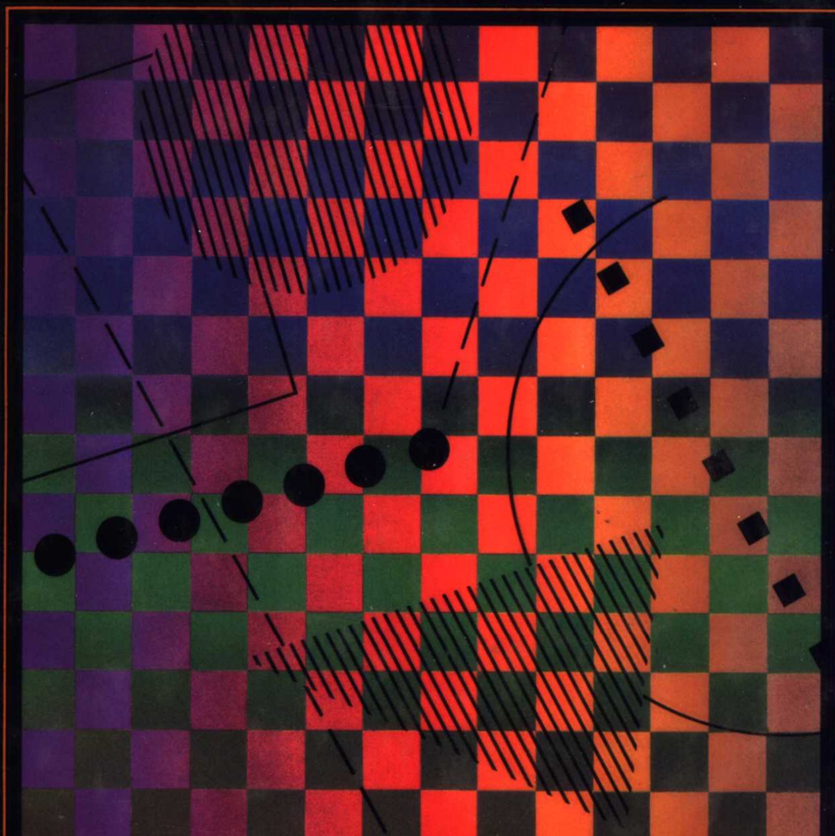
ALGEBRA AND TRIGONOMETRY
WITH ANALYTIC GEOMETRY



KARL J. SMITH

Algebra and Trigonometry

WITH ANALYTIC GEOMETRY

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This Book Is Dedicated
with Love to My Wife,
Linda Ann Smith

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Formulas

Distance between (x_1, y_1) and (x_2, y_2): $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$, where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$.

Slope of the line passing through (x_1, y_1) and (x_2, y_2): $m = \Delta y/\Delta x$.

Pythagorean Theorem: The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

Quadratic Formula: If $ax^2 + bx + c = 0$, $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Discriminant: $b^2 - 4ac$
$b^2 - 4ac < 0 \rightarrow$ *no real* solutions
$b^2 - 4ac = 0 \rightarrow$ *one real* solution
$b^2 - 4ac > 0 \rightarrow$ *two real* solutions

Arithmetic Sequence: $a_n = a_1 + (n - 1)d$

Geometric Sequence: $g_n = g_1 r^{n-1}$

Arithmetic Series: $A_n = \dfrac{n}{2}[2a_1 + (n - 1)d]$

Geometric Series: $G_n = \dfrac{g_1(1 - r^n)}{1 - r}$; $\quad G = \dfrac{g_1}{1 - r}$ if $|r| < 1$

Binomial Expansion: $(a + b)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$, where $\dbinom{n}{k} = \dfrac{n!}{k!(n - k)!}$

Forms of a Complex Number z:

Rectangular form: $z = a + bi$
Polar form: $z = r \text{ cis } \theta$
$\qquad = r(\cos \theta + i \sin \theta)$
Conversions: $a = r \cos \theta \qquad r = \sqrt{a^2 + b^2}$
$\qquad b = r \sin \theta \qquad \theta' = \tan^{-1} \left|\dfrac{b}{a}\right|$, where θ' is the reference angle for θ

De Moivre's Formula: If n is a natural number, $(r \text{ cis } \theta)^n = r^n \text{ cis } n\theta$.

Complex Root Theorem: If n is a positive integer, $(r \text{ cis } \theta)^{1/n} = \sqrt[n]{r} \text{ cis} \dfrac{1}{n}(\theta + 360°k)$, where $k = 0, 1, 2, \ldots, (n - 1)$.

