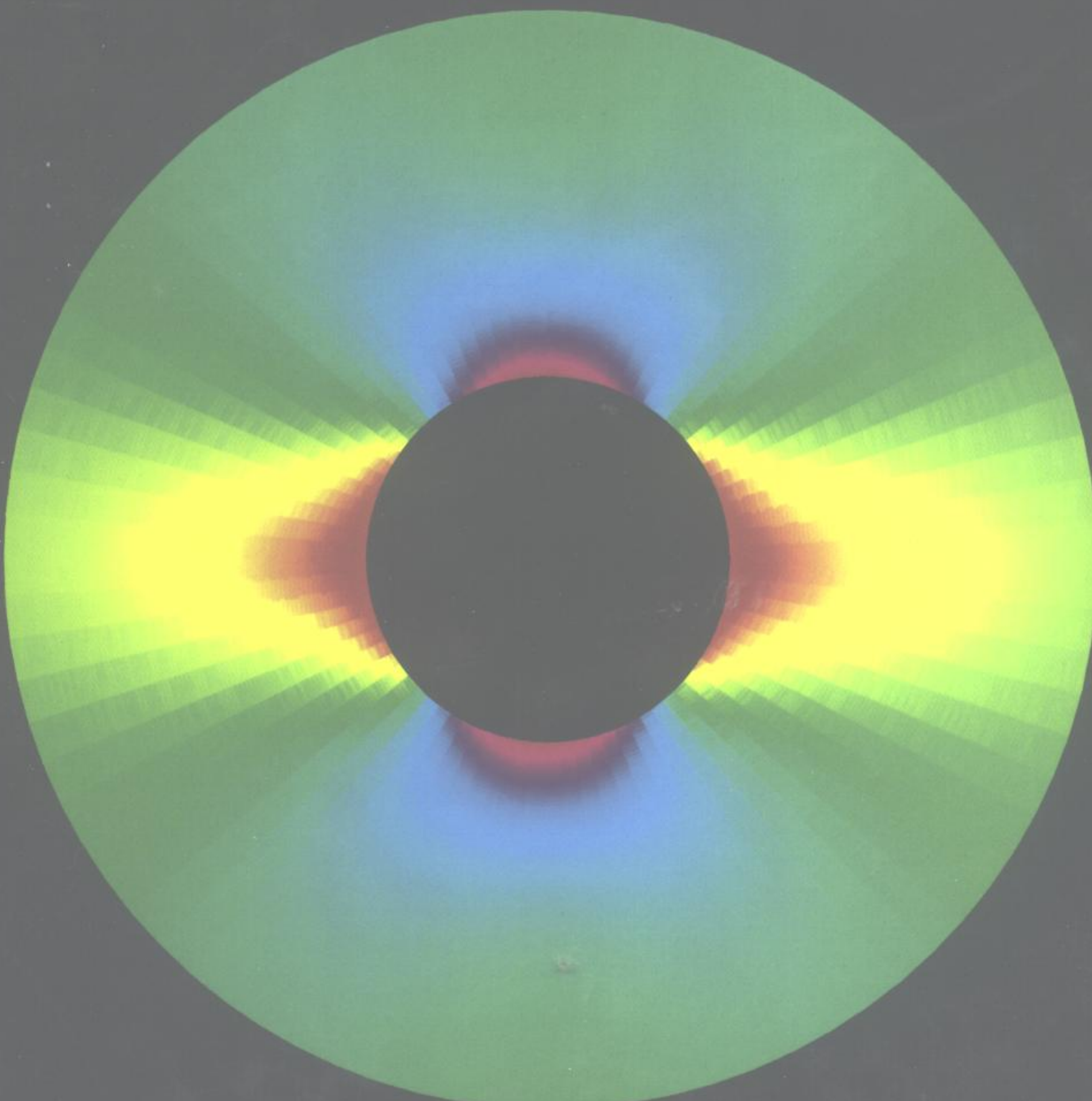
MECHANICS *of* MATERIALS

David Roylance



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MECHANICS OF MATERIALS

PREFACE

This text is intended to meet the needs of undergraduate courses in mechanics of materials, required of students in many engineering curricula. It is written primarily for use as an introduction to the area, but, depending on which topics are emphasized, it can be used in more advanced levels as well.

Mechanics of materials has been taught pretty much the same way for many years, using topics and methods described beautifully over 50 years ago in classic texts by Stephen Timoshenko. There are now many texts based on his approach, which emphasizes deriving formulas and working problems involving stresses and deformations in simple structures built with linear elastic materials. This text retains many of these traditional topics, which are fundamental to engineering practice. However, this text is a definite departure from the Timoshenko approach and should be considered by instructors willing to consider something a little different.

