MODERN OPTICS

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Preface

This textbook is designed for use in a standard physics course on optics. The book is the result of a one-semester elective course that has been taught to juniors, seniors, and first-year graduate students in physics and engineering at Duke University for several years. Students who take this course should have completed an introductory physics course and math courses through differential equations. Electricity and magnetism can be taken concurrently.

Modern Optics differs from the classical approach of most textbooks on this subject in that its treatment of optics includes some material that is not found in more conventional textbooks. These topics include nonlinear optics, guided waves, Gaussian beams, and light modulators. Moreover, a selection of optional material is provided for the instructor so that the course content can reflect the interest of the instructor and the students. Basic derivations are included to make the book appealing to physics departments, and design concepts are included to make the book appealing to engineering departments. Because of the material covered here, the electrical engineering and biomedical engineering departments at Duke have made the corresponding optics course a prerequisite for some of their advanced courses in optical communications and medical imaging.

Before the 1960s, the only contact that the average person had with optics was a camera lens or eyeglasses. Geometric optics was quite adequate for the design of these systems, and it was natural to emphasize this aspect of optics in a curriculum. The approach used introduced the students to the theory and to examples of the application of the theory, accomplished by a description of a large variety of optical instruments. The reason for this approach was that lens design is found to be quite tedious, and the optimization of a lens design is more easily described than accomplished.

Today the student is exposed to many more optical systems. Everyone encounters supermarket scanners, copying machines, compact disk players, holograms, and discussions of fiber optic communications. In the research environment, lasers, optical modulators fiber optic interconnects, and nonlinear optics have become important tools. Upon graduation, many students will be called on to participate in the use or the development of these modern optical systems. An elementary discussion of geometrical optics and a review of classical optical instruments will not adequately prepare the student for these demands.

This book was written to provide both a fundamental study of the principles of optics and an exposure to actual optics engineering problems and solutions. To include new material has meant that some of the topics covered in classical texts had to be removed. A large portion of the conventional treatment of geometrical optics was deleted, along with a discussion of classical optical systems. In their place were a geometrical optics discussion of fiber optics and a discussion of holography. Rather than describe a host of optical systems, a few optical systems, such as the Fabry–Perot interferometer, are examined using a variety of theories. This book emphasizes diffraction and the use of Fourier theory to describe the operation of an optical system.

To allow the development of a one-year course in modern optics, a number of topics have been added or expanded. A discussion of electrooptic and magnetooptic effects is used to introduce optical modulators, and a discussion of nonlinear optics is constructed around second harmonic generation. Because of the importance of birefringence in optical modulators and nonlinear optics, an expanded discussion of optical anisotropy has been included. This is a departure from most texts that ignore anisotropy because of the need to use tensors. In modern optics, anisotropy is an important design tool, and its treatment allows a discussion of the design of optical modulators and phase matching in nonlinear materials.

The first two chapters review wave theory and electromagnetic theory. Except for the section on polarization in Chapter 2, these chapters could be used as reading assignments for well-prepared students. Chapter 3 discusses reflection and refraction and utilizes the boundary conditions of Maxwell's equations to obtain the fraction of light reflected and refracted at a surface.

Chapter 4 discusses interference of waves and describes several instruments that are used to measure interference. Two of the interferometers, Young's two-slit experiment and the Fabry–Perot interferometer, should receive emphasis in discussions of this chapter because of the role they play in later discussions. An appendix to this chapter provides a brief introduction to some of the design techniques that are used to produce multilayer interference filters. All of the appendices in the book are included to fill in the gaps in students' knowledge and to provide some flexibility for the instructor. The appendices may, therefore, be ignored or used as the subject matter for special assignments.