Cramer's Rule: If

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases} \quad \text{then} \quad x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D} \quad (D \neq 0)$$

D is the determinant of the coefficients and the constants b_1, b_2, b_3 replace the coefficients x, y, z for D_x, D_y, and D_z, respectively.

Matrices

$A + B = [a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} = [a_{ij} + b_{ij}]_{m \times n}$

$cA = c[a_{ij}]_{m \times n} = [ca_{ij}]_{m \times n}$

$I = [a_{ij}]_{n \times n}$, where $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

$AB = [a_{ij}]_{m \times n}[b_{ij}]_{n \times p} = \left[\displaystyle\sum_{k=1}^{n} a_{ik}b_{kj}\right]_{m \times p}$

$IA = AI = A$

$A^{-1}A = AA^{-1} = I$

Principle of Mathematical Induction: If a given proposition $P(n)$ is true for $P(1)$ and if the truth of $P(k)$ implies its truth for $P(k + 1)$, then $P(n)$ is true for all positive integers n.

Functions

Definition: A function f of a real number x is a rule that assigns a single real number $f(x)$ to each x in the domain of the function. This definition can be characterized as a mapping or as a set of ordered pairs.

Constant Function: $f(x) = a$

Linear Function: $f(x) = mx + b$

Standard form:	$Ax + By + C = 0$
Point-slope form:	$y - k = m(x - h)$
Slope-intercept form:	$y = mx + b$
Two-point form:	$y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$ or $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$
Intercept form:	$\dfrac{x}{a} + \dfrac{y}{b} = 1$
Horizontal line:	$y = k$
Vertical line:	$x = h$

Quadratic Function: $f(x) = ax^2 + bx + c,\ a \neq 0$

Absolute Value Function: $f(x) = |x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Polynomial Function: $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0,\ a_n \neq 0$

Rational Function: $f(x) = P(x)/D(x)$, where $P(x)$ and $D(x)$ are polynomial functions and $D(x) \neq 0$.
Asymptotes: Let $f(x) = P(x)/D(x)$, where $P(x)$ and $D(x)$ have no common factors. Then:
1. The *vertical line* given by $x = r$ is an asymptote if $D(r) = 0$.
2. The *horizontal line* given by $y = k$ is an asymptote if $f(x) \to k$ as $|x| \to \infty$.
3. The *slant line* given by $y = mx + b$ is an asymptote if $f(x) = mx + b + \dfrac{R(x)}{D(x)}$; that is, if the degree of P is one more than the degree of D.

Exponential Function: $f(x) = b^x \quad (b > 0, b \neq 1)$
Laws of exponents: Let a and b be nonzero real numbers and let m and n be any real numbers, except that the form 0^0 and division by zero are excluded.

FIRST LAW: $b^m \cdot b^n = b^{m+n}$

SECOND LAW: $\dfrac{b^m}{b^n} = b^{m-n}$

THIRD LAW: $(b^n)^m = b^{mn}$

FOURTH LAW: $(ab)^m = a^m b^m$

FIFTH LAW: $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

Logarithmic Function: $f(x) = \log_b x \quad (b > 0, b \neq 1)$
Laws of logarithms:

FIRST LAW: $\log_b AB = \log_b A + \log_b B$

SECOND LAW: $\log_b \dfrac{A}{B} = \log_b A - \log_b B$

THIRD LAW: $\log_b A^p = p \log_b A$

Log of both sides theorem: $\log_b A = \log_b B \Leftrightarrow A = B$

Change of base theorem: $\log_a x = \dfrac{\log_b x}{\log_b a}$

Trigonometric Functions: Let θ be an angle in standard position with a point $P(x, y)$ on the terminal side a distance of $r = \sqrt{x^2 + y^2}$ from the origin $(r \neq 0)$. Then the trigonometric functions are defined as follows:

$\cos\theta = \dfrac{x}{r} \qquad \sin\theta = \dfrac{y}{r} \qquad \tan\theta = \dfrac{y}{x} \quad (x \neq 0)$

$\sec\theta = \dfrac{r}{x} \quad (x \neq 0) \qquad \csc\theta = \dfrac{r}{y} \quad (y \neq 0) \qquad \cot\theta = \dfrac{x}{y} \quad (y \neq 0)$

