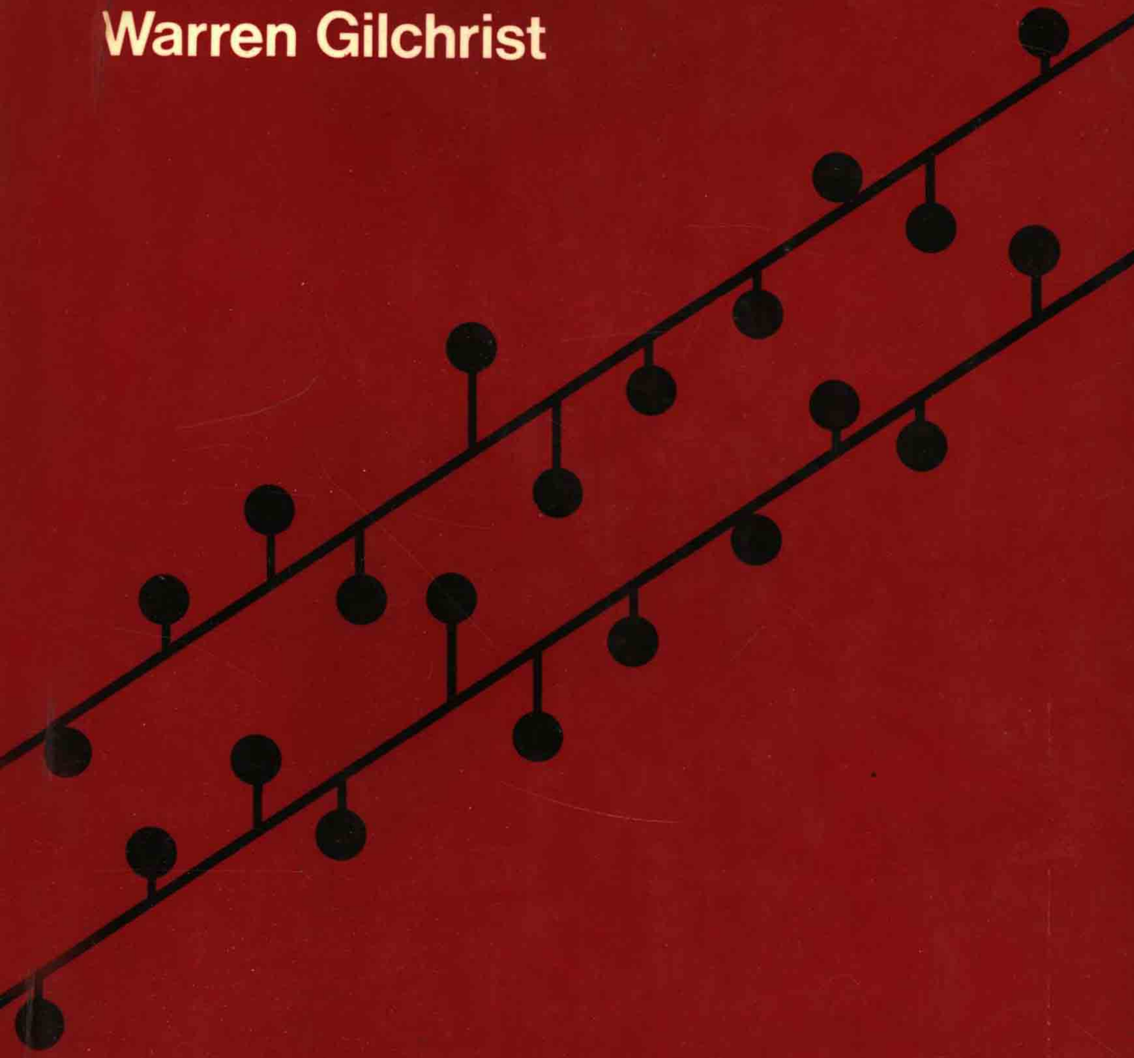
Statistical Forecasting

Warren Gilchrist



Statistical Forecasting

Warren Gilchrist

Head of Department of Mathematics and Statistics,
Sheffield City Polytechnic

A Wiley–Interscience Publication

Reprinted September 1977

Library of Congress Cataloging in Publication Data:

Gilchrist, Warren Garland, 1932–
Statistical forecasting.