The treatment of geometrical optics, presented in Chapter 5, is not traditional. It was through the reduction of traditional subject matter that space was obtained to introduce more modern topics. A brief introduction to the matrix formalism used in lens design is presented, and its use is demonstrated by analyzing a confocal Fabry–Perot resonator. Geometrical optics and the concept of interference are used to analyze the propagation of light in a fiber. This introduction to fiber optics is then extended through the use of the Lagrangian formulation to propagation in a graded-index optical fiber. The first part of the chapter demonstrates the formal connection between geometric optics and wave theory. Most students would rather not cover this material and, therefore, it is usually omitted. The connection between the matrix equations and the more familiar lens equations is established in

Appendix 5-A. Because of their importance in the Graduate Records Exam, aberrations are treated in Appendix 5-B.

The Fourier theory in Chapter 6 is presented as a review of and refresher on the subject. It is an important element in the discussion of the concept of coherence in Chapter 8, and Fraunhofer diffraction in Chapter 10. The discussions of optical signal processing, Appendix 10-B, and imaging, Appendix 10-C, draw heavily on Fourier theory.

The discussion of dispersion given in Chapter 7 could be delayed and combined with the other chapters on material interactions (Chapters 13 and 15). It is included here to justify the discussion of coherence in Chapter 8. The discussion of dispersion in materials had as its objective the development by the student of a unifying view of the interaction of light and matter.

The development of coherence theory in Chapter 8 is built around applications of the theory to spectroscopy and astronomy. It is a very difficult subject, but building the theory around the methods used to measure coherence should make the subject more intelligible.

Both the Fresnel and Gaussian wave formalism of diffraction are introduced in Chapter 9. The Gaussian wave formalism is used to analyze a Fabry–Perot cavity and thin lens. This chapter can be skipped, and the material introducing the Fresnel–Huygens integral can be covered in a single lecture.

The Fresnel formalism is expanded and discussed in Chapters 10 and 11. Fraunhofer diffraction is treated from a linear-system viewpoint in Chapter 10, and applications of the theory to signal processing and imaging are presented in Appendices 10-B and 10-C. These two appendices are the most important in the book. Fresnel Diffraction is introduced in Chapter 11, where it is used to interpret Fermat's principle and analyze zone plates and pinhole cameras. In Chapter 12, Fresnel theory is used to discuss the operation of a hologram. Chapter 12 also includes a simple quasigeometric theory that is used to highlight the fundamental properties of a hologram.

Chapter 13 uses the introduction of polarizers and retarders as a basis for the development of the theory of the propagation of light in anisotropic materials. The treatment of anisotropic materials is expanded over the conventional presentation to allow an easy transition into the discussion of light modulators in Chapter 14. The many geometrical constructions used in the discussion of anisotropy are confusing to everyone. To try to make the material understandable, only one construction is used in Chapter 13. To provide the student with reference material to aid in reading other books and papers, the other constructions are discussed in the appendices.

The discussion of modulators in Chapter 14 provides an application-based introduction to electro- and magnetooptic interactions. The design of an electrooptic modulator provides the student with an example of the use of tensors. The material interactions presented in Chapters 14 and 15 require the use of tensors, a subject normally avoided in an undergraduate curriculum. Tensor notation has been used in this book because it is key in the understanding of many optical devices. Some familiarity with tensors removes much of the "magic" associated with the design of modulators and the application of phase matching discussed in Chapter 15.

The subject of nonlinear optics in Chapter 15 is developed by using examples based on frequency doubling. Only a few brief comments are made about third-order nonlinearities. The additional discussion of third-order processes is best presented by using a quantum mechanical viewpoint.

It was thought that this would be best done in a separate course. The material presented in this chapter would prepare the student to immediately undertake a course in nonlinear optics.

