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Quantitative
GEOGRAPHY

Quantitative Geography

Techniques and Theories in Geography

JOHN P. COLE

*Lecturer in Geography,
Nottingham University*

CUCHLAINE A. M. KING

*Reader in Geography,
Nottingham University*

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Preface

The purpose of this book is to present in one volume a large number of techniques and theories used in geography. For the most part it is fairly elementary in scope. Indeed it starts each topic assuming that the reader knows nothing or very little about it. In this way it is hoped that many people will be able to obtain a background that will enable them both to appreciate the so-called 'quantitative' papers now appearing in many geographical periodicals, and also to practise techniques and develop theories in their own work.

When using this book the reader is asked to abandon temporarily all his preconceived ideas about the scope and content of geography. He will then be more receptive to the mathematics and statistics introduced in chapters 2 and 3 as an essential basis for material discussed in the rest of the book. A large number of step by step worked examples are given, and numerous maps and diagrams have been included to help those who learn more quickly with the help of visual illustrations.

The order in which the topics are arranged was given much thought. It was based as far as possible on the idea that some topics have to be covered and learned before others can be started. There are however many ways in which the 15 chapters could have been arranged. Clearly therefore it will sometimes be desirable for the reader to go back and reconsider earlier chapters or sections after having read later ones. To some extent the chapters may be considered self-contained entities and read independently, and the book may prove useful as a kind of handbook to be referred to from time to time.

In connexion with the scope and contents of the book, the following points should be noted. Firstly, there is very little about the nature of geography, though some references are given on this matter in chapter 1. Secondly, obviously even in a book this size it has not been possible to cover all techniques and theories in geography. Some have been given considerable attention while others have been neglected. Thirdly, both the choice of techniques and theories, and the choice of examples, have been dictated by the limitations of knowledge and specialisms of the authors. Geomorphology and political geography are over represented at the expense of other branches. Fourthly, both physical and human geography are considered. Often similar methods of approach can be used. Cross-fertilization between these major branches of geography can greatly enrich the whole subject, and help to

maintain the unity of geography at a time of increasing specialization. Fifthly, of the many examples in the book, a considerable number are fictitious; some are unrealistic in order to be simple.

Briefly, next, a few words on why a book of this kind is necessary. Awareness of place, relative location, distance and other underlying concepts in geography have presumably long been part of the experience of man. Geography as a discipline has however been widely regarded by people outside professional geography itself in a very limited sense. In particular, it is seen and referred to by many either as an inventory of locations or positions of things in the world, or as the physical background of areas being studied for particular purposes. It is to the credit of a limited number of geographers, mainly American, that an attempt has been made to turn geography into the science of areal or spatial relationships. There have been various isolated attempts to do this previously, but since the Second World War, and particularly since about the mid-1950s, there has been a more conscious movement to borrow techniques and theories from such diverse disciplines as statistics, psychology, physics and engineering, and to use them in geography.

One purpose of this book, then, is to bring together some of the things that have been done during the so-called quantitative revolution in geography, and to make them accessible to others. The authors feel that while geography is not a vocational subject, it has nevertheless a great deal to offer in an applied sense (see chapter 14) and as a way of seeing things. They are of the opinion that a quantitative approach is highly desirable in geography and beneficial to it.

The authors are grateful to many colleagues and other friends for ideas and advice of various kinds. Above all, they are greatly indebted to Professor C. W. J. Granger, of the Department of Mathematics, Nottingham University, for his continual encouragement and advice on statistical matters and his comments on geographical matters. Professor R. Emerson, of the University of East Anglia, gave much help and encouragement. Professor T. Hagerstrand of the University of Lund, Sweden, helped in the organization of the book and in many other ways. Mr. J. M. Bates of the Department of Economics, Nottingham University, very kindly read chapter 3 and Miss R. E. Gent similarly checked chapter 2. The authors also wish to thank Mr. G. A. Smith for agreeing to the inclusion of a great deal of material, in the collecting and processing of which he collaborated. It is perhaps unfair to select a few names from those of the many students at Nottingham who have become interested in quantitative methods, but discussion with the following has been most useful: Dr. John Andrews, Mrs. K. M. Bonniface, Mr. D. J. Blair, Mr. R. S. Andrew, Miss M. R. Tinning and Miss J. Brindley. Finally, the authors would like to thank Professor K. C. Edwards for the encouragement he has given to the development of quantitative techniques in the Geography Department at Nottingham University.

