MATHEMATICAL METHODS FOR SCIENTISTS AND ENGINEERS

Linear and Nonlinear Systems

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Linear and Nonlinear Systems

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PREFACE

As you begin to read a new textbook, it is reasonable to ask in what way this text is different from others already available. And is the difference desirable? One has a wealth of excellently presented material to draw upon in order to learn about some particular topic, so a new book is obliged to acknowledge and use this information.

Almost half of the text is devoted to the study of nonlinear systems. This acknowledges the critical new role that nonlinear phenomena are playing in modern science and technology. The path we have taken is to develop in Part I our grasp of linear systems with particular emphasis on asymptotic methods. Then, in Part II, we rely on the developed techniques when we focus our attention on weakly nonlinear oscillatory systems and nonlinear difference equations. The two parts merge so as to bring about a balance between the need to master techniques that are important for the study of traditionally included material with the necessity to be prepared for future developments. In order to keep this text to a manageable size, some time-honored topics have been omitted. This recognizes that over the past decades much of the material traditionally taught in mathematical methods courses has been assimilated into science courses, allowing the applications to be closely tied to mathematical techniques. Thus, for example, students can learn the special functions of math physics in electricity and magnetism courses, where the solution of Maxwell's equations in the standard geometries requires the use of trigonometric functions, Bessel functions, and Legendre polynomials. By contrast, material such as complex variables and matrix theory have evolved into courses in their own right. With these thoughts in mind, I decided on a selection of topics that balances techniques that are important to master, but that are not usually discussed at length in subject matter courses with methods that are so essential that I cannot omit them. I emphasize perturbation, or approximation, techniques that are relatively easy to use and that are applicable to a broad spectrum of problems that occur in science. I assume that the readers have a firm background in treating problems that admit a solution in a closed form and have become the foundation of undergraduate courses in the physical sciences and engineering. For example, I expect that readers know about the exponential function, the simple harmonic oscillator, the conservation of energy, and principles of mechanics, etc., as well as integral and differential calculus, elementary differential equations, and elementary matrix theory. I omit any discussion of complex variables and I do not develop any methods that rely on their use. This is done to make the discussion accessible to a larger audience and with the recognition that interested students usually take an independent course in complex variables.

I emphasize that this is not a math text, and thus I omit rigorous proofs and, in addition, permit a certain degree of carelessness in restrictions and conditions. For example, only matrices with distinct eigenvalues are considered since the case of degenerate eigenvalues is unusual and many times causes needless complexities in the proofs. I mean by this that there are advantages in assuming that our matrices have distinct eigenvalues since they can then be brought to diagonal form. If a particular matrix has degenerate eigenvalues, we can think of introducing a perturbation that gives a small separation to the eigenvalues. In most cases, this does not substantially change the problem.

The text is designed to help the reader develop tools of analysis that can be used to treat problems that do not admit a simple solution in a closed form, but that have their origin in solvable problems with which the reader is already familiar. I have adopted a repetitive style in which the same thing is phrased in a variety of ways, in part to help the reader catch at a later time something that was missed the first time around. It also recognizes that, in a textbook, there is a definite exchange of control between the author and the reader since the latter chooses the pace to proceed when reading the text. My choice of the material to emphasize and reemphasize serves as guideposts for the reader.

I envision the text as containing approximately enough material for a one-year course in mathematical methods for advanced undergraduates or beginning graduate students in physics, chemistry, engineering, biology, and other sciences. The examples are drawn from applications in these fields and cast in a language or form that reveals the underlying or fundamental aspects of the problem under study. There is also some discussion of incorrect techniques and inadequate methods so as to alert the reader to wrong avenues of attack or possible traps. A few basic examples are used throughout so as to enable the reader to have robust and familiar paradigms as a guide. I have used the computer software packages *Lotus 1-2-3* (Lotus Development Corp.) and *Math CAD* (MathSoft, Inc.) to generate some of the numerical and graphical solutions. In my opinion, it is essential for all of us to develop

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expertise at using such programs to generate solutions and to improve our calculational insight.

After presenting some preliminary material, I discuss some properties of matrices and introduce the gamma function, which forms the foundation of our calculational techniques by enabling us to evaluate the most commonly occurring integrals. This is followed by a discussion of related functions and the concept of asymptotic expansions and approximations. This leads to the Euler–MacLaurin sum expansion, a powerful technique to evaluate a large class of sums and one that has the capability to often cast results in a form usable either in conjunction with a computer or a desk calculator. I then introduce the Laplace method of evaluation of integrals that depend upon a large parameter. This technique is discussed in some detail so as to enable the interested reader to study, independently, the related method of stationary phase and the method of steepest descent, both of which require an understanding of complex variables. This section of the text concludes with a discussion of the asymptotic behavior of second-order linear differential equations and an introduction to the related perturbation theory.