Enough material has been included for a one-year course in optics. Chapters 2 to 4, 6, 9 (excluding Gaussian waves), and 10 contain the core material and could be used in a one-quarter course. By adding Chapter 7 and 8 along with Appendices 10-B and 10-C, a one-semester course can be created. The instructor can alter the subjects discussed from year to year by adding topics such as Appendix 4-A, the guided wave discussion of Chapter 5, or the discussion of holography in Chapter 12 in place of Appendices 10-B and 10-C. A less demanding one-semester course can be created by ignoring Chapter 8 and by substituting Chapter 5, Appendix 4-A, or possibly Chapter 12. In anticipation of developing skill and knowledge, the subject matter and problems increase in difficulty as the student moves through the book.

A number of people provided help in the preparation of this book. Those who provided photos or drawings are identified in the figure captions. Their generosity is most appreciated. Dr. Frank DeLucia provided the initial motivation for writing the book. Many ideas and concepts are the result of breakfast discussions with A. VanderLugt. The book would never have gone past the note stage without the equation writer, MacΣqn, written by Dennis Venable. Thomas Stone provided ideas, photographic skills, and encouragement during the preparation of most of the photos in this book. His enthusiasm kept me working.

A very special thanks must go to Nicholas George. He loaned me equipment and lab space to prepare many of the photos. His encouragement prevented me from shelving the project, and his technical discussions provided me with an improved understanding of optics.

Robert D. Guenther

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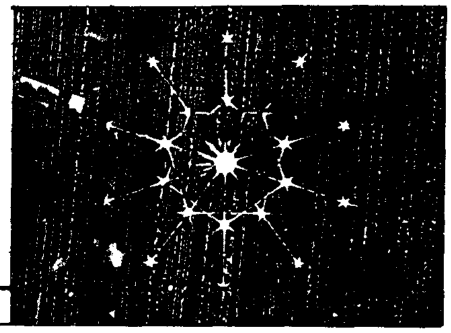

Wave Theory

The theory of wave motion is an important mathematical model in many areas of physics. A large number of seemingly unrelated phenomena can be explained using the solution of the wave equation, the basic equation of wave theory. The wave theory is a fundamental part of modern quantum theory and the solutions of the wave equation are used to explain a number of classical phenomena. Familiarity with the wave theory developed in the study of light will aid in the understanding of such diverse physical processes as water waves, vibrating drums and strings, traffic dynamics, and seismic waves.

Mathematically, the basis of wave theory is a second order, partial differential equation called the wave equation. In this chapter, a vibrating string will be used as an illustration to aid in visualizing the various aspects of the wave theory. Initially, a traveling wave on a string will be used to find the functional form of a one-dimensional wave and to derive the wave equation. Following a discussion of the energy and momentum associated with the traveling wave, the one-dimensional model associated with the string illustration will be expanded to three dimensions. The displacement of the wave discussed in this chapter is assumed to be a scalar function and the theory is called a *scalar wave theory*. In the next chapter, the vector wave theory will be discussed.

Christian Huygens (1629–1695) developed the wave theory of light in 1678. **Isaac Newton (1642–1727)** proposed a counter theory based on a particle view of light. Newton's scientific stature resulted in only a few scientists during the 18th century, for example **Leonard Euler (1707–1783)** and **Benjamin Franklin (1706–1790)**, accepting the wave theory and rejecting the particle theory of Newton. In 1801 **Thomas Young (1773–1829)** and in 1814 **Augustin Jean Fresnel (1788–1827)** utilized experiments to demonstrate interference and diffraction of light and presented a theoretical explanation of the experiments through the use of the wave theory. Fresnel was able to explain rectilinear propagation using the wave theory, thereby removing Newton's main objection to the wave theory. The acceptance of

Fresnel's theory was very slow, and the final rejection of Newton's theory did not come until the measurement of the speed of light in water and air by **Jean Bernard Léon Foucault (1819–1868)**. The velocity measurements were a key element in the rejection of Newton's theory because the particle theory required the speed of light in a medium to exceed the speed of light in a vacuum in order to explain refraction. The measurements by Foucault showed the propagation velocity in a vacuum to exceed the velocity in water.

