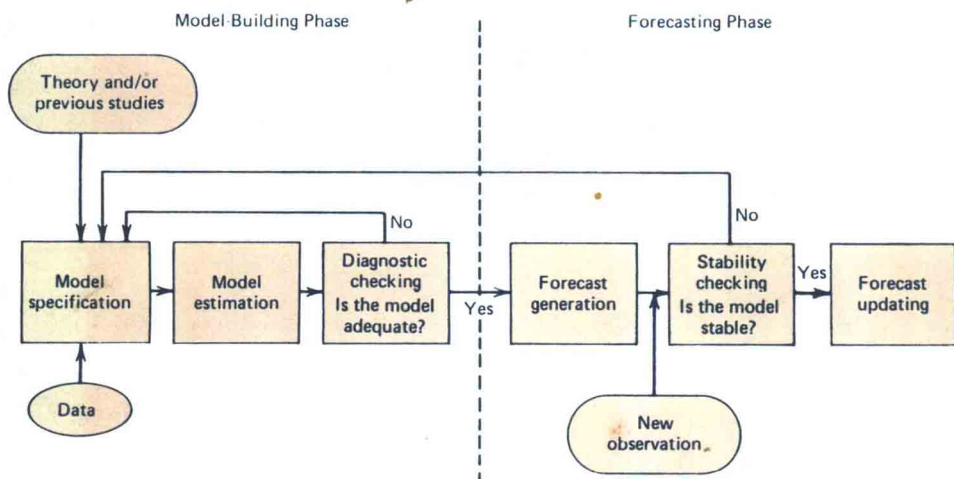


STATISTICAL METHODS FOR FORECASTING

Bovas Abraham and
Johannes Ledolter



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Statistical Methods for Forecasting

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Preface

Forecasting is an important part of decision making, and many of our decisions are based on predictions of future unknown events. Many books on forecasting and time series analysis have been published recently. Some of them are introductory and just describe the various methods heuristically. Certain others are very theoretical and focus on only a few selected topics.

This book is about the statistical methods and models that can be used to produce short-term forecasts. Our objective is to provide an intermediate-level discussion of a variety of statistical forecasting methods and models, to explain their interconnections, and to bridge the gap between theory and practice.

Forecast systems are introduced in Chapter 1. Various aspects of regression models are discussed in Chapter 2, and special problems that occur when fitting regression models to time series data are considered. Chapters 3 and 4 apply the regression and smoothing approach to predict a single time series. A brief introduction to seasonal adjustment methods is also given. Parametric models for nonseasonal and seasonal time series are explained in Chapters 5 and 6. Procedures for building such models and generating forecasts are discussed. Chapter 7 describes the relationships between the forecasts produced from exponential smoothing and those produced from parametric time series models. Several advanced topics, such as transfer function modeling, state space models, Kalman filtering, Bayesian forecasting, and methods for forecast evaluation, comparison, and control are given in Chapter 8. Exercises are provided in the back of the book for each chapter.

This book evolved from lecture notes for an MBA forecasting course and from notes for advanced undergraduate and beginning graduate statistics courses we have taught at the University of Waterloo and at the University of Iowa. It is oriented toward advanced undergraduate and beginning graduate students in statistics, business, engineering, and the social sciences.

A calculus background, some familiarity with matrix algebra, and an intermediate course in mathematical statistics are sufficient prerequisites.

Most business schools require their doctoral students to take courses in regression, forecasting, and time series analysis, and most offer courses in forecasting as an elective for MBA students. Courses in regression and in applied time series at the advanced undergraduate and beginning graduate level are also part of most statistics programs. This book can be used in several ways. It can serve as a text for a two-semester sequence in regression, forecasting, and time series analysis for Ph.D. business students, for MBA students with an area of concentration in quantitative methods, and for advanced undergraduate or beginning graduate students in applied statistics. It can also be used as a text for a one-semester course in forecasting (emphasis on Chapters 3 to 7), for a one-semester course in applied time series analysis (Chapters 5 to 8), or for a one-semester course in regression analysis (Chapter 2, and parts of Chapters 3 and 4). In addition, the book should be useful for the professional forecast practitioner.