Conic Sections

$y = mx + b$	is the **line** with slope m and y intercept b.
$y - k = m(x - h)$	is the **line** with slope m passing through (h,k).
$y - k = a(x - h)^2$	is the **parabola** with vertex (h,k) opening upward if $a > 0$ and downward if $a < 0$.
$x - h = a(y - k)^2$	is the **parabola** with vertex (h,k) opening to the right if $a > 0$ and to the left if $a < 0$.
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	is the **ellipse** with center at $(0,0)$, x intercepts $\pm a$, y intercepts $\pm b$, and foci $(\pm c,0)$ with constant sum $2a$: $(a^2 - b^2 = c^2)$.
$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$	is the **ellipse** with center at $(0,0)$, x intercepts $\pm b$, y intercepts $\pm a$, and foci $(0,\pm c)$ with constant sum $2a$: $(a^2 - b^2 = c^2)$.
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	is the **hyperbola** with center at $(0,0)$, x intercepts $\pm a$, foci $(\pm c,0)$, and constant difference $2a$: $(a^2 + b^2 = c^2)$.
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	is the **hyperbola** with center at $(0,0)$, y intercepts $\pm a$, foci $(0,\pm c)$, and constant difference $2a$: $(a^2 + b^2 = c^2)$.
	$y = \pm \frac{b}{a}x$ are the **asymptotes** in both cases.

These standard conic sections can be translated or rotated:

Translation substitution:

$x' = x - h$

$y' = y - k$

Rotation substitution:

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

where $\cot 2\theta = (A - C)/B$

(see page 484)

Polynomials

Theorems: Let $P(x)$ and $Q(x)$ be polynomial functions.

Zero Factor Theorem: If $P(x)Q(x) = 0$, then either (or both) $P(x) = 0$ or $Q(x) = 0$.

Remainder Theorem: If $P(x)$ is divided by $x - r$ until a remainder independent of x is obtained, then the remainder is equal to $P(r)$.

Intermediate Value Theorem for Polynomial Functions: If $P(x)$ is a polynomial function on $[a,b]$ such that $P(a) \neq P(b)$, then P takes on every value between $P(a)$ and $P(b)$ over the interval $[a,b]$.

Factor Theorem: If r is a root of the polynomial equation $P(x) = 0$, then $x - r$ is a factor of $P(x)$. Also, if $x - r$ is a factor of $P(x)$, then r is a root of the polynomial equation $P(x) = 0$.

Root Limitation Theorem: A polynomial function of degree n has, at most, n distinct roots.

Location Theorem: If P is a polynomial function such that $P(a)$ and $P(b)$ are opposite in sign, then there is at least one real root on the interval $[a,b]$.

Rational Root Theorem: If $P(x)$ has integer coefficients and has a rational root p/q (where p/q is reduced), then p is a factor of a_0 and q is a factor of a_n.

Upper and Lower Bound Theorem: If $a > 0$ and all the numbers in the last row have the same sign in the synthetic division of $P(x)$ by $x - a$, then a is an upper bound for the roots of $P(x) = 0$. If $b < 0$ and the numbers in the last row alternate in sign in the synthetic division of $P(x)$ by $x - b$, then b is a lower bound for the roots of $P(x) = 0$.

Descartes' Rule of Signs: Let $P(x)$ be written in descending powers of x.

1. The number of positive real zeros is equal to the number of sign changes or is equal to that number decreased by an even integer.
2. The number of negative real zeros is equal to the number of sign changes in $P(-x)$ or is equal to that number decreased by an even integer.

Fundamental Theorem of Algebra: If $P(x)$ is a polynomial of degree ≥ 1 with complex coefficients, then $P(x) = 0$ has at least one complex root.

Number of Roots Theorem: If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has exactly n roots (if roots are counted according to their multiplicity).

Trigonometry

Trigonometric Functions

Let θ be an angle in standard position with a point $P(x,y)$ on the terminal side a distance of r from the origin ($r \neq 0$). Then the trigonometric functions are defined by

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}\ (x \neq 0)$$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}\ (y \neq 0)$$

$$\tan\theta = \frac{y}{x}\ (x \neq 0) \qquad \cot\theta = \frac{x}{y}\ (y \neq 0)$$

Inverse Trigonometric Functions

Inverse Function	*Domain*	*Range*
$y = \text{Arccos}\ x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \text{Arcsin}\ x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \text{Arctan}\ x$	all reals	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \text{Arccot}\ x$	all reals	$0 < y < \pi$

Exact Values

function \ angle θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.	0	undef.
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{2}}$	2	undef.	-1	undef.
$\csc\theta$	undef.	2	$\frac{2}{\sqrt{2}}$	$\frac{2}{\sqrt{3}}$	1	undef.	-1
$\cot\theta$	undef.	$\frac{3}{\sqrt{3}}$	1	$\frac{\sqrt{3}}{3}$	0	undef.	0