'A Wiley–Interscience publication.'
1. Time-series analysis. 2. Prediction theory.
I. Title.
QA280.G54 519.5'4 76-13504

ISBN 0 471 99402 2 (Cloth)
ISBN 0 471 99403 0 (Paper)

Typeset in IBM Century by
Preface Ltd, Salisbury, Wilts
and printed in Great Britain by
The Pitman Press, Bath

Preface

From the dawn of recorded history, and probably before, man has sought to forecast the future. Indeed, the ability to foresee the consequences of actions and events is one of the defining properties of 'mind'. Where the phenomenon being forecast was a purely physical one, such as the occurrence of midsummer's day or of an eclipse, man was able from very early times to obtain very accurate forecasts. Initially such forecasts were derived on a purely empirical basis. Methods were found that worked without any basic understanding as to why they worked. Later, as greater understanding of the phenomena developed theoretical models were developed which enabled forecasts to be obtained on a more reasoned basis. During the last century interest focused on a number of phenomena, such as economic variation and sunspot cycles, where

(a) there were available series of observations taken over a period of time, called time-series, upon which forecasts could be based;
(b) purely mathematical rules were found to be inadequate to describe the phenomena since they involved features of a chance nature.

Over the last fifty years approaches to this type of problem have been developed which seek to allow for the influence of chance and for the unknown complexity of these situations. Thus, forecasting methods were developed which were essentially statistical in nature. These were based on using statistical techniques to fit models of a wide variety of types to past time-series data and on forecasting the future from these models.

The aim of this book is to provide the reader with an understanding of the methods and practice of statistical forecasting. The structure of the book is based on the structure of the activities involved in the practice of statistical forecasting. Figure P.1 summarizes this structure. Part I of the book provides a general introduction to some of the basic concepts and terms in statistical forecasting. Part II deals with the development of statistical forecasting methods applicable to each of a variety of different situations. The approach to classifying this part into chapters has been to classify the situations; e.g. one chapter considers

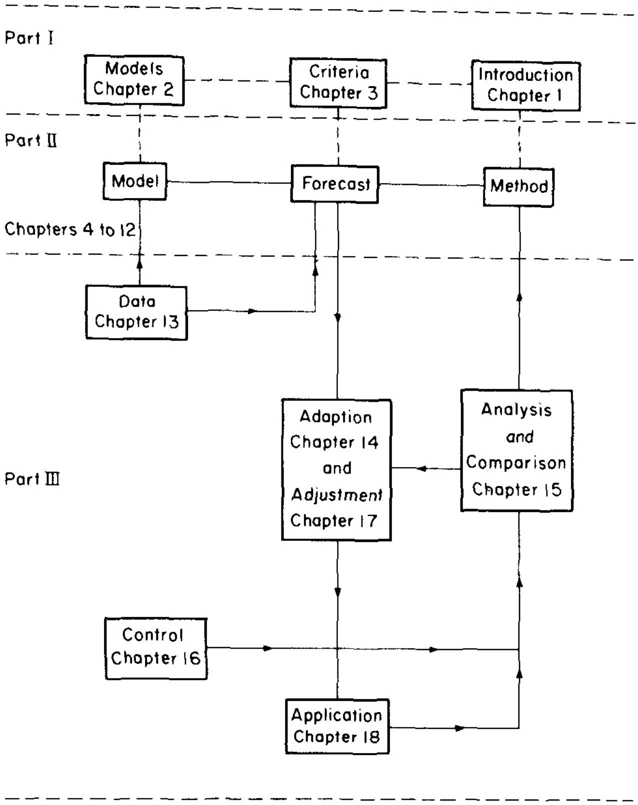


Figure P.1 The structure of the book and of statistical forecasting

trending time series, another seasonally varying time series. We then consider the various methods that might be used in each situation. In practice there is considerably more to statistical forecasting than putting data into a formula to obtain a forecast. Part III deals with these further facets. In Chapter 13 we look at the data that is needed for forecasting, where it might come from and how it may be treated. Chapter 14 looks at ways in which the forecasting methods of Part II may be adapted to apply to a wider range of applications and to give a better quality of forecast. In Chapter 15 we examine a wide range of methods for analysing and comparing forecasting methods. Chapter 16 has the same basic aim, but is conerned more with the routine control of an operational forecasting system. In Chapter 17 we use some of the properties of forecasts, that might be identified using the methods of Chapters 15 and 16, to create even better forecasts. Always we seek to bear in mind that forecasts are produced for practical purposes that may influence the way we obtain and use our forecast in the first place.