We are grateful to a number of friends who helped in the preparation of this book. We are glad to record our thanks to Steve Brier, Bob Hogg, Paul Horn, and K. Vijayan, who commented on various parts of the manuscript. Any errors and omissions in this book are, of course, ours. We appreciate the patience and careful typing of the secretarial staff at the College of Business Administration, University of Iowa and of Marion Kaufman and Lynda Hohner at the Department of Statistics, University of Waterloo. We are thankful for the many suggestions we received from our students in forecasting, regression, and time series courses. We are also grateful to the *Biometrika* trustees for permission to reprint condensed and adapted versions of Tables 8, 12 and 18 from *Biometrika Tables for Statisticians*, edited by E. S. Pearson and H. O. Hartley.

We are greatly indebted to George Box who taught us time series analysis while we were graduate students at the University of Wisconsin. We wish to thank him for his guidance and for the wisdom which he shared so freely. It is also a pleasure to acknowledge George Tiao for his warm encouragement. His enthusiasm and enlightenment has been a constant source of inspiration.

We could not possibly discuss every issue in statistical forecasting. However, we hope that this volume provides the background that will allow the reader to adapt the methods included here to his or her particular needs.

B. ABRAHAM
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Waterloo, Ontario
Iowa City, Iowa
June 1983

Contents

1	INTRODUCTION AND SUMMARY	1
1.1	Importance of Good Forecasts	1
1.2	Classification of Forecast Methods	2
1.3	Conceptual Framework of a Forecast System	3
1.4	Choice of a Particular Forecast Model	4
1.5	Forecast Criteria	5
1.6	Outline of the Book	5
2	THE REGRESSION MODEL AND ITS APPLICATION IN FORECASTING	8
2.1	The Regression Model	9
2.1.1	Linear and Nonlinear Models,	10
2.2	Prediction from Regression Models with Known Coefficients	12
2.3	Least Squares Estimates of Unknown Coefficients	13
2.3.1	Some Examples,	13
2.3.2	Estimation in the General Linear Regression Model,	16
2.4	Properties of Least Squares Estimators	20
2.5	Confidence Intervals and Hypothesis Testing	25
2.5.1	Confidence Intervals,	25
2.5.2	Hypothesis Tests for Individual Coefficients,	26
2.5.3	A Simultaneous Test for Regression Coefficients,	26
2.5.4	General Hypothesis Tests: The Extra Sum of Squares Principle,	27
2.5.5	Partial and Sequential F Tests,	29

2.6	Prediction from Regression Models with Estimated Coefficients	30
2.6.1	Examples, 31	
2.7	Examples	33
2.8	Model Selection Techniques	41
2.9	Multicollinearity	45
2.10	Indicator Variables	49
2.11	General Principles of Statistical Model Building	52
2.11.1	Model Specification, 53	
2.11.2	Model Estimation, 54	
2.11.3	Diagnostic Checking, 54	
2.11.4	Lack of Fit Tests, 56	
2.11.5	Nonconstant Variance and Variance-Stabilizing Transformations, 58	
2.12	Serial Correlation among the Errors	60
2.12.1	Serial Correlation in a Time Series, 61	
2.12.2	Detection of Serial Correlation among the Errors in the Regression Model, 63	
2.12.3	Regression Models with Correlated Errors, 66	
2.13	Weighted Least Squares	74
	Appendix 2 Summary of Distribution Theory Results	77
3	REGRESSION AND EXPONENTIAL SMOOTHING METHODS TO FORECAST NONSEASONAL TIME SERIES	79
3.1	Forecasting a Single Time Series	79
3.2	Constant Mean Model	81
3.2.1	Updating Forecasts, 82	
3.2.2	Checking the Adequacy of the Model, 83	
3.3	Locally Constant Mean Model and Simple Exponential Smoothing	85
3.3.1	Updating Forecasts, 86	
3.3.2	Actual Implementation of Simple Exponential Smoothing, 87	
3.3.3	Additional Comments and Example, 89	