I have included a number of topics not found in most mechanics of materials texts, such as time-dependent effects and fracture phenomena. Such coverage is needed because today's engineers use many methods and materials not treated by the traditional texts. Beyond these "modern" topics, university instruction is most effective and interesting when it includes at least some coverage of "context" material. Several topics in this text, such as finite element methodology and statistical aspects of materials properties, may be scheduled as separate subjects later in the engineering curriculum. I feel that an introductory overview of these topics is valuable both for students who do not study these subjects later and for those who do. It makes the subjects easier to understand if they are taken later, and it helps put the traditional, introductory material in its proper context even for those who do not study the more advanced subjects.

In what is probably the most striking departure from traditional texts, I have sought to relate the mechanics of materials to the chemistry and microstructure of modern engineered materials, building on materials topics in chemistry and other subjects in the engineering science core curriculum. An appreciation of the relations between a material's processing, microstructure, properties, and eventual performance is extremely useful in mechanical design. Intuition is the designer's most important tool, and an important objective in teaching mechanics of materials is to help the student develop a sense of how the materials actually respond to mechanical loading. Coverage should encompass the materials in use today, to include those with appreciable time dependency and anisotropy in their mechanical response, in addition to the linear elastic materials usually studied. Formal concepts in stress and strain are important, but so is an appreciation of the molecular and microscopic response of materials to these stresses. This is my natural bias as a professor of materials engineering, but I am convinced that this mindset is useful to all engineers working with materials.

Today's design engineers use computer software for much of their work, and this text takes advantage of symbolic manipulation and spreadsheet methods in addition to programmed code. (Several examples employ the MAPLE™ package, which is available at steep student discounts on many campuses.) The computer allows the student to sidestep many tedious manipulations that have only marginal educational value and to see more easily the underlying mechanics. The programmed codes for stress transformations and finite element calculations included in the text diskette fall short of commercial standards in graphics-based user friendliness, but will allow the student to begin to acquire expertise in numerical modeling.

I have tried to describe the various physical concepts simply and at some length initially, and then gradually generalize the treatment both geometrically and mathematically. The issue of mathematical level and notation is an important one, as the more concise methods found in most of the professional literature tend to be daunting to the beginning student. This is true even of students with good mathematical skills, since the problem in most engineering subjects is not so much carrying out the various mathematical operations—such as algebraic manipulations and differentiation—as grasping mathematics as a *descriptive language* for engineering problems.

However, it is vital that students become comfortable with this language, just as medical students must grasp the jargon of their field to become effective physicians. I have made greater use of vector and matrix notation than in traditional texts, out of the belief that the extra abstraction is more than compensated by the value of immersion in the proper technical language. Further, the geometrical aspects of mechanics problems can lead to lengthy, tedious expressions that tend to distract the reader when only scalar algebraic methods are used, in spite of their being more direct.

The mathematical level of the text is not overly demanding. Most of the topics and problems require only algebraic manipulation and numerical evaluation, although a few topics—such as polymer viscoelasticity and fracture theory—employ mathematical techniques that are not needed in traditional mechanics of materials texts. Laplace transforms and convolution integrals are examples. However, these techniques are included in most first subjects in differential equations, which the student is assumed to have taken before or perhaps concurrently with this subject. These are not difficult techniques, especially if used in conjunction with symbolic manipulation software, and neither students nor instructors should shy away from them.

The text begins with a survey of the response of materials to simple tension. Here the geometry is simple, so that the basics of stress and strain and materials response can be described with scalar expressions. This permits a broad range of response to be covered—including energy-controlled elasticity of metals and ceramics, entropic elasticity of rubber, viscoelasticity of leathery polymers, and anisotropy of composite materials—leading to a walk along the stress-strain curve to preview the ways materials respond to increasingly higher stresses. This sets the stage for coverage in subsequent chapters of the structural types used in most mechanical designs: trusses, pressure vessels, torsion rods, beams, plates, and laminates. As the dimensionality increases, appropriate mathematical expressions and notational forms are introduced and used to preserve the basic simplicity of the concepts. After the mathematical foundations of solid mechanics have been established through this step-by-step process, an overview of closed-form, experimental, and finite element stress analysis can be presented in a natural and succinct way.