Identities

Fundamental Identities

1. $\sec\theta = \dfrac{1}{\cos\theta}$
2. $\csc\theta = \dfrac{1}{\sin\theta}$
3. $\cot\theta = \dfrac{1}{\tan\theta}$
4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
5. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
6. $\cos^2\theta + \sin^2\theta = 1$
7. $\tan^2\theta + 1 = \sec^2\theta$
8. $1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

9. $\cos(\frac{\pi}{2} - \theta) = \sin\theta$
10. $\sin(\frac{\pi}{2} - \theta) = \cos\theta$
11. $\tan(\frac{\pi}{2} - \theta) = \cot\theta$

Opposite-Angle Identities

12. $\cos(-\theta) = \cos\theta$
13. $\sin(-\theta) = -\sin\theta$
14. $\tan(-\theta) = -\tan\theta$

Sum and Difference Identities

15. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
    $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
16. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
    $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
17. $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
    $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double-Angle Identities

18. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
    $= 2\cos^2\theta - 1$
    $= 1 - 2\sin^2\theta$
19. $\sin 2\theta = 2\sin\theta\cos\theta$
20. $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities

21. $\cos\frac{1}{2}\theta = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$
22. $\sin\frac{1}{2}\theta = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$
23. $\tan\frac{1}{2}\theta = \dfrac{1 - \cos\theta}{\sin\theta}$
    $= \dfrac{\sin\theta}{1 + \cos\theta}$

Product Identities

24. $2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$
25. $2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$
26. $2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
27. $2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$

Sum Identities

28. $\cos\alpha + \cos\beta = 2\cos\left(\dfrac{\alpha + \beta}{2}\right)\cos\left(\dfrac{\alpha - \beta}{2}\right)$
29. $\cos\alpha - \cos\beta = -2\sin\left(\dfrac{\alpha + \beta}{2}\right)\sin\left(\dfrac{\alpha - \beta}{2}\right)$
30. $\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha + \beta}{2}\right)\cos\left(\dfrac{\alpha - \beta}{2}\right)$
31. $\sin\alpha - \sin\beta = 2\sin\left(\dfrac{\alpha - \beta}{2}\right)\cos\left(\dfrac{\alpha + \beta}{2}\right)$

PREFACE

This book is designed to prepare students for the study of calculus by presenting comprehensive coverage of the traditional topics of college algebra and trigonometry. There is no magic key to success with this book; you will need to spend time with the concepts and problems on a daily basis, and you will need to work hard. Try to read the text before the lecture, work through the examples with a pencil and paper, and ask questions. Above all, you will need to work many problems. The book, however, makes it easy to know what is important:

- Important terms are presented in **boldface type**.
- Important ideas are enclosed in boxes.
- Common pitfalls and helpful hints and explanations are shown in *italic*.
- WARNINGS are given to call your attention to common mistakes.
- Color is used in a functional way to help you "see" what to do next.
- The most important goals of each section are listed as objectives at the end of each chapter.
- Answers to the odd-numbered problems are provided to let you know if you are not on the right track; many solutions and hints are also provided and more than 470 graphs are presented in the answer section alone to provide additional examples.
- Problems are graded and presented in pairs or triplets so that you can practice some of the simpler problems before progressing to the more challenging ones.
- Review problems are provided for each chapter objective, and cumulative reviews are given as practice tests.

The success of any textbook depends upon the strength of its problem sets. I have devoted an extraordinary amount of effort to writing the problems for this text. The problem sets are divided into A, B, and C problems, according to the level of difficulty. A wide variety of applied problems is provided to help you develop the type of problem-solving ability you will need in calculus.

A great deal of flexibility is possible in the instructor's selection of topics presented in this book. For this reason, more material is provided here than can be used in a single semester or quarter. For example, an entire trigonometry course is presented in Chapters 8 and 9. Complex numbers can easily be skipped or included. Optional topics include variation (Sec. 4.6), graphing techniques (Chapter 5), partial fractions (Sec. 6.6), matrix algebra (Sec. 10.2, 10.3), Cramer's Rule (Sec. 10.5, 10.6), linear programming (Sec. 10.7) and additional topics in algebra (Chapter 11). Graphing techniques (Chapter 5) are not required to study conic sections (Sec. 5.5, 5.6) or polynomial and rational functions (Chapter 6) or conic sections (Chapter 12).