J. P. COLE
C. A. M. KING

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1

Introduction

'If the earth were uniform — well polished, like a billiard ball — there probably would not be any such science as geography ...'

J. Gottmann (1951)

'Without the hard little bits of marble which are called "facts" or "data" one cannot compose a mosaic; what matters, however, are not so much the individual bits, but the successive patterns into which you arrange them, then break them up and rearrange them.'

A. Koestler (1964)

1.1 Introduction

The subject matter that comes within the field of geography is very diverse. This great diversity makes it essential that the geographer should be able to arrange his data systematically in order to make use of them in his own special way. The geographical way of looking at things differs from that of some other sciences, although the subject matter may be the same. The essential element in nearly all geographical work is spatial variation, as is so admirably suggested by Gottman's statement quoted above. The importance of this aspect will be stressed throughout the book.

The purpose of chapter 1 is to draw attention to some of the basic characteristics of geographical data and to consider ways of looking at this material. Section 1.2 discusses various definitions of the field of geography. It is not possible to set rigid bounds to this field, but suggestions can be made to help to delimit the very broad field of geography. Section 1.3 draws attention to some of the basic characteristics of geographical data. Attempts are made to systematize the data for ease of handling and to draw attention to similarities between apparently unlike situations in geography. Section 1.4 deals briefly with some of the criticisms levelled at traditional geography and with attempts to provide answers to them. In section 1.5 the need for a scientific approach is stressed. Section 1.6 outlines the sequence of topics adopted in this book to explore some of the ways in which geography can be studied.

1.2 The content of geography

The field of geography is the earth's surface*. This however is also the field of many other sciences. Geography is the science that is mainly concerned

* The term surface is used here not in a strictly mathematical sense, but synonymously with shell (see glossary).

with the distribution of elements that occur on the earth's surface and with the variations of the distributions through time and in space. Not all the distributions that occur however are necessarily within the field of geography. Moreover, geography does not have a particular set of objects, but shares those of other disciplines. Some limits can be set to this broad field, both by considering space and by considering the scope of the enquiry or study.

(1) The space limitations of geography can be fixed more easily than the scope of the enquiry. It has already been said that geography is concerned with the surface of the earth. The interior of the earth is therefore excluded. The core zone is the sphere of the geophysicist, and the geographer is only very indirectly concerned with this zone. The core is surrounded by the uniform mantle, and this in turn is separated from the crust by the Moho discontinuity. This boundary surface provides a reasonable lower limit to the earth's surface for most geographical purposes. The upper limit can be defined by the upper limit of the atmosphere, within which are confined the weather phenomena that have such an important effect on the earth's surface. With the wide occurrence of artificial satellites however it has now become important to reconsider the upper limit. Beyond this upper limit lie the solar system and the stars. Although the solar system has a great effect on the earth's surface, it does not lie directly in the geographical field. Indirectly its influence must often be taken into account in geographical problems. The stars lie beyond the scope of geography and can be left to the astronomers. In this sphere also, however, geographers may use the stars for surveying and fixing positions on the earth. Thus the solar system and the core of the earth have an effect on the earth's surface and from this point of view are relevant to geography, but in themselves they are not part of geography.

(2) It is more difficult to define the limitations of scope than of space. Geography is concerned with the distribution of different elements on the earth's surface. The larger elements, such as countries, towns or mountain ranges, usually have geographical relevance. The limit to the geographical study of these elements is usually reached where they become increasingly subdivided into smaller entities. As long as they are undivided units they often have geographical relevance. When however an object cannot be further subdivided into like units, its study often becomes more relevant to another science. Geography, therefore, deals mainly with whole objects and with organisms which exist as units and which, as such, are discrete and may be able to move or be moved as one body. This size limitation excludes nearly all objects that can only be studied with a microscope. Geography is therefore mainly concerned with objects of medium size or large size and not of very small or microscopic size.

The limit of size varies from one branch of geography to another. It is often related to the competence of the investigator. Some examples will illustrate how geographical data merge into those of other disciplines as the size of the object is reduced. Plant associations and entire single plants are studied in biogeography, but the dissection of plants is left to the botanists. The same applies to the animal in relation to the zoologist. Geographers are

concerned with man as a whole body, but not with the way in which he is put together, this being the field of anatomy and physiology. Similarly towns form the basis of urban geography. They can be divided into buildings which are still whole units and can still be studied by geographers. However, the interior layout of the building is more the concern of the architect than of the geographer.