The final two chapters of the book describe the response of materials to stresses beyond the elastic limit. Yield and plastic flow are very materials dependent, and the engineer has access to a wide variety of materials processing methods to tailor and optimize the yielding process. This is one of the common grounds of mechanical and materials engineers, and is of great practical as well as theoretical value. The final chapter treats modern methods of designing for fracture and fatigue, including the elements of statistical variability, fracture mechanics, and time-dependent damage accumulation.

LIST OF SYMBOLS

A	Area; free energy; Madelung constant
$\mathbf{A}$	Transformation matrix
$\mathcal{A}$	Plate extensional stiffness
a	Length; transformation matrix element; crack length
a_T	Time-temperature shifting factor
B	Design allowable for strength
$\mathbf{B}$	Matrix of derivatives of interpolation functions
$\mathcal{B}$	Plate coupling stiffness
b	Width; thickness
$\bar{b}$	Burgers' vector
C	Stress optical coefficient; compliance
$\mathcal{C}$	Viscoelastic compliance operator
c	Numerical constant; length; speed of light, wavespeed
C.V.	Coefficient of variation
$\mathbf{D}$	Stiffness matrix; flexural rigidity of plate
$\mathcal{D}$	Plate bending stiffness
d	Diameter; distance; grain size
E	Modulus of elasticity; electric field
E^*	Activation energy
$\mathcal{E}$	Viscoelastic stiffness operator
e	Electronic charge
e_{ij}	Deviatoric strain
F	Force
f_s	Form factor for shear
G	Shear modulus
$\mathcal{G}$	Viscoelastic shear stiffness operator, strain energy release rate
$\mathcal{G}_c$	Critical strain energy release rate
g	Acceleration of gravity
GF	Gage factor for strain gages
H	Brinell hardness

h	Depth of beam
I	Moment of inertia; stress invariant
$\mathbf{I}$	Identity matrix
J	Polar moment of inertia
K	Bulk modulus; global stiffness matrix; stress intensity factor
$\mathcal{K}$	Viscoelastic bulk stiffness operator
k	Spring stiffness; element stiffness; shear yield stress; Boltzmann's constant
L	Length, beam span
$\mathbf{L}$	Matrix of differential operators
$\mathcal{L}$	Laplace transform
M	Bending moment
N	Crosslink or segment density; moiré fringe number; interpolation function; cycles to failure
$\mathbf{N}$	Traction per unit width on plate
N_A	Avogadro's number
$\mathcal{N}$	Viscoelastic Poisson operator
n	Refractive index; number of fatigue cycles
$\hat{\mathbf{n}}$	Unit normal vector
P	Concentrated force
P_f	Fracture load; probability of failure
P_s	Probability of survival
p	Pressure; moiré gridline spacing
Q	Force resultant; first moment of area
q	Distributed load
R	Radius; reaction force; strain or stress rate; gas constant; electrical resistance
$\mathbf{R}$	Reuter's matrix
r	Radius; area reduction ratio
S	Entropy; moiré fringe spacing; total surface energy; alternating stress
$\mathbf{S}$	Compliance matrix
s	Laplace variable; standard deviation
SCF	Stress concentration factor
T	Temperature; tensile force; stress vector; torque
$\mathbf{T}$	Traction vector
T_g	Glass transition temperature
t	Time; thickness
t_f	Time to failure
U	Strain energy
U^*	Strain energy per unit volume
$\mathbf{u}$	Displacement vector
$\tilde{u}$	Approximate displacement function
V	Shearing force; volume; voltage
V^*	Activation volume
v	Velocity

W	Weight; work
u, v, w	Components of displacement
x, y, z	Rectangular coordinates
X	Standard normal variable
α, β	Curvilinear coordinates
α_L	Coefficient of linear thermal expansion
γ	Shear strain; surface energy per unit area; weight density
ΔV	Change in volume
δ	Deflection, Dirac function
δ_{ij}	Kronecker delta
ϵ	Normal strain
$\boldsymbol{\epsilon}$	Strain pseudovector
ϵ_{ij}	Strain tensor
$\boldsymbol{\epsilon}_T$	Thermal strain vector
η	Viscosity
θ	Angle; angle of twist per unit length
$\boldsymbol{\kappa}$	Curvature
λ	Extension ratio, wavelength
ν	Poisson's ratio
ρ	Mass density; electrical resistivity; radius of curvature
Σ_{ij}	Distortional stress
σ	Normal stress
$\boldsymbol{\sigma}$	Stress pseudovector
σ_{ij}	Stress tensor
σ_e	Endurance limit
σ_f	Failure stress (ultimate tensile strength)
σ_m	Mean stress
σ_t	True stress
σ_Y	Yield stress
τ	Shear stress; relaxation time
ϕ	Airy stress function
ξ	Dummy length or time variable
Ω	Configurational probability
ω	Angular frequency
∇	Gradient operator

