INTERMEDIATE ALGEBRA

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+
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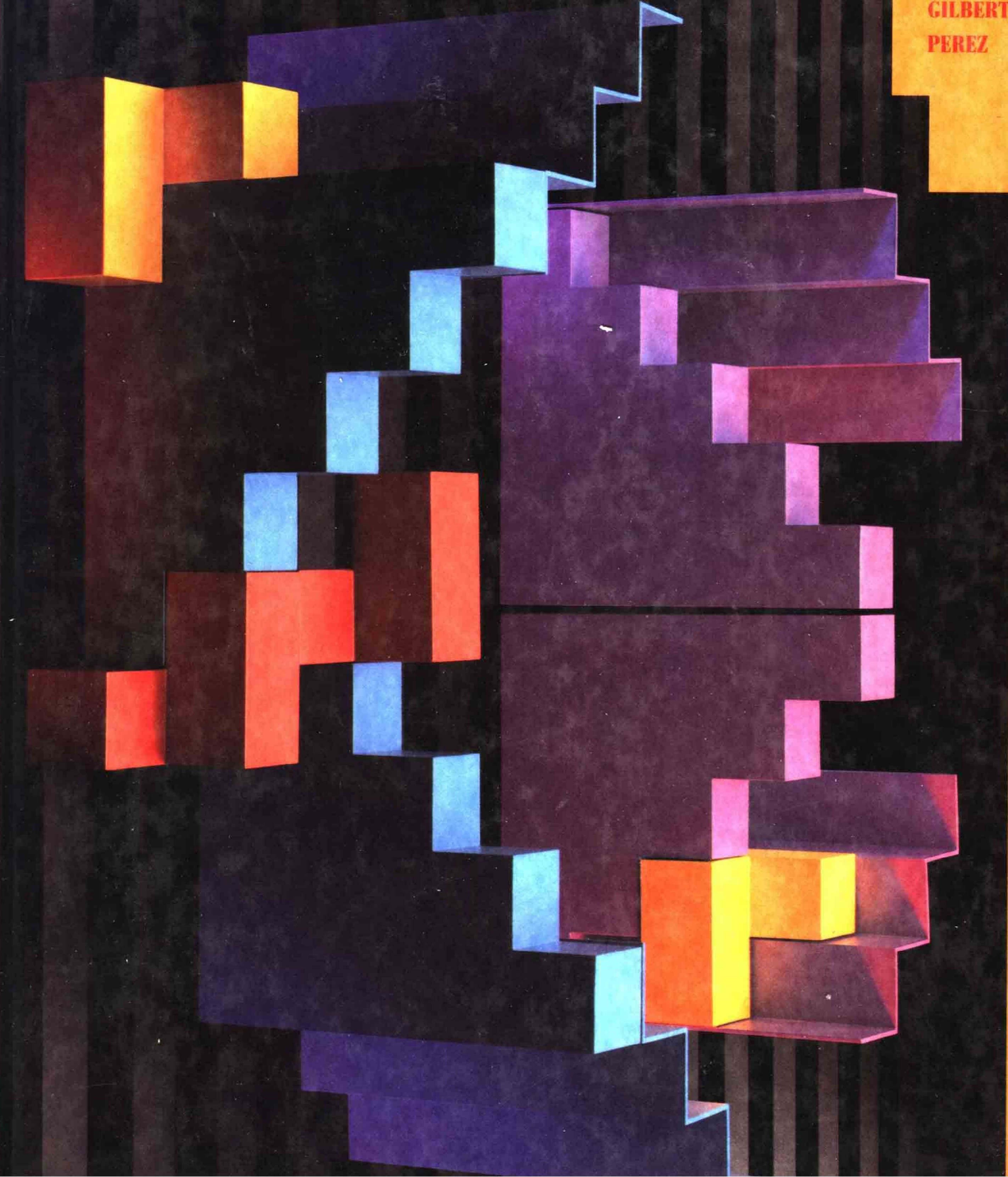

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To our parents,

who have always encouraged

and supported us

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TO THE INSTRUCTOR

This text is designed for the student who has completed a course in introductory algebra. For this student, much of the material in the first three or four chapters will be review; however, we have found that the student can benefit from reviewing some fundamental concepts. While you may choose not to spend class time on some or all of this material, having it at the beginning of the book helps the student easily find a point he or she may want to back up to, in order to review.