3.4	Regression Models with Time as Independent Variable	95
3.4.1	Examples,	96
3.4.2	Forecasts,	98
3.4.3	Updating Parameter Estimates and Forecasts,	100
3.5	Discounted Least Squares and General Exponential Smoothing	101
3.5.1	Updating Parameter Estimates and Forecasts,	102
3.6	Locally Constant Linear Trend Model and Double Exponential Smoothing	104
3.6.1	Updating Coefficient Estimates,	107
3.6.2	Another Interpretation of Double Exponential Smoothing,	107
3.6.3	Actual Implementation of Double Exponential Smoothing,	108
3.6.4	Examples,	110
3.7	Locally Quadratic Trend Model and Triple Exponential Smoothing	120
3.7.1	Implementation of Triple Exponential Smoothing,	123
3.7.2	Extension to the General Polynomial Model and Higher Order Exponential Smoothing,	124
3.8	Prediction Intervals for Future Values	125
3.8.1	Prediction Intervals for Sums of Future Observations,	127
3.8.2	Examples,	127
3.8.3	Estimation of the Variance,	129
3.8.4	An Alternative Variance Estimate,	132
3.9	Further Comments	133
4	REGRESSION AND EXPONENTIAL SMOOTHING METHODS TO FORECAST SEASONAL TIME SERIES	135
4.1	Seasonal Series	135
4.2	Globally Constant Seasonal Models	139
4.2.1	Modeling the Seasonality with Seasonal Indicators,	140
4.2.2	Modeling the Seasonality with Trigonometric Functions,	149

4.3	Locally Constant Seasonal Models	155
4.3.1	Locally Constant Seasonal Models Using Seasonal Indicators,	158
4.3.2	Locally Constant Seasonal Models Using Trigonometric Functions,	164
4.4	Winters' Seasonal Forecast Procedures	167
4.4.1	Winters' Additive Seasonal Forecast Procedure,	167
4.4.2	Winters' Multiplicative Seasonal Forecast Procedure,	170
4.5	Seasonal Adjustment	173
4.5.1	Regression Approach,	174
4.5.2	Smoothing Approach,	174
4.5.3	Seasonal Adjustment Procedures,	179
Appendix 4	Computer Programs for Seasonal Exponential Smoothing	182
	EXPSIND. General Exponential Smoothing with Seasonal Indicators,	182
	EXPHARM. General Exponential Smoothing with Trigonometric Forecast Functions,	185
	WINTERS1. Winters' Additive Forecast Procedure,	188
	WINTERS2. Winters' Multiplicative Forecast Procedure,	190
5	STOCHASTIC TIME SERIES MODELS	192
5.1	Stochastic Processes	192
5.1.1	Stationary Stochastic Processes,	194
5.2	Stochastic Difference Equation Models	197
5.2.1	Autoregressive Processes,	199
5.2.2	Partial Autocorrelations,	209
5.2.3	Moving Average Processes,	213
5.2.4	Autoregressive Moving Average (ARMA) Processes,	219
5.3	Nonstationary Processes	225
5.3.1	Nonstationarity, Differencing, and Transformations,	225

5.3.2	Autoregressive Integrated Moving Average (ARIMA) Models,	231
5.3.3	Regression and ARIMA Models,	237
5.4	Forecasting	238
5.4.1	Examples,	240
5.4.2	Prediction Limits,	246
5.4.3	Forecast Updating,	247
5.5	Model Specification	248
5.6	Model Estimation	250
5.6.1	Maximum Likelihood Estimates,	250
5.6.2	Unconditional Least Squares Estimates,	253
5.6.3	Conditional Least Squares Estimates,	257
5.6.4	Nonlinear Estimation,	258
5.7	Model Checking	261
5.7.1	An Improved Approximation of the Standard Error,	262
5.7.2	Portmanteau Test,	263
5.8	Examples	263
5.8.1	Yield Data,	264
5.8.2	Growth Rates,	267
5.8.3	Demand for Repair Parts,	270
Appendix 5	Exact Likelihood Functions for Three Special Models	273
I.	Exact Likelihood Function for an ARMA(1, 1) Process,	273
II.	Exact Likelihood Function for an AR(1) Process,	278
III.	Exact Likelihood Function for an MA(1) Process,	279
6	SEASONAL AUTOREGRESSIVE INTEGRATED MOVING AVERAGE MODELS	281
6.1	Multiplicative Seasonal Models	283
6.2	Autocorrelation and Partial Autocorrelation Functions of Multiplicative Seasonal Models	285
6.2.1	Autocorrelation Function,	286
6.2.2	Partial Autocorrelation Function,	291
6.3	Nonmultiplicative Models	291