This interrelation between forecast and application is explored in Chapter 18.

The general aim of the book is to give a systematic account of the methods of statistical forecasting in common use and of the practical aspects of their use; consequently, the level of mathematics and statistics assumed of the reader has been kept to that which the writer's teaching experience suggests as appropriate to people whose interest is in obtaining forecasts for practical purposes. This is mainly limited to the basic ideas of statistics and the ability to follow simple algebraic arguments. Where more than this is used, the section is marked with an * and can be omitted without detriment to the understanding of later sections. Where certain ideas are used that might be unfamiliar to a significant proportion of readers, these are discussed in the appendices. Wherever possible, the discussion of each forecasting method is developed and illustrated by example in such a way that the reader may then be able to use the method himself. There are, however, a number of methods whose mathematical development has been regarded as beyond the scope of this book, but computer programs are commonly available for their use. The author's aim in these cases has therefore been to discuss and illustrate the basic concepts of the methods. Thus the reader should be able to use such computer programs with understanding of their principles though not of their technical details.

A short list of relevant references which are referred to in the text are given at the end of each chapter.

Acknowledgements

I would like to express my thanks to the many people who have discussed forecasting problems with me over the years and those who have given me comments on early drafts of this book. In particular, I would like to thank N. Booth, O. Oliver, V. G. Gilchrist and also numerous students who have helped me develop suitable approaches to many of the topics in this book. Finally, I would like to thank the Social Science Research Council for their support of work on the analysis of forecasting methods that underlies, particularly, Chapter 15.

W. G. GILCHRIST

Contents

PART I PRELIMINARIES

PART II FORECASTING FOR SOME BASIC MODELS

PART III THE FORECASTING PROCESS

Part I

Preliminaries

Chapter 1

An introduction to forecasting

1.1 Everybody forecasts

Every time we make an appointment we are making a forecast about our future ability to keep that appointment. At the 'see you in ten minutes' level we do not even think about the forecast, though for more long-term appointments we try to think of any obvious things that might stop the meeting from occurring as forecast. The more we think about our forecast, the more likely it is to be right, for we will not be so likely to overlook some prior engagement or some other aspect of the situation. In this example we might even try to assess our confidence in our forecast by the amount of surprise we would show when the arrangement falls through. Such acts of unconscious or semiconscious forecasting are part of our everyday lives; they are also part of the lives of those who are involved with industrial or business problems. The good manager is not so much one who can minimize the effects of past mistakes but rather one who can successfully manage the future. Consider, for example, the following list of questions:

What will next month's sales be?
How much should be produced this month?
How big a stock should be kept?
How much material should be bought?
When should the material be bought?
What should the sales target be?
Should the labour force be increased?
What price should be charged?
How many operators are required?
What will the profit be?

To obtain the best anwers to these questions, we would need to be able to see into the future. To be able to give the best practical answers to these questions, we need to be able to forecast the future. Our forecasts may be wrong, but they must be made. In the past most answers to questions of the above type have been based on unconscious or semiconscious forecasts. In these forecasts it was frequently assumed that the future would be just like the recent past. The growing interest in many aspects of forecasting is based on the belief that conscious and careful thought cannot fail to help improve our forecasting skill and thus our ability to get answers to the above types of question. It may be in some cases that a thorough scientific forecasting study might not lead to much better forecasts than the old 'by guess and by gosh' methods. Even so, such a study will enable those involved to have a better understanding of their situation and thus improve their control of it

There are many methods of statistical forecasting, none of which can be put forward as providing all the answers to everybody's forecasting problems. The techniques discussed in this book are chosen because they have found the widest application. The emphasis of the book, however, is not on specific techniques but on general methods and attitudes towards forecasting. In this way it is hoped that the reader will be better able to apply statistical forecasting methods to his unique problems. New techniques and applications of forecasting are being published every month, and thus if this book is regarded simply as a compendium of forecasting techniques it would very soon become obsolete. If, however, the reader treats it as a text on general methods and attitudes in statistical forecasting, then he should be able to incorporate the new developments in this rapidly expanding field into his basic framework of knowledge.