I then direct the reader's attention, in the second part of the book, to a lengthy and detailed discussion of weakly nonlinear oscillating systems that have their origin in the simple harmonic oscillator. It has become common practice to devote almost all of our curricular attention to linear systems and to neglect nonlinear problems. This is done in part because a systematic discussion of linear systems is possible through the powerful techniques of superposition, classification of solutions, and final determination of coefficients.

On the other hand, nonlinear problems are interesting and also occur in diverse areas of science. They can be studied by a variety of powerful and understandable techniques that are within the capabilities of advanced undergraduate or beginning graduate students. My goal is to introduce the reader to nonlinear problems and the associated models that occur in diverse fields of science and engineering.

My approach is to first analyze the harmonic oscillator (with and without damping) by a technique that prepares us for some of the new phenomena that appear in nonlinear problems. I then go on to discuss the logistic equation, which is a simple nonlinear model that occurs in situations where one is trying to understand growth with limited resources. This prepares us for the tackling of models of weakly nonlinear oscillatory systems. I emphasize the weak character of the problems we are treating because they have a natural small parameter that we use to order terms in our perturbation expansion. I begin by discussing the classical methods of analysis due to Lindstedt and Poincaré. Then I develop the methods of averaging and multiple time scales, the two most universally applicable and commonly used methods of analysis.

I include in our discussion of nonlinear systems problems that require some preparation before they are in the proper form to be analyzed by the techniques we have developed. I conclude the study of nonlinear systems with a brief introduction to topics associated with the notion of chaos. This is accomplished through an analysis of one-dimensional quadratic maps and a derivation of the Feigenbaum numbers.

It is my experience that we are so thoroughly trained or brainwashed by linear problems that we are reluctant to let go of the intuition we have gained in this area when studying nonlinear problems, even though we know that this intuition can be both misleading and incorrect. We find, generally, that a perturbation expansion of nonlinear problems exhibits a relationship between the lowest-order approximation and the correction terms that is unfamiliar from our study of linear problems. Thus, we need some help in getting started in the right direction. For example, the simple harmonic oscillator, while being the fundamental starting point for the analysis of the Duffing oscillator, does not prepare us for the treatment of secular terms that are the new aspect of oscillatory motion, which can in this case be traced to the nonlinearity. It is essential to struggle to change our perspective from linear to nonlinear thinking if only because so much of modern science is concerned with nonlinear problems. I hope that the effort that has been put into this aspect of the text provides an adequate starting point for this venture.

Bibliographies are given at the end of each of the chapters of Part I and following Chapters 15, 16, and the Appendix.

I want to thank the students at Stony Brook, particularly Cherian Varughese, Usha Ravi, and Fernando Camilo, for sharing with me in the development of this text. It is my first extensive writing project and it took a long time, almost 20 years of classroom experience to bring it to fruition. This was in great part because my ideas regarding what was essential and what was not have changed significantly during this period. The audience has also changed and many excellent texts have appeared. My ideas firmed about ten years ago and I have used this period to try them out in the classroom and bring them into what I hope is a coherent and interesting form.

I wish to thank my colleagues Cliff Swartz, Arnie Feingold, and David Fox for their interest in this project. Cliff, an experienced author, gave me lots of much needed advice regarding how to get started and how to transform lecture notes into a text. Arnie read the first half of the manuscript with some care. He did not agree with everything either as regards emphasis or style, but he listened to my ideas and points of view and tried to help me avoid errors in logic and pedagogy as well as in substance. David read the second half and helped me to express my ideas with more clarity than I am used to. I have also greatly benefited from the comments on the second part of the manuscript by my research collaborator Juan Lin of Washington College, with whom I have talked endlessly regarding many aspects of nonlinear systems. Yair Zarmi of the Ben Gurion University of the Negev and the Jacob Blaustein Institute for Desert Research at Sede Boqer, Israel, invited me to the Institute to participate in its research program as well as to give lectures on nonlinear dynamics. This interaction led to Yair's incisive reading of the material on nonlinear oscillatory systems. I would also like to thank Madan Lal Mehta of C. E. N. Saclay, who over the past years has shared his ideas with me and both read the manuscript and gave me his valued comments. I have also enjoyed my frequent conversations with Lee Segel of the Weizmann Institute. He explained a number of things to me and encouraged me to clarify many of my ideas. Finally, I greatly appreciate the help I received from Jeffrey Kahn and Meng Zhou on all matters relating to computing; from Lois Koh who did the illustrations; from Bea Shube, recently retired from John Wiley and Sons, who encouraged me to get the book written; and from Marion Mastauskas and Heike Gustafson who worked with me to transform the typed manuscript into a form acceptable to the publisher.

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PART I

LINEAR SYSTEMS

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