In a study of dynamic situations, processes, not discrete objects, are involved. Geomorphologists are interested in landscape evolution, but in some cases the study of the processes involved must be left to the physicist or the chemist. The study of climate is relevant to geography, but this leads to a study of meteorology, which when pursued in depth lies within the domain of physics. These limits are more often set by the competence of the worker. The transport network, another dynamic situation, is a legitimate field of study of the geographer, but he is not concerned with the workings of the vehicles that use the network, this aspect being the engineer's concern.

The indivisible unit may be taken as the smallest object that is really relevant to geography. The geographer may then study the interaction of independent units and their effect on each other. Each unit is normally a member of another more generalized class of units. The relationship between the units and their associated classes of units is considered in chapter 13, when classification is discussed. The lowest member of the system of classification is usually the indivisible unit.

(3) Distributions of individual units on the earth's surface can normally be considered to have geographical interest. Distributions within the units, for example the distribution of veins and arteries in the body, or certain other distributions, such as the distribution of stars in the sky, are not geographical. However such non-geographical distributions can often be made use of in geography. They can be used as an analogy to assist in the understanding of geographical distributions. J. Hutton (1795) for instance, used the analogy between the circulation of blood in the human body and the circulation of water on the earth. Other examples are discussed in chapter 11. It is also possible for the operation to be reversed. The geographical attitude to distributions could have useful analogies in non-geographical distributional problems. The principle of working with analogies from one discipline to another is one of the basic features of *General Systems Theory*, which is discussed in chapter 11.

1.3 The nature of geographical data

(a) *Spatial, temporal and other variations*

It was suggested in section 1.2 that one of the most important distinguishing features of geography is its concern with *spatial* variations. A distinction is sometimes made between spatial, temporal and other variations. This is illustrated by three examples in figure 1.1, each occurring on a line.

A spatial variation, at least from the point of view of the geographer, implies relative location of objects in space. Relative location may be expressed by distance along a line from a point of origin, by two co-ordi-

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nates on a surface, or by three co-ordinates in 3-dimensional space. A 2-dimensional example is the earth's surface, on which each place has a particular relationship to every other place. Thus each object and observation studied in geography has a unique location in space at any given time, and from its unique location each is in a special relationship to all other objects or observations in space.

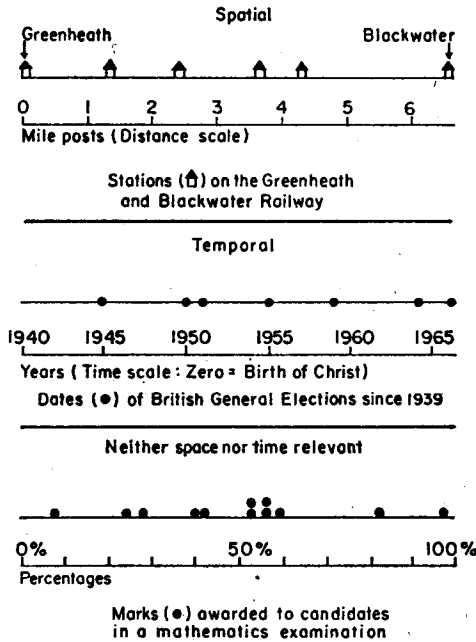


Figure 1.1 Spatial, temporal and other variations on a line.

Figure 1.1 shows simple examples of spatial and temporal variations and of a case of variation in which relative location is not relevant either in space or in time. On the imaginary railway, the stations are placed at clearly distinguishable points along a distance scale. On the temporal scale, the elections are similarly placed in order of occurrence in time. The marks in the mathematics examination were however simultaneously obtained by the candidates in a given room during a given period; in this case space and time are irrelevant for each candidate, although the room does of course occupy space and the exam covers a period of time.

To illustrate the nature of variations further, some information is given in the table below about ten fictitious school children who sit in the front row in a classroom. In practice it is more likely that one would try to obtain information about considerably more than ten items in a study of the kind shown, but to keep the example reasonably small, only ten have been used.

Moreover, the information is more appropriate to the field of education than to geography. The purpose of the example is however to show that some variations are essentially spatial in character and that others are not.