TRAVELING WAVES

Before the equation of motion of a wave is discussed, a mathematical expression for a wave will be obtained. We will assume that a disturbance propagates without change along a string and that each point on the string undergoes simple harmonic motion (see Appendix 1-A for a brief review of harmonic motion). This assumption will allow us to obtain a simple mathematical expression for a wave that will be used to define the parameters characterizing a wave.

A guitar string is plucked creating a pulse that travels to the right and left along the x axis at a constant speed c. In Figure 1-1, the pulse traveling toward the right is shown. The pulse's amplitude is defined as $y \equiv f(x,t)$ and equals y_1 at position x_1 and time t_1. This amplitude travels a distance $c(t_2 - t_1)$ to the right of x_1 and is described mathematically by

$$y \equiv f(x, t)$$

Assume the pulse does not change in amplitude as it propagates

$$f(x_1, t_1) = f(x_2, t_2)$$

where $x_2 = x_1 + c(t_2 - t_1)$. If the function has the form

$$y = f(ct - x) \tag{1-1}$$

then the requirement that the pulse does not change is satisfied because

$$f(x_1, t_1) = f(ct_1 - x_1)$$

$$f(x_2, t_2) = f(ct_2 - x_2) = f[ct_2 - x_1 - c(t_2 - t_1)]$$

$$= f(ct_1 - x_1)$$

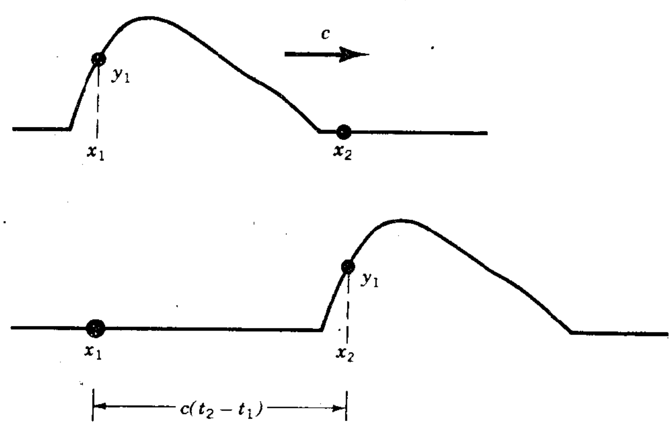

FIGURE 1-1. Propagation of a pulse on a guitar string. The amplitude does not change as the pulse propagates along the string.

Using the same reasoning, we can show that an unchanging pulse traveling to the left, along the x axis, with speed c is described by

$$y = g(x + ct)$$

The expression $y = f(ct - x)$ is a shorthand notation to denote a function that contains x and t only in the combination $(ct - x)$, i.e., the function can contain combinations of the form $2(ct - x)$, $(t \pm x/c)$, $(x - ct)$, $(ct - x)^2$, $\sin(ct - x)$, etc., but not expressions such as $(2ct - x)$ or $(ct^2 - x^2)$.

To the assumption of an unchanging propagating disturbance is now added the requirement that each point on the guitar string oscillate transversely, i.e., perpendicular to the direction of propagation, with simple harmonic motion. The string in Figure 1-1 lies along the x axis and the harmonic motion will be in the y direction. The point on the string at the origin ($x = 0$) undergoes simple harmonic motion with amplitude Y and frequency ω (the angular frequency $\omega = 2\pi\nu$ will be used throughout this book; the linear frequency ν is defined in Appendix 1-A). The equation describing the motion of the origin is

$$y = Y \cos \omega t$$

The origin acts as a source of a continuous train of pulses (a wave train) moving to the right.

A function of $(ct - x)$ that will reduce to harmonic motion at $x = 0$ is

$$y = f(ct - x) = Y \cos\left[\frac{\omega}{c}(ct - x)\right]$$

This is called a *harmonic wave*.