Because inexpensive calculators are readily available, the computational aspects of the logarithmic functions in Chapter 7 have been minimized. Likewise, the presentation of the trigonometric functions in Chapter 8 assumes that calculators are available. In case they are not, alternative solution techniques using tables are also provided.

Three main types of logic are used on calculators: arithmetic logic, algebraic logic (recognizes the order-of-operations convention from algebra), and RPN logic (operation entered last). It is suggested that you use a calculator with algebraic or RPN logic. RPN calculators are characterized by an ENTER or SAVE key. If you are using a calculator with arithmetic logic, you must use parentheses or pick out the operations to be performed first. For example, consider the problem $2 + 3 \cdot 4$, which illustrates the differences among the three types of logic:

ALGEBRAIC LOGIC	RPN LOGIC	ARITHMETIC LOGIC
2	2	3
+	ENTER	×
3	3	4
×	ENTER	=
4	4	+
=	×	2
	+	=

If you input 2 + 3 × 4 = on a calculator with arithmetic logic, you will obtain the incorrect answer 20. For this reason, a calculator with algebraic or RPN logic is recommended.

The correct output is 14.

Many thanks go to the reviewers of this book: Jan Cole; Gerald Goff, Oklahoma State University; Lynda Morton, University of Missouri; and Phillip Schmidt, SUNY—The College at New Paltz; as well as to my students who worked through several preliminary versions. I am also indebted to Donna Szott, who checked and rechecked every problem and example in this book. Thanks also go to Delores Howard, who did an excellent typing job.

I would also like to thank all the people who helped to make this book a reality: my wife and family, who need to live with my schedule; Jeremy Hayhurst, Amy Mayfield, Joan Marsh, and Vernon Boes of Brooks/Cole; and Carl Brown and Cece Munson, who made the everyday production of this book a pleasure.

Karl J. Smith
Sebastopol, CA

CONTENTS

*Sections that require complex numbers.

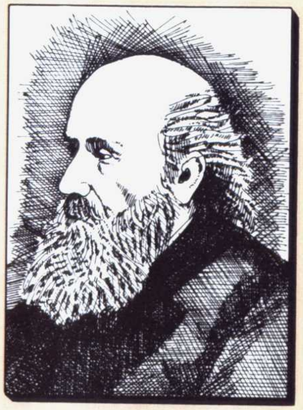
†If you would like an Algebra–Trigonometry book with less review material, the author has an alternate form of this book that combines Chapters 1–3 into a single chapter. The title is *Precalculus Mathematics: A Functional Approach*, and it is available from Brooks/Cole Publishing Company.

1

Fundamental Concepts

There are three ruling ideas, three so to say, spheres of thought, which pervade the whole body of mathematical science, to some one or other of which, or to two or all three of them combined, every mathematical truth admits of being referred; these are the three cardinal notions, of Number, Space and Order. Arithmetic has for its object the properties of number in the abstract. In algebra, viewed as a science of operations, order is the predominating idea. The business of geometry is with the evolution of the properties of space, or of bodies viewed as existing in space.

J. J. SYLVESTER
Philosophical Magazine
Vol. 24 (1844), p. 285



J. J. Sylvester (1814–1897)

It is appropriate to begin our study of algebra and trigonometry with a quotation from J. J. Sylvester, one of the most colorful men in the history of mathematics. Sylvester's writings are flowery and eloquent. He was able to make the dullest subject bright, fresh, and interesting. His enthusiasm and zest for life were evident in every line of his writings. He was, however, a perfect fit for the stereotype of an absent-minded mathematics professor.

Born and educated in England, he accepted an appointment as Professor of Mathematics at the University of Virginia in 1841. After only three months, he resigned because of an altercation with a student. He returned to England penniless, but in 1876 he came back to America to accept a position at Johns Hopkins University. The seven years he spent at Johns Hopkins were the happiest and most productive of his life. During his stay there he founded, in 1878, the *American Journal of Mathematics.*

It is in the spirit of Sylvester that I write this book for you, the student. I have tried to deal with your frustrations and concerns and write a book that is bright, fresh, and interesting. I have tried to make the material easy to read and understand. If I have overlooked anything, I hope that you don't dismiss this as the work of a typical absent-minded professor; please take the time to write me a letter or a postcard and tell me.

As you begin your study of algebra and trigonometry, I want to remind you that learning mathematics requires systematic and regular study. Do not expect it to come to you in a flash, and do not expect to cram just before an examination. Take it in small steps, one day at a time, and you will find success at your doorstep.