In deciding in what order to present the topics in our book, we were aware of varying needs in intermediate algebra classrooms. Our reasons for choosing the present order of topics include the following:

1. The properties of exponents are presented in three chapters—3, 4, and 5. The student is given the first treatment of the product laws of exponents in Chapter 3 and then has ample opportunity to work with them before the next discussion in Chapter 4. Here the student receives the remainder of the definitions and laws governing integer exponents, and he or she is given opportunity to work with them in several sections. Finally, the presentation of rational exponents and radicals in Chapter 5 reinforces the laws of exponents further.
2. Solving factorable quadratic equations was placed in Chapter 3 so that equations leading to quadratic equations could be included in sections 4.8 (rational equations), 4.9 (applications of rational equations), and 5.5 (radical equations).
3. Graphing functions and equations in two variables are introduced early, in Chapter 6, so that these important concepts for college algebra can be reinforced several times in chapters 7, 8, 9, and 10.

Our motivation for writing this text was the traditionally high attrition rate in college algebra classes. We have attempted to write a book that will help a student with a weak algebra background bridge the gap between intermediate and college algebra. To do this, we have included the following features:

1. We have attempted to address the student in a conversational style to make the material less formal and distant without, however, sacrificing

mathematical integrity. We also have injected some humor to make the subject less dry to a student who typically views math with a combination of fear and boredom.

2. The examples are thoroughly worked out, including every step that a student should need in order to follow the solutions.

3. Boxes containing common student errors are provided to warn the student of mistakes to avoid.

4. All of the important definitions, properties, formulas, and so on, are repeated in the chapter reviews to help the student remember them. In addition, the review exercises contain an ample number of problems, including all the different types of problems covered in the chapter. A timed chapter test also helps the student determine if he or she has mastered the concepts of that chapter (see, for example, the Chapter 4 review).

5. The exercises include a large number of problems, ranging from simple drill problems to more challenging ones (see, for example, the problems at the end of Section 7.2).

6. Applications, while primarily presented in four sections devoted entirely to word problems, are included wherever appropriate throughout the text (see, for example, the exercises in sections 3.1, 6.2, 7.2, and 10.3). In addition, applications are used at times to motivate the introduction of a new topic (see, for example, the beginning of Section 8.3).

7. Some more advanced topics are included, but at an introductory level, such as functions and higher order equations.

Upon completing this book, a student should be ready not only for college algebra, but also for trigonometry.

We have provided an Instructor's Manual, containing four forms of tests for each chapter, two midterms, and two final examinations. Answers to the tests are provided in an answer key in the back of the manual. The Instructor's Manual also provides answers to all even-numbered exercises in the text, plus a chart suggesting sample allotments of class periods for each section of the text. A *Study Guide with Selected Solutions*, by Vicky Lymbery and Ellen T. Wood, with additional examples and exercises, is also available.

The test bank for *Intermediate Algebra* is also available in a computerized format entitled MICRO-PAC© GENIE for use on the IBM-PC or 100-percent compatible machines. It allows you to select items in a variety of ways and print them quickly and easily. Since GENIE combines word processing and graphics with data base management, it also permits you to create your own questions—even those with mathematical notations or geometric figures—as well as to edit those provided in the test bank. To obtain additional information on MICRO-PAC© GENIE, contact your

Wadsworth-Brooks/Cole sales representative or phone Helga Newman at (415) 595-2350.

As with any book there are many people to thank, without whom this project would never have been completed. We are grateful for the professional assistance and encouragement received from Heather Bennett, Anne Scanlan-Rohrer, Merle Sanderson, Michele Judge, Andrea Cava, and Debbie McDaniel. Of course, we are also grateful to all the people at Wadsworth who helped make this book a reality. The following people reviewed the manuscript, in part or in its entirety: Beverly Abila, Rio Hondo College; Carol Achs, Mesa Community College; Judy Cain, Tompkins Cortland Community College; Leland Fry, Kirkwood Community College; Alice Grandgeorge, Manchester Community College; Alice Hagood, Alvin Community College; Daniel Hostetler, University of Cincinnati; Bill E. Jordan, Seminole Community College; John Kennedy, Santa Monica College; William Neal, Fresno City College; Wayne Rich, Utah State University; and Ellen T. Wood, Stephen F. Austin State University. They offered countless suggestions, which helped shape the manuscript into its present form. We are extremely appreciative of the help Ken Seydel, Karl Seydel, and Karen Seydel gave us in checking the exercises and the solutions for mistakes. We would like to thank our colleagues at North Harris County College and San Antonio College for their assistance and encouragement. We would especially like to thank the administration at North Harris County College for allowing us to field-test the book there.