Table 1.1 Front row of class in Melford school

	I	II	III	IV	V	VI	VII	VIII
William Bacon	BR	E	3	M	5	108	8/3	950
Joseph Cunningham	BR	E	5	M	10	96	8/6	960
John Hemlock	BR	E	7	M	1	116	8/4	1110
Mavis Holt	BR	E	9	F	2	112	9/0	210
Edward Knight	BR	E	11	M	4	114	8/4	230
Joan Myers	BR	E	13	F	6	101	8/8	670
Jean Primrose	BR	E	15	F	7	95	8/2	400
Frank Smith	BR	E	17	M	8	93	8/11	820
Roger Treadgold	BR	E	19	M	3	114	8/11	1440
Anthony Yates	BR	E	21	M	9	102	8/10	90

- I Nationality (BR = British)
- II Native language (E = English)
- III Distance in feet of seat in class from left hand wall of classroom
- IV Sex (M = Male, F = Female)
- V Teacher's ranking of the children according to ability
- VI Result of IQ test adjusted for age but not yet revealed to teacher
- VII Age in years and months to nearest month
- VIII Distance of child's home in yards from school gate along shortest feasible route

Eight pieces of information are given about each child. I and II, nationality and native language do not vary; they are therefore 'constants', though potentially they could become variables if non-British or non-English speaking children happened to be in the set under consideration. III is a variable with spatial implications. Note, however, that as the desks in the class are spaced at regular intervals apart, the variation is *regular*. Moreover, the spatial arrangement of objects in a room (regular or irregular) is not generally considered by geographers to be of direct importance to them. IV is a biological variable (and therefore neither a spatial nor a temporal variable). It has only a yes/no answer. V and VI are educational (intelligence assessment) variables, and like IV, have no spatial or temporal connotations. It may be noted in anticipation of the discussion of levels of data later in the book that V has its information ranked (with intervals of one unit), while VI has its information with different intervals between the marks. VII is of temporal interest since the ages of the children can be plotted on a time scale in relation to a base month, for example that in which the eldest was born. Of the eight columns of data, VIII is of particular interest to the geographer since it is of a spatial nature. The geographer might be interested solely in the distance each child lives from school (regardless of direction) as a measure of effort and/or time required to journey between school and home. He might however also be interested in the actual distribution of the

homes of the ten children in area, not only in relation to the school, but also in relation to each other and even to all or some part of the general setting, say a town, in which the school is situated. Figure 1.2 compares variable VII, plotted on a time scale, with variable VIII plotted both on a distance line and in area.

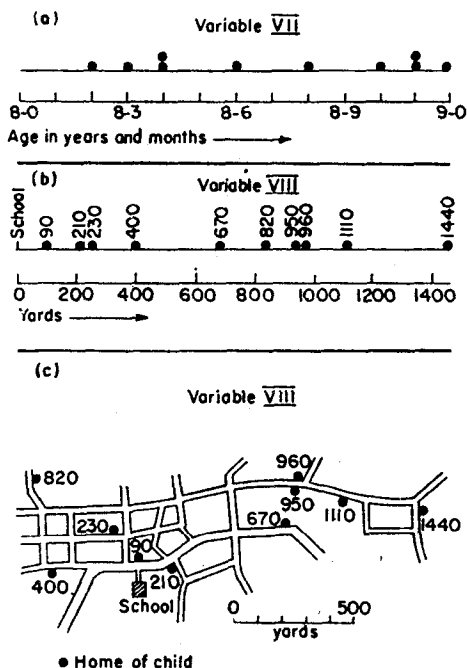


Figure 1.2 Examples of variations in time and in space. VII and VIII refer to information given about ten children in the table in the text. (a) Variations on a time scale. (b) Distances from a point (the school entrance). (c) Relative distances and directions from a point (school entrance) of ten children.

VIII is the only variable in the table that is of direct interest to the geographer. But in analysing the facts given by variable VIII the geographer might use some or all of the rest of the information available. Suppose, further, that several children had some other language, rather than English, for their native tongue. The urban geographer might then be interested to know if these non-English speaking children all lived in one part of the town, or if they were dispersed among the English speaking children. Thus the geographer can use non-spatial data in conjunction with spatial data.

In table 1.1 several kinds of information are given about the children. There is some confusion over the difference between quantitative and qualitative types of information. Levels or scales of data, nominal, ordinal, interval and ratio, are discussed in chapters 2 and 3. Here, attention is drawn

to the possibility of converting apparently qualitative information into a quantitative, numerical form if only in terms of 1 to represent presence of some feature and 0 to represent its absence. Thus male and female can be expressed as 1 and 0, male or not male.