CONTENTS

TENSILE RESPONSE OF MATERIALS

CHAPTER 1

1.1 INTRODUCTION

The progress of human development is often reckoned in terms of the materials used by society, with the Stone Age in prehistory and progressing through bronze, iron, and successively more sophisticated materials. Early people used materials largely as they were produced by nature, but today we have come to rely increasingly on *engineered* materials: materials we have optimized for various applications by careful selection and processing. For centuries the relations between processing and properties were developed and used empirically, but this approach becomes cumbersome and expensive as requirements grow increasingly complex. As can be depicted by the tetrahedron shown in Fig. 1.1, modern materials science and engineering deals with a general rational approach to materials optimization: A material's *processing* is chosen so as to develop a desired *microstructure;* the microstructure controls *properties;* and finally, the material's properties are important in dictating the *performance* of the structure in which the material is used.

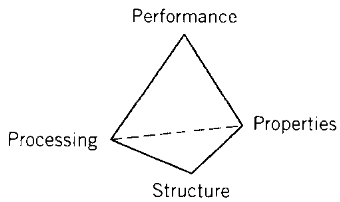

FIGURE 1.1 Modern materials science and engineering.

Most engineering designs involve selection and manipulation of materials, and many of these designs are largely or partly *mechanical:* They require that the resulting structure support applied loads without fracture or excessive deformation. This requires that the designer be able to determine the magnitude and direction of internal forces that may cause rupture or slippage of molecular bonds and to provide enough material of suitable strength to ensure that these events do not occur. The techniques required to accomplish these tasks constitute the major part of the engineering science base in mechanical design.

As we study mechanics of materials, we must keep in mind that real designs must satisfy a number of criteria in addition to mechanical reliability. Cost is almost always very important: Material costs money, and the designer must use only enough material to satisfy the strength requirements. Other important criteria might include thermal and electrical requirements; toxicity and flammability of the material, which could endanger both the user and manufacturing personnel; and the environmental impact associated with both use and eventual disposal. At its core, however, the design problem for load-bearing structures involves ensuring the mechanical integrity of the material, and this aspect of the design process is the major goal of this text.

Analysis or design problems in the mechanics of materials generally involve two major areas:

- Determination of the internal forces set up within the material by loads or displacements imposed on it; this is a largely mathematical undertaking, termed *stress analysis*. These internal forces are often independent of the choice of material used in the structure, and it is often possible to carry out this analysis without much specific knowledge of the material itself. This is the *mechanics* in "mechanics of materials."
- Understanding the material's response to these internal forces. The material may stretch or distort, this deformation may be reversible or permanent, or the material may fracture in any of several ways. This part of the problem is most certainly materials-specific; it is the *materials* in "mechanics of materials."

This chapter will outline some of the basic concepts underlying both of these aspects. This will be done by describing the internal force distribution set up by a simple tensile load and how materials can respond to these forces. Subsequent chapters will extend these concepts to geometrically more complicated situations and gradually introduce the mathematical language used by the literature of the field to describe them.

1.2 TENSILE STRENGTH AND TENSILE STRESS

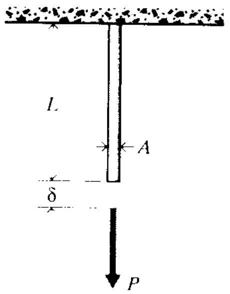

FIGURE 1.2
The tension test.

Perhaps the most natural test of a material's mechanical properties is the *tension test*, in which a strip or cylinder of the material, having length L and cross-sectional area A, is anchored at one end and subjected to an axial load P, acting along the specimen's long axis, at the other (see Fig. 1.2). As the load is increased gradually, the axial deflection δ of the loaded end will increase also. Eventually the test specimen breaks or does something else catastrophic, often fracturing suddenly into two or more pieces. (Materials can fail mechanically in many different ways; for instance, recall how blackboard chalk, a piece of fresh wood, and bouncing putty break.) As engineers we naturally want to understand such matters as how δ is related to P and what ultimate fracture load we might expect in a specimen of different size than the original one. As materials technologists we wish to understand how these relationships are influenced by the constitution and microstructure of the material.