6.4	Model Building	293
6.4.1	Model Specification,	293
6.4.2	Model Estimation,	299
6.4.3	Diagnostic Checking,	299
6.5	Regression and Seasonal ARIMA Models	299
6.6	Forecasting	302
6.7	Examples	306
6.7.1	Electricity Usage,	306
6.7.2	Gas Usage,	308
6.7.3	Housing Starts,	310
6.7.4	Car Sales,	310
6.7.5	Demand for Repair Parts,	313
6.8	Seasonal Adjustment Using Seasonal ARIMA Models	317
6.8.1	X-11-ARIMA,	317
6.8.2	Signal Extraction or Model-Based Seasonal Adjustment Methods,	318
Appendix 6	Autocorrelations of the Multiplicative $(0, d, 1)(1, D, 1)_{12}$ Model	319
7	RELATIONSHIPS BETWEEN FORECASTS FROM GENERAL EXPONENTIAL SMOOTHING AND FORECASTS FROM ARIMA TIME SERIES MODELS	322
7.1	Preliminaries	322
7.1.1	General Exponential Smoothing,	322
7.1.2	ARIMA Time Series Models,	323
7.2	Relationships and Equivalence Results	327
7.2.1	Illustrative Examples,	329
7.3	Interpretation of the Results	330
Appendix 7	Proof of the Equivalence Theorem	331
8	SPECIAL TOPICS	336
8.1	Transfer Function Analysis	336
8.1.1	Construction of Transfer Function-Noise Models,	338

8.1.2	Forecasting,	344	
8.1.3	Related Models,	348	
8.1.4	Example,	348	
8.2	Intervention Analysis and Outliers		355
8.2.1	Intervention Analysis,	355	
8.2.2	Outliers,	356	
8.3	The State Space Forecasting Approach, Kalman Filtering, and Related Topics		359
8.3.1	Recursive Estimation and Kalman Filtering,	361	
8.3.2	Bayesian Forecasting,	363	
8.3.3	Models with Time-Varying Coefficients,	364	
8.4	Adaptive Filtering		368
8.5	Forecast Evaluation, Comparison, and Control		370
8.5.1	Forecast Evaluation,	372	
8.5.2	Forecast Comparison,	373	
8.5.3	Forecast Control,	374	
8.5.4	Adaptive Exponential Smoothing,	377	
REFERENCES			379
EXERCISES			386
DATA APPENDIX			418
TABLE APPENDIX			426
AUTHOR INDEX			435
SUBJECT INDEX			437

CHAPTER 1

Introduction and Summary

Webster's dictionary defines *forecasting* as an activity "to calculate or predict some future event or condition, usually as a result of rational study or analysis of pertinent data."

1.1. IMPORTANCE OF GOOD FORECASTS

The ability to form good forecasts has been highly valued throughout history. Even today various types of fortune-tellers claim to have the power to predict future events. Frequently their predictions turn out to be false. However, occasionally their predictions come true; apparently often enough to secure a living for these forecasters.

We all make forecasts, although we may not recognize them as forecasts. For example, a person waiting for a bus or parents expecting a telephone call from their children may not consider themselves forecasters. However, from past experience and from reading the bus schedule, the person waiting for the bus expects it to arrive at a certain time or within a certain time interval. Parents who have usually received calls from their children every weekend expect to receive one during the coming weekend also.

These people form expectations, and they make forecasts. So does a bank manager who predicts the cash flow for the next quarter, or a control engineer who adjusts certain input variables to maintain the future value of some output variable as close as possible to a specified target, or a company manager who predicts sales or estimates the number of man-hours required to meet a given production schedule. All make statements about future events, patterning the forecasts closely on previous occurrences and assuming that the future will be similar to the past.