Finally, our families and friends offered much encouragement from the beginning. Gilbert Perez would especially like to thank his wife Dottie, not only for her love and support, but also for doing the bulk of the typing.

TO THE STUDENT

Have your ever watched a tennis match on television and marveled at how the pros made it look so easy? You might have noticed that your math instructor makes algebra look easy. And yet in both cases, when you try to play tennis, or work some algebra problems, you find it not so easy. What's the reason? A tennis pro practices for hours each day. He or she played tennis for years before becoming good enough to play professionally. Your math instructor has been working with math for *years*, spending many hours a day in preparation for class. This is not to imply that you should strive for, or expect to achieve, the level of proficiency of your math instructor; but it does imply that if you want to do well in a math course you have to practice it for hours and that the more you practice, the better you should be. If you don't want to waste your time and money registering for the same algebra course semester after semester, here's what you should do:

1. *Attend class regularly and ask questions.* There are almost no students who are capable of learning algebra without going to class. You need to see and hear the instructor's explanations. While you can't expect to understand everything, you should have an idea of how to work the problems. If not, *ask!*

2. *Do your homework soon after class.* You will forget most of what was discussed in class within a few hours after class unless you do your homework during this time. Waiting until the last minute before the next class meeting to do your homework is the worst thing you can do.

3. *Read the book carefully.*
   a. It is best to read each section before the material is discussed in class. Try to follow the examples and work some of the exercises.
   b. After class, reread the section carefully. Read the statement of the problem in each example and try to solve the problem before reading the solution given in the example.
   c. It is unlikely that your instructor will assign all of the exercises for homework. Some time after you have done your homework, try to work some of the unassigned exercises. Note that the exercises marked with asterisks are more difficult than the others. Also, exercises requiring a calculator are marked by a calculator symbol:

   d. We have boxed some common mistakes that many students make. Make a special note of these when you work the exercises. They are marked by this symbol: 
   e. The answers to the odd-numbered exercises are in the back of the book. You should not look at the answers until you have finished working each problem. Remember, on a quiz or test you don't have the answers to refer to.
   f. When you have completed a chapter, use the chapter reviews to go back over the important properties, formulas, and so on, that were introduced in that chapter. Exercises from each section are provided for you to get more practice. When you think you have reviewed the chapter sufficiently, take the timed chapter test provided.

4. If you need additional help, a *Study Guide with Selected Solutions*, by Vicky Lymbery and Ellen T. Wood (Stephen F. Austin State University), is available with extra examples and exercises.

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CHAPTER 1

Fundamental Concepts

1·1 Sets

One of the most basic concepts in any mathematics course is that of a *set*. A set is a collection of objects, for example, the set that contains the counting numbers from 1 to 5 inclusive is written

$$\{1, 2, 3, 4, 5\}$$

The numbers within the set are called *elements* or *members* of the set, and they are set apart by commas and written within braces, { }, which is called *set notation*. We often name a set by using a capital letter; for example,

$$A = \{1, 2, 3, 4, 5\}$$

When we mention set A, we know that we are referring to the set $\{1, 2, 3, 4, 5\}$.

If we want to indicate that 2 is an element of a set A, we write

$$2 \in A$$

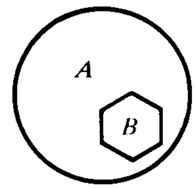
If we want to indicate that 8 is not an element of A, we write

$$8 \notin A$$

Consider the set $B = \{2, 4\}$. Each element of set B is also an element of set A, so we say that set B is a *subset* of set A and write

$$B \subseteq A$$

The concept that B is contained within A is sometimes illustrated as follows:



On the other hand, consider set $C = \{1, 3, 5, 7\}$. Since $7 \in C$ but $7 \notin A$, set C is not a subset of set A, which is written

$C \nsubseteq A$

Note the difference between the use of $\in$ and $\subseteq$. We precede $\in$ by an element and we precede $\subseteq$ by a set.

EXAMPLE 1·1·1

Consider the sets $F = \{1, 3, 5, 7, 9\}$, $G = \{3, 5\}$, and $J = \{3, 4, 5\}$. Determine whether each of the following statements is true or false.