(b) The objects and spaces studied by the geographer

The geographer is interested in the arrangement of objects in space. The objects he studies are also studied in other disciplines (e.g. plants and animals in biology). Moreover, the geographer does not have a complete monopoly of the study of space. This interests equally for example the astronomer, geologist and architect. The geographer, however, handles both the objects and the spaces in a particular way, often making drastic simplifications and abstractions. Some of these must now be noted.

Both the spaces in which the objects are arranged, and the objects themselves exist in the 'real' world in 3-dimensional space. Although more sophisticated measures may be used, the shape of a given space or object may be described very approximately by a comparison of the length of its longest axis with the length of the other two axes at right angles to the longest axis. Very broadly, it is possible to have spaces or objects (which occupy spaces) that have all three axes roughly equal (ball or cube), two long and one short (paving slab) or one long and two short (pencil or rod). These are illustrated in figure 1.3a.

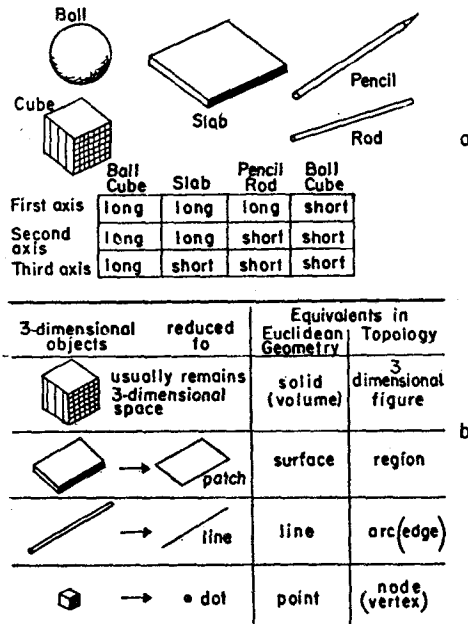


Figure 1.3 The shape of objects and spaces. (a) Long and short axes.

(b) The elimination of dimensions.

Often in geography it is adequate and convenient to overlook one, two or even three of the dimensions (see figure 1.3*b*). Thus the short axis or axes may be overlooked. The slab may therefore be thought of as a 2-dimensional surface and the rod as a line. The small compact object may still be reduced to a dot of no dimensions by analogy with a point in geometry, if it is sufficiently small in relation to the space in which it is situated. Further aspects of 3-dimensional objects and space will be dealt with in chapter 9.

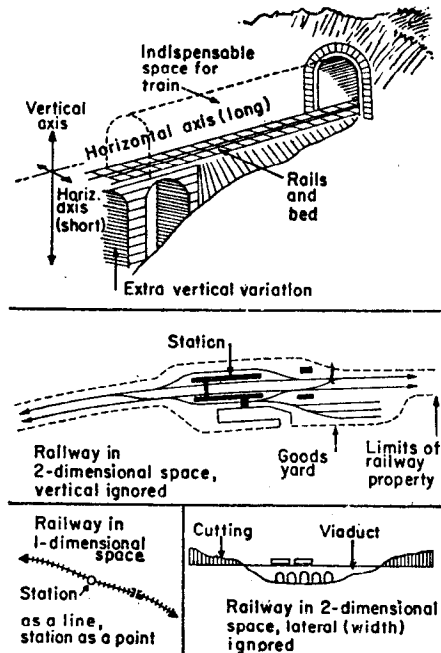


Figure 1.4 A railway as an example of the reduction of a 3-dimensional space to a line in geography.

The example of a railway illustrates what has been said so far (see figure 1.4). The railway, with all its attachments, forms a space, and the trains using it are objects. The railway clearly exists in three dimensions. It varies both in width (one horizontal axis) and in depth (vertical axis). Stations, sidings and other features give variations in lateral extent, while viaducts and cuttings give variations in vertical extent. The unbounded but indispensable space above the rails, through which the train has to pass, makes the space in which a railway operates 3-dimensional throughout. Usually, however, the geographer greatly reduces the scale of a railway before looking at it, and then loses sight of, or interest in, anything but its long axis or length. In other words, the railway, really a greatly elongated 3-dimensional space,