Since future events involve uncertainty, the forecasts are usually not perfect. The objective of forecasting is to reduce the forecast error: to

produce forecasts that are seldom incorrect and that have small forecast errors. In business, industry, and government, policymakers must anticipate the future behavior of many critical variables before they make decisions. Their decisions depend on forecasts, and they expect these forecasts to be accurate; a forecast system is needed to make such predictions. Each situation that requires a forecast comes with its own unique set of problems, and the solutions to one are by no means the solutions in another situation. However, certain general principles are common to most forecasting problems and should be incorporated into any forecast system.

1.2. CLASSIFICATION OF FORECAST METHODS

Forecast methods may be broadly classified into *qualitative* and *quantitative* techniques. *Qualitative* or *subjective* forecast methods are intuitive, largely educated guesses that may or may not depend on past data. Usually these forecasts cannot be reproduced by someone else, since the forecaster does not specify explicitly how the available information is incorporated into the forecast. Even though subjective forecasting is a nonrigorous approach, it may be quite appropriate and the only reasonable method in certain situations.

Forecasts that are based on mathematical or statistical models are called *quantitative*. Once the underlying model or technique has been chosen, the corresponding forecasts are determined automatically; they are fully reproducible by any forecaster. Quantitative methods or models can be further classified as deterministic or probabilistic (also known as stochastic or statistical).

In *deterministic* models the relationship between the variable of interest, Y , and the explanatory or predictor variables X_1, \dots, X_p is determined exactly:

$$Y = f(X_1, \dots, X_p; \beta_1, \dots, \beta_m) \quad (1.1)$$

The function f and the coefficients β_1, \dots, β_m are known with certainty. The traditional “laws” in the physical sciences are examples of such deterministic relationships.

In the social sciences, however, the relationships are usually *stochastic*. Measurement errors and variability from other uncontrolled variables introduce random (stochastic) components. This leads to *probabilistic* or *stochastic models* of the form

$$Y = f(X_1, \dots, X_p; \beta_1, \dots, \beta_m) + \text{noise} \quad (1.2)$$

where the noise or error component is a realization from a certain probability distribution.

Frequently the functional form f and the coefficients are not known and have to be determined from past data. Usually the data occur in time-ordered sequences referred to as *time series*. Statistical models in which the available observations are used to determine the model form are also called *empirical* and are the main subject of this book. In particular, we discuss regression and single-variable prediction methods. In *single-variable forecasting*, we use the past history of the series, let's say z_t , where t is the time index, and extrapolate it into the future. For example, we may study the features in a series of monthly Canadian consumer price indices and extrapolate the pattern over the coming months. Smoothing methods or parametric time series models may be used for this purpose. In *regression forecasting*, we make use of the relationships between the variable to be forecast and the other variables that explain its variation. For example, we may forecast monthly beer sales from the price of beer, consumers' disposable income, and seasonal temperature; or predict the sales of a cereal product by its price (relative to the industry), its advertising, and the availability of its coupons. The standard regression models measure instantaneous effects. However, there are often lag effects, where the variable of interest depends on present and past values of the independent (i.e., predictor) variables. Such relationships can be studied by combining regression and time series models.

1.3. CONCEPTUAL FRAMEWORK OF A FORECAST SYSTEM

In this book we focus our attention exclusively on quantitative forecast methods. In general, a quantitative forecast system consists of two major components, as illustrated in Figure 1.1. At the first stage, the *model-building stage*, a forecasting model is constructed from pertinent data and available theory. In some instances, theory (for example, economic theory) may suggest particular models; in other cases, such theory may not exist or may be incomplete, and historical data must be used to specify an appropriate model. The tentatively entertained model usually contains unknown parameters; an estimation approach, such as least squares, can be used to determine these constants. Finally, the forecaster must check the adequacy of the fitted model. It could be inadequate for a number of reasons; for example, it could include inappropriate variables or it could have misspecified the functional relationship. If the model is unsatisfactory, it has to be respecified, and the iterative cycle of model specification–estimation–diagnostic checking must be repeated until a satisfactory model is found.