A number of different notations are used for a harmonic wave; the one used in this book involves a constant

$$k = \frac{\omega}{c} \tag{1-2}$$

called the *propagation constant* or the *wave number* and is written

$$y = Y \cos(\omega t - kx) \tag{1-3}$$

The values of x for which the phase $(\omega t - kx)$ changes by 2π is the *spatial period* and is called the *wavelength* λ. Let $x_2 = x_1 + \lambda$, so that

$$\omega t - kx_2 = \omega t - kx_1 - k\lambda = \omega t - kx_1 - 2\pi$$

thus

$$k = \frac{2\pi}{\lambda} \tag{1-4}$$

since $k = \omega/c = 2\pi\nu/c$, we also have the relationship $c = \nu\lambda$.

To determine the speed of the wave in space, a point on the wave is selected and the time it takes to go some distance is measured. This is equivalent to asking how fast a given value of phase propagates in space. Assume that in the time $\Delta t = (t_2 - t_1)$, the disturbance y_1 travels a distance $\Delta x = (x_2 - x_1)$, as is shown in Figure 1-1. Since the disturbance at the two points is the same, i.e., y_1, then the phases must be equal

$$\omega t - kx = \omega(t + \Delta t) - k(x + \Delta x)$$

$$\frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

In the limit as $\Delta t \to 0$, we obtain the *phase velocity*

$$c \equiv \frac{dx}{dt} = \frac{\omega}{k}$$

The adjective "phase" is used because this velocity describes the motion of a preselected phase of the wave. Another method that can be used to obtain the propagation speed associated with a wave is to define the phase velocity using the result from partial differential calculus

$$\left(\frac{\partial x}{\partial t}\right)_y = -\frac{\left(\frac{\partial y}{\partial t}\right)_x}{\left(\frac{\partial y}{\partial x}\right)_t} = \frac{\omega}{k}$$

This equation may be verified by applying it to **(1-3)**.

WAVE EQUATION

To generate the differential equation of motion of a wave propagating along a string, we must look at a small section of the string as a pulse passes by. We are going to assume that we only have small amplitude pulses so that the tension in the string is not changed appreciably as the pulse passes by. As a consequence of this assumption, we have $\partial y/\partial x << 1$; therefore, the deflected string shown in Figure 1-2 makes an angle θ with the horizontal such that $\cos\theta \approx 1$ and $\sin\theta \approx \tan\theta = \partial y/\partial x$ (we use partial derivatives because the deflection is a function of both time and position; in this derivation, we hold time constant). With these approximations, the components of tension at position x in Figure 1-2 are

$$T_x = T\cos\theta \approx T$$

$$T_y = T\sin\theta \approx T\left(\frac{\partial y}{\partial x}\right)$$

At position $x + \Delta x$ in Figure 1-2, the slope is also small since $\Delta\theta$ is small $[\cos(\theta + \Delta\theta) \approx 1]$, resulting in

$$T_x + \Delta T_x \approx T$$

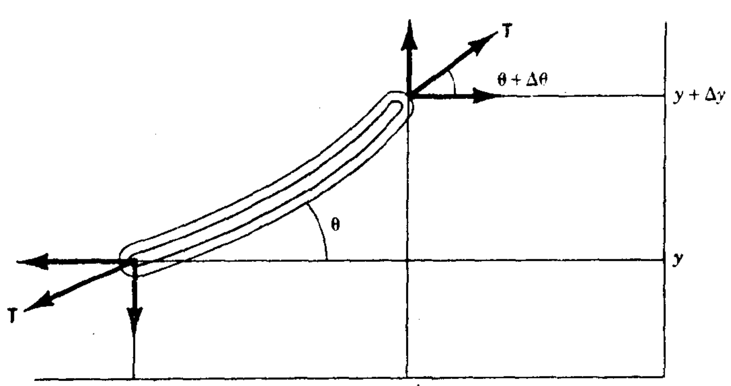

FIGURE 1-2. The string is deflected as the pulse passes by. The tension T is decomposed into components in the x and y directions.