CHAPTER OVERVIEW This chapter reviews the fundamental concepts you will need for your study of algebra and trigonometry. The focus in this chapter is a review of the various sets of numbers and their properties that you will encounter as you progress through the course. This chapter has 14 specific objectives that you will need to master. They are listed at the end of the chapter on pages 39–41.

1.1 REAL NUMBERS

In mathematics, when we speak about numbers we need to have clearly in mind the types of numbers under consideration. For example, if someone says to you, "pick a number," it is unlikely that you would choose $\sqrt{5}$ or $\frac{\pi}{2}$. When we say "consider a number" in this course, however, we would want to include choices such as these. These numbers are called **real numbers**. You have been introduced to various **sets** of numbers in your previous mathematics courses, but it is worthwhile to review them here.

Natural numbers: $N = \{1, 2, 3, 4, \ldots\}$

Whole numbers: $W = \{0, 1, 2, 3, 4, \ldots\}$

Integers: $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rationals: $Q = \{$all Quotients $\frac{p}{q}$ where p is an integer and q is a nonzero integer$\}$
These are numbers whose decimal representations either terminate or repeat.

Irrationals: $Q' = \{$numbers whose decimal representations do not terminate or repeat$\}$

Real numbers: $R = \{$numbers which are in Q or in $Q'\}$*

A capital letter is usually used to name a set. The set of natural numbers is often referred to by the letter N; the letter I is used to refer to the set of integers; and Q (for quotients) refers to the set of rationals. One method of designating a set is to enclose a list of its **members**, or **elements**, in braces, $\{\ \}$, and to use three dots, if needed, to indicate that the numbers continue in the pattern shown. The set is said to be **finite** if the number of elements in the set is a counting number or less than some counting number. If it is not finite, it is said to be **infinite**. A set with no members is called the **empty set**, or **null set**, and is labeled $\{\ \}$ or ϕ.

This course will focus attention on the set of real numbers, which is easily visualized by using a **coordinate system** called a **number line**, as shown in Figure 1.1. A **one-to-one correspondence** is established between all real numbers and all points on such a number line:

1. Every point on the line corresponds to precisely one real number.
2. For each real number, there corresponds one and only one point.

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* Later in this book we will consider a set which includes the real numbers but also numbers which are not real numbers. This set is called the set of **complex numbers**. For now, however, we will limit ourselves to the set of real numbers.

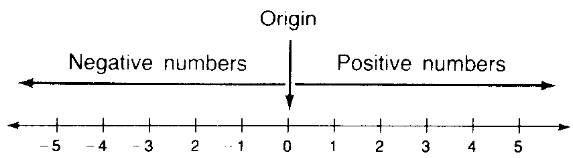


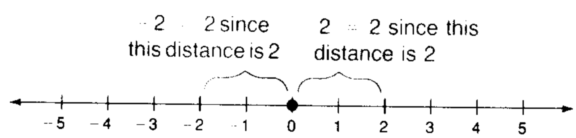
Figure 1.1 A real number line

A point associated with a particular number is called the **graph** of that number, and the number is called the **coordinate** of the point. Numbers associated with points to the right of the origin are called **positive real numbers**, and those associated with points to the left are called **negative real numbers**. A number line is also a convenient way of ordering any two real numbers. If a point whose coordinate is a lies to the right of a point whose coordinate is b on a number line, then ***a* is greater than *b***, which is written $a > b$. For example, $6 > 3$ and $-4 > -10$. If a point whose coordinate is b is to the left of a point whose coordinate is a, then we say that ***b* is less than *a***, or $b < a$. For example, $3 < 6$ and $-10 < -4$. The other symbols of comparison are summarized below:

$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b

The symbols $>$, $\geq$, $<$, and $\leq$ are referred to as the **inequality symbols**.

A concept called **absolute value** is also associated with points on a number line. The symbol $|a|$ is read "the absolute value of a" and means the distance between the graph of a and the origin. For example:



EXAMPLE 1 **a.** $|5| = 5$ **b.** $|-5| = 5$ **c.** $\left|-\frac{1}{2}\right| = \frac{1}{2}$

d. $|\sqrt{5}| = \sqrt{5}$ **e.** $-|-5| = -5$ (↑ The opposite of the absolute value of −5) **f.** $-|\pi| = -\pi$ (↑ The opposite of the absolute value of π) ■

Since we will want to use the notion of absolute value in a variety of contexts, it is important to state a formal definition that is equivalent to the above geometric characterization that used distances.