At the second stage, the *forecasting stage*, the final model is used to obtain the forecasts. Since these forecasts depend on the specified model,

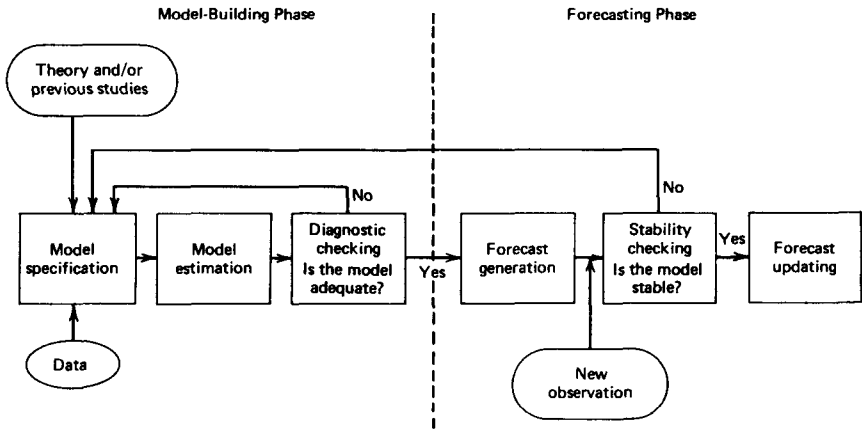


Figure 1.1. Conceptual framework of a forecasting system.

one has to make sure that the model and its parameters stay constant during the forecast period. The stability of the forecast model can be assessed by checking the forecasts against the new observations. Forecast errors can be calculated, and possible changes in the model can be detected. For example, particular functions of these forecast errors can indicate a bias in the forecasts (i.e., consistent over- or underpredictions). The most recent observation can also be used to update the forecasts. Since observations are recorded sequentially in time, updating procedures that can be applied routinely and that avoid the computation of each forecast from first principles are very desirable.

1.4. CHOICE OF A PARTICULAR FORECAST MODEL

Among many other forecast criteria, the choice of the forecast model or technique depends on (1) what degree of accuracy is required, (2) what the forecast horizon is, (3) how high a cost for producing the forecasts can be tolerated, (4) what degree of complexity is required, and (5) what data are available.

Sometimes only crude forecasts are needed; in other instances great accuracy is essential. In some applications, inaccuracy can be very costly; for example, inaccurate forecasts of an economic indicator could force the Federal Reserve Board to boost its lending rate, thus creating a chain of undesirable events. However, increasing the accuracy usually raises substantially the costs of data acquisition, computer time, and personnel. If a small loss in accuracy is not too critical, and if it lowers costs substantially, the

simpler but less accurate model may be preferable to the more accurate but more complex one.

The forecast horizon is also essential, since the methods that produce short-term and long-term forecasts differ. For example, a manufacturer may wish to predict the sales of a product for the next 3 months, while an electric utility may wish to predict the demand for electricity over the next 25 years.

A forecaster should try building simple models, which are easy to understand, use, and explain. An elaborate model may lead to more accurate forecasts but may be more costly and difficult to implement. Ockham's razor, also known as the principle of parsimony, says that in a choice among competing models, other things being equal, the simplest is preferable.

Another important consideration in the choice of an appropriate forecast method is the availability of suitable data; one cannot expect to construct accurate empirical forecast models from a limited and incomplete data base.

1.5. FORECAST CRITERIA

The most important criterion for choosing a forecast method is its accuracy, or how closely the forecast predicts the actual event. Let us denote the actual observation at time t with z_t and its forecast, which uses the information up to and including time $t - 1$, with $z_{t-1}(1)$. Then the objective is to find a forecast such that the future forecast error $z_t - z_{t-1}(1)$ is as small as possible. However, note that this is a future forecast error and, since z_t has not yet been observed, its value is unknown; we can talk only about its expected value, conditional on the observed history up to and including time $t - 1$. If both negative (overprediction) and positive (underprediction) forecast errors are equally undesirable, it would make sense to choose the forecast such that the *mean absolute error* $E|z_t - z_{t-1}(1)|$, or the *mean square error* $E[z_t - z_{t-1}(1)]^2$ is minimized. The forecasts that minimize the mean square error are called *minimum mean square error (MMSE) forecasts*. The mean square error criterion is used here since it leads to simpler mathematical solutions.

1.6. OUTLINE OF THE BOOK

This book is about the statistical methods and models that can be used to produce short-term forecasts. It consists of four major parts: regression, smoothing methods, time series models, and selected special topics.