1.P1}, C_{2.P1}$ | the first and second principal plane sections of the traveling part surface $P_1$ |
| $C_{1.P2}, C_{2.P2}$ | the first and second principal plane sections of the generated part surface $P_2$ (the enveloping surface) |
| $Cnf_R(P_1/P_2)$ | indicatrix of conformity for two smooth regular part surfaces $P_1$ and $P_2$ at a current contact point $K$ |
| $Cnf_k(P_1/R_2)$ | indicatrix of conformity that is converse to the indicatrix $Cnf_R(P_1/P_2)$ |
| $\mathscr{E}$ | a characteristic line |
| $E_{P1}, F_{P1}, G_{P1}$ | fundamental magnitudes of first order of the smooth regular part surface $P_1$ |
| $E_{P2}, F_{P2}, G_{P2}$ | fundamental magnitudes of first order of the smooth regular part surface $P_2$ |
| $\mathscr{G}_P$ | *Gaussian* curvature of a part surface $P$ at a point $m$ |
| $\mathscr{M}_P$ | mean curvature of a surface $P$ at a point $m$ |
| $K$ | point of contact of two smooth regular part surfaces $P_1$ and $P_2$ (or a point within a line of contact of the part surfaces $P_1$ and $P_2$) |
| $L_c$ (or *LC*) | line of contact between two regular part surfaces $P_1$ and $P_2$ |
| $L_{P1}, M_{P1}, N_{P1}$ | fundamental magnitudes of second order of the smooth regular part surface $P_1$ |
| $L_{P2}, M_{P2}, N_{P2}$ | fundamental magnitudes of second order of the smooth regular part surface $P_2$ |
| $O_{P1}$ | axis of rotation of the part surface $P_1$ |
| $O_{P2}$ | axis of rotation of the part surface $P_2$ |
| $O_{pa}$ | axis of rotation of the plane of action, *PA* |
| *PA* | plane of action |
| $P_{ln}$ | axis of instant rotation of two regular part surfaces $P_1$ and $P_2$ in relation to one another |
| $\mathbf{Rc}\,(PA \mapsto \mathscr{G})$ | the operator of rolling/sliding (the operator of transition from the plane of action, *PA*, to the gear, $\mathscr{G}$, in crossed-axis gearing) |
| $\mathbf{Rc}\,(PA \mapsto \mathscr{P})$ | the operator of rolling/sliding (the operator of transition from the plane of action, *PA*, to the pinion, $\mathscr{P}$, in crossed-axis gearing) |

| | |
|---|---|
| $\mathbf{Rl}_x(\varphi_y, Y)$ | the operator of rolling over a plane ($Y$-axis is the axis of rotation, $X$-axis is the axis of translation) |
| $\mathbf{Rl}_z(\varphi_y, Y)$ | the operator of rolling over a plane ($Y$-axis is the axis of rotation, $Z$-axis is the axis of translation) |
| $\mathbf{Rl}_y(\varphi_x, X)$ | the operator of rolling over a plane ($X$-axis is the axis of rotation, $Y$-axis is the axis of translation) |
| $\mathbf{Rl}_z(\varphi_x, X)$ | the operator of rolling over a plane ($X$-axis is the axis of rotation, $Z$-axis is the axis of translation) |
| $\mathbf{Rl}_x(\varphi_z, Z)$ | the operator of rolling over a plane ($Z$-axis is the axis of rotation, $X$-axis is the axis of translation) |
| $\mathbf{Rl}_y(\varphi_z, Z)$ | the operator of rolling over a plane ($Z$-axis is the axis of rotation, $Y$-axis is the axis of translation) |
| $\mathbf{Rr}_u(\varphi, Z)$ | the operator of rolling of two coordinate systems |
| $\mathbf{Rs}\,(A \mapsto B)$ | the operator of the resultant coordinate system transformation, say from a coordinate system $A$ to a coordinate system $B$ |
| $\mathbf{Rt}\,(\varphi_x, X)$ | the operator of rotation through an angle $\varphi_x$ about the $X$-axis |
| $\mathbf{Rt}\,(\varphi_y, Y)$ | the operator of rotation through an angle $\varphi_y$ about the $Y$-axis |
| $\mathbf{Rt}\,(\varphi_z, Z)$ | the operator of rotation through an angle $\varphi_z$ about the $Z$-axis |
| $R_{1.P1}$, $R_{2.P1}$ | the first and second principal radii of the gear tooth flank $P_1$ |
| $R_{1.P2}$, $R_{2.P2}$ | the first and second principal radii of the gear tooth flank $P_2$ |
| $\mathbf{Sc}_x(\varphi_x, p_x)$ | the operator of screw motion about the $X$-axis |
| $\mathbf{Sc}_y(\varphi_y, p_y)$ | the operator of screw motion about the $Y$-axis |
| $\mathbf{Sc}_z(\varphi_z, p_z)$ | the operator of screw motion about the $Z$-axis |
| $\mathbf{Tr}\,(a_x, X)$ | the operator of translation at a distance $a_x$ along the $X$-axis |
| $\mathbf{Tr}\,(a_y, Y)$ | the operator of translation at a distance $a_y$ along the $Y$-axis |
| $\mathbf{Tr}\,(a_z, Z)$ | the operator of translation at a distance $a_z$ along the $Z$-axis |
| $U_{P1}$, $V_{P1}$ | curvilinear (*Gaussian*) coordinates of a point of a smooth regular part surface $P_1$ |
| $U_{P2}$, $V_{P2}$ | curvilinear (*Gaussian*) coordinates of a point of a smooth regular part surface $P_2$ |
| $\mathbf{U}_{P1}$, $\mathbf{V}_{P1}$ | tangent vectors to curvilinear coordinate lines on a smooth regular part surface $P_1$ |
| $\mathbf{U}_{P2}$, $\mathbf{V}_{P2}$ | tangent vectors to curvilinear coordinate lines on a smooth regular part surface $P_2$ |
| $\mathbf{V}_\Sigma$ | vector of the resultant motion of the smooth regular part surface $P_1$ in relation to a reference system that the smooth regular part surface $P_2$ will be associated with |
| $d_{cnf}^{\min}$ | minimal diameter of the indicatrix of conformity $Cnf_R(P_1/P_2)$ for two smooth regular part surfaces $P_1$ and $P_2$ at a current contact point $K$ |
| $k_{1.P1}$, $k_{2.P1}$ | the first and second principal curvatures of the smooth regular part surface $P_1$ |
| $k_{1.P2}$, $k_{2.P2}$ | the first and second principal curvatures of the smooth regular part surface $P_2$ |
| $\mathbf{n}_P$ | unit normal vector to a smooth regular part surface $P$ |
| $p_{sc}$ | screw parameter (reduced pitch) of instant screw motion of the part surface $P_1$ in relation to the part surface $P_2$ |