1. $G \subseteq F$

True. Each element of G is also an element of F.

2. $J \subseteq F$

False. 4 is an element of J, but 4 is not an element of F.

3. $J \subseteq J$

True. Each element of J is also an element of J.

NOTE ▶ *Every set is a subset of itself.* Thus if A is a set, $A \subseteq A$.

An interesting set is the *empty set* or *null set*, which contains no elements. It is denoted by either $\{\ \}$ or ϕ. (ϕ is the Greek letter *phi*, pronounced "fee.") An example of the empty set is the set of counting numbers between 2 and 3. Consider the set $A = \{1, 2\}$. We can say that $\phi \subseteq A$ because if ϕ were not a subset of A, then there would have to be an element of ϕ that is not in A; but there aren't any elements in ϕ! We can say that *the empty set is a subset of any set.*

Up to now, we have considered only *finite sets*, sets which have a limited number of elements. The set that contains all the counting numbers is written

$N = \{1, 2, 3, 4, \ldots\}$

This is an *infinite set* because the number of elements in this set is not limited. We write only the first few numbers, establishing a pattern, and then write the three dots (called ellipses) to indicate that this pattern continues forever. Another example of an infinite set is the set of all even counting numbers:

$E = \{2, 4, 6, 8, \ldots\}$

We can also use the three dots when we are dealing with a finite set that has many elements. For example, the set of the first 100 counting numbers can be written

$P = \{1, 2, 3, 4, \ldots, 100\}$

EXAMPLE 1·1·2

Consider the sets $N = \{1, 2, 3, \ldots\}$, $E = \{2, 4, 6, 8, \ldots\}$, and $A = \{1, 2, 3, 4, 5\}$. Determine whether each of the following statements is true or false.

1. $E \subseteq N$

True. Each element of E is also an element of N.

2. $24 \in E$

True. 24 is even and is therefore an element of E.

3. $N \subseteq A$

False. N contains numbers like 6, 7, 8, ... that are not elements of A.

4. $\phi \subseteq A$

True. The empty set is a subset of every set.

Two sets are *equal* if they contain exactly the same elements. For example, the sets

$X = \{2, 7, 9\}$ and $Y = \{7, 2, 7, 9\}$

are equal because each contains the same elements, 2, 7, and 9, even though they are listed in different orders and one of the elements is repeated in set Y.

Suppose we are interested in the set of elements that two sets D and G have in common. This set is called the *intersection* of D and G and is denoted by

$$D \cap G$$

Other times we might be interested in putting the elements of sets D and G together into one set. This set is called the *union* of D and G and is denoted by

$$D \cup G$$

For example, suppose you are looking for a job. You might find that you can divide the prospective jobs into two sets. Let's label one set M (for money), which will represent the set of jobs that pay well. Another set might be the set of jobs in a desirable location, which we will label L. When you start looking for a job, you might want one that satisfies both conditions. The set that contains these jobs is the intersection of the two sets,

$$M \cap L$$

To be in this set, a job must be both in set M (pay well) *and* in set L (good location). But suppose, after looking for some time, you cannot find a job that satisfies both requirements. You might be willing to settle for a job that satisfies either one condition *or* the other. In that case, the set of jobs you would consider is the union of the two sets M and L,

$$M \cup L$$

We have taken all of the jobs in set M and all of the jobs in set L and formed one large set.

As another example, let $S = \{1, 2, 3, 5\}$ and $T = \{-2, 0, 2, 4\}$. Then the intersection of S and T is $S \cap T = \{2\}$. The union of S and T is $S \cup T = \{1, 2, 3, 5, -2, 0, 2, 4\} = \{-2, 0, 1, 2, 3, 4, 5\}$. You can think of the intersection as where the two sets meet, like the intersection of two streets (see Figure 1.1.1). The union of sets resembles the union of states in one country (see Figure 1.1.2).

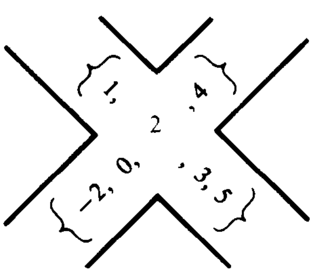


FIGURE 1·1·1

EXAMPLE 1·1·3

Consider the sets $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 3, 4\}$, $C = \{2, 4, 6, 8, \ldots\}$, $D = \{4, 8, 12, 16, \ldots\}$, $F = \{\text{all females}\}$, and $M = \{\text{all males}\}$. Find the following.