One of the pivotal historical developments in our understanding of material mechanical properties was the realization that the strength of a uniaxially loaded specimen is related to the magnitude of its *cross-sectional area*. This notion is reasonable when we consider the strength to arise from the number of chemical bonds connecting one cross section with the one adjacent to it, as depicted in Fig. 1.3, where each bond is visualized as a spring with a certain stiffness and strength. Obviously, the number of such bonds will increase proportionally with the section's area. The axial strength of a piece of blackboard chalk will therefore increase as the *square* of its diameter. In contrast, increasing the *length* of the chalk will not make it stronger (in fact it will likely become weaker, because the longer specimen will be statistically more likely to contain a strength-reducing flaw).

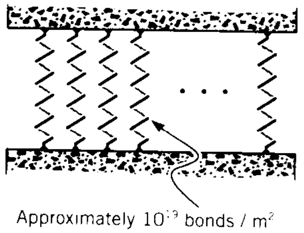

FIGURE 1.3 Interplanar bonds.

Galileo (1564–1642)[1] is said to have used this observation to note that giants, should they exist, would be very fragile creatures. Their strength would be greater

[1] Galileo, *Two New Sciences*, English translation by H. Crew and A. de Salvio, The Macmillan Co., New York, 1933. Also see S.P. Timoshenko, *History of Strength of Materials*, McGraw-Hill, New York, 1953.

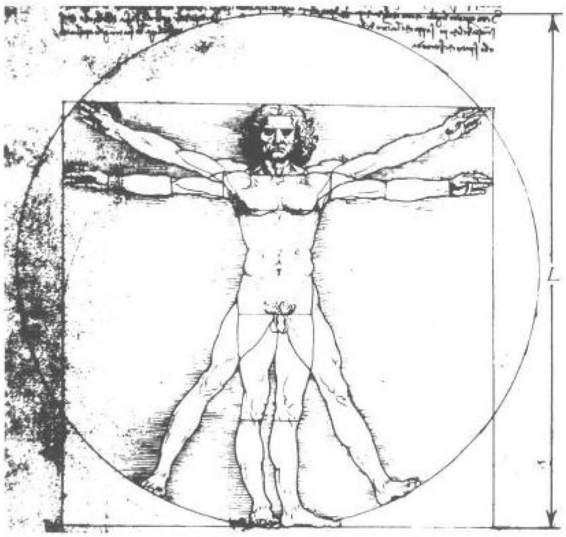

FIGURE 1.4 Strength scales with L^2, but weight scales with L^3. (Bettman Archive.)

than ours, because the cross-sectional areas of their skeletal and muscular systems would be larger by a factor related to the square of their height (denoted L in Fig. 1.4). On the other hand, their *weight*, and thus the loads they must sustain, would increase as their volume—that is, by the *cube* of their height. A simple fall would probably do them great damage. Conversely, the "proportionate" strength of the famous arachnid mentioned weekly in the *Spider-Man* comic strip is mostly just this same size effect. There's nothing magical about the muscular strength of insects, but the ratio of L^2 to L^3 works in their favor when strength per body weight is reckoned (see Fig. 1.5). This cautions us that simple scaling of a previously proven design is not a safe design procedure. A jumbo jet is not just a small plane scaled up; if that were done, the load-bearing components would be too small in cross-sectional area to support the much greater loads they would be called upon to resist.

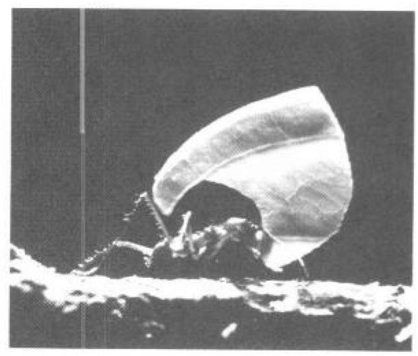

FIGURE 1.5 Small animals can have very high strength-to-weight ratios, as this ant makes clear. (Paul McCormick/The Image Bank.)

When reporting the strength of materials loaded in tension, it is customary to account for this effect of area by dividing the breaking load by the cross-sectional area:

$$\boxed{\sigma_f = \frac{P_f}{A_0}} \tag{1.1}$$

where σ_f is the *ultimate tensile stress*, often abbreviated as UTS; P_f is the load at fracture; and A_0 is the original cross-sectional area. (Some materials exhibit substantial reductions in cross-sectional area as they are stretched, and using the original rather than final area gives the so-called *engineering* strength.) The units of stress are obviously load per unit area: N/m^2 (also called pascals, or Pa) in the SI system, and lb/in^2 (or psi) in units still used commonly in the United States.

EXAMPLE 1.1

In many design problems the loads to be applied to the structure are known at the outset, and we wish to compute how much material will be needed to support them. As a very simple case, consider using a steel rod, circular in cross-sectional shape