1. $F \cup M$

$F \cup M = \{\text{all people}\}$. Every person is either male or female.

EXERCISES 1·1

Consider the sets $A = \{5, 10, 15, 20, \ldots\}$, $B = \{10, 20, 30, 40, \ldots\}$, $C = \{25, 50, 75, 100\}$, and $D = \{100, 200, 300, \ldots, 1000\}$. Determine whether each of the following statements is true or false.

1. $A \subseteq B$
2. $B \subseteq A$
3. $100 \in A$
4. $100 \in B$
5. $75 \in B$
6. $700 \in B$
7. $A \subseteq A$
8. $\phi \subseteq A$
9. $15 \subseteq A$
10. $\{20\} \in B$
11. $C \subseteq A$
12. $C \subseteq B$
13. $D \subseteq A$
14. $D \subseteq B$
15. $700 \in D$
16. $A \cap B = B$
17. $A \cup B = A$
18. $A \cap C = \phi$
19. $B \cap D = \phi$
20. $\phi \cap C = \phi$

Consider the sets $M = \{1, 2, 3, \ldots, 10\}$, $N = \{1, 2, 3, 4, \ldots\}$, $D = \{1, 3, 5, 7, \ldots\}$, and $P = \{2, 4, 6, 8, 10\}$. Find the following.

21. $M \cap N$
22. $M \cap D$
23. $M \cap P$
24. $M \cup N$
25. $M \cup P$
26. $M \cup D$
27. $N \cup D$
28. $N \cap D$
29. $D \cap P$
30. $D \cup P$
31. $N \cup P$
32. $N \cap P$
33. $M \cap M$
34. $M \cup M$
35. $M \cup \phi$
36. $M \cap \phi$
***37.** $(M \cap D) \cup P$
***38.** $(M \cap N) \cap D$
***39.** $(M \cup D) \cap P$
***40.** $(M \cap P) \cup (D \cap P)$
***41.** List all the subsets of the set $\{1, 2, 3\}$.

1·2 Natural Numbers and Integers

In algebra there are instances when we wish to deal with only a specific set of numbers. In this section we will look at two sets of numbers, the natural numbers and the integers. We will consider other specific sets of numbers in the next three sections.

The set of counting numbers $\{1, 2, 3, 4, \ldots\}$ in mathematics is generally called the set of *natural numbers* and is labeled with an N. The set of *integers* includes the natural numbers, the negatives of the natural numbers, and zero. It is usually labeled with an I:

$$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

Note the relationship between these two sets:

$$N \subseteq I$$

In this section we will review the rules covering the arithmetic of integers; however, we first need to introduce the concept of *absolute value.*

Definition 1·2·1 The absolute value of a number x, denoted by $|x|$, is defined as follows:

- $|x|$ is x if x is greater than or equal to zero
- $|x|$ is $-x$ if x is less than zero.

Change 19 to -19 and -40 to 40, and change the operations before them to addition:

$$\begin{aligned} -8 - 19 + 29 - (-40) + (-11) &= -8 + (-19) + 29 + 40 + (-11) \\ &= -8 + (-19) + (-11) + 29 + 40 \\ &= -38 + 69 \\ &= 31 \end{aligned}$$

When we are working addition problems, the numbers we add are called *terms*. For instance, in the problem $7 + (-16) = -9$, the terms are 7 and -16. The answer, -9, is called the *sum*. When we are working multiplication problems, the numbers we multiply are called *factors*. For example, in the problem $5 \cdot 8 = 40$, the factors are 5 and 8. The answer, 40, is called the *product*.

Rule 4	Examples
The product of two integers with the same sign is positive.	$16 \cdot 5 = 80$ $(-3)\cdot(-14) = 42$

Rule 5	Examples
The product of two integers with opposite signs is negative.	$13\cdot(-3) = -39$ $(-4)\cdot 16 = -64$

When multiplying more than two integers, we multiply two integers at a time, starting at the left and multiplying the product each time by the next integer until all the integers have been multiplied.

EXAMPLE 1·2·3

Perform the following multiplications.

1. $(-5)\cdot(3)\cdot(-2)\cdot(-4)$

$$\begin{aligned} &= (-15)\cdot(-2)\cdot(-4) \\ &= (30)\cdot(-4) \\ &= -120 \end{aligned}$$