1.2 Methods of forecasting

In this section we shall define three general methods of forecasting that involve very different approaches. It is not intended to imply that these are the only types available, or that any specific method must automatically be classifiable as one of these, but these are introduced to provide a general background.

1.2.1 Intuitive methods

These are the classical methods of forecasting. They are based essentially on the individual's feeling for the situation, or on the expert's opinion. Often one is forced into these methods by the complexity of the situation and the sparceness of reliable and relevant data. One obvious way of seeking to improve these methods is to replace the single expert by a small group or larger number of people. For example, in forecasting sales one might use the summarized views of the sales force or of a committee of the area sales managers.

Alternatively, a sample survey of the views of potential customers might yield more reliable information.

1.2.2 Causal methods

These methods are based on trying to forecast effects on the basis of knowledge of their causes. In many forecasting situations the causes are economic and hence most of the work in this field is also of an economic nature. Economic relationships between causes and effects are studied; as the effects occur after the causes direct knowledge of the causes leads to a forecast of the effects. But if, as often occurs, the time lag between cause and effect is very small, we may still have to forecast the causes, since information on the causes may not become available until some time after the effects have occurred. The causes used for forecasting are then chosen as those that can be forecast in a more reliable fashion than the effects. If the mechanism of the relationship is known, quite elaborate forecasting methods may be devised. Often, however, the nature of the relationship is obscure and we are forced to use some purely empirical form of relationship. Chapter 6 gives a discussion of some of the relevant forecasting methods. There are a considerable number of books dealing with economic forecasting; the references list some examples.

1.2.3 Extrapolative methods

These methods are all based on the extrapolation into the future of features shown by relevant data in the past. The methods are usually of a mathematial or statistical nature and provide the basis for most of this book. Such methods are sometimes referred to in books on economic forecasting as naive methods. However, to understand any situation we must start by being naive, and consider the simplest methods and approaches. When we understand these thoroughly, we may then progress to the higher realms.

1.3 The basis of forecasting

To attempt a scientific study of forecasting the future might seem a hopeless exercise, since each problem is obviously unique. As we look more closely at specific examples of forecasting, however, we find some common priniciples. Let us consider some examples.

Example 1.1

Most political policy-makers are concerned with forecasting the future as it would be if no new actions were taken. They then consider the *structure* of society as they see it and take action to introduce policies that will improve society according to their political criteria. To be able to decide what action to take, they must be able to identify structures in society that will respond to their policies. It is not unknown for the implemented policies to fail. This might be due to an incorrect identification of the structure or, alternatively, to a lack of *stability* in that structure. To correctly make such forecasts and define policy, the policy-maker needs to be able to identify the appropriate *structure* and have some confidence in its *stability* over the appropriate period.

Example 1.2

H. T. Davis (1941) points out that had the mass of the sun not been so great, relative to the planets, statistical methods would have had to be used to investigate the physical laws of gravitation and to predict future movements of the planets. As things are, the gravitational pull of the sun on a planet is so powerful that the planet's orbit is almost the same as it would be if the other planets did not exist. From observations on this orbit the *structure* of the relationship between sun and planet can be inferred and the inverse square law of gravitation deduced as the model that fits most of what is observed. The great *stability* of the situation enables very accurate forecasts of the future positions of the planets to be made. Had the mass of the sun been smaller, its effects would not have dominated those of the other planets and much more complicated orbits would have occurred. The consequence of this complication would have been that, though the inverse square law would still be the basic law, it would have been much harder to find it from the observed data. Further, forecasting the future positions of the planets would have been much more difficult, as the simplicity and stability of orbit we have would not have occurred.

Example 1.3

As a third example we will consider some of the forms taken by a set of data which might correspond to orders received by a firm each week. Table 1.1 gives the appropriate data. Suppose the form of the data is that given in line (a) of the table. Here the orders come from just one customer whose demand is constant. The data show a very simple *structure*, constant demand, and a perfectly *stable* behaviour. We would thus feel fairly confident in forecasting orders of 6,000 items for week 13. Suppose, however, the data took the form shown in line (b) of Table 1.1. Here the occurrence of 9,000 units of demand at week 5 leads to some doubt about the stability of the structure shown in the remaining weeks. Hence we might still forecast 6,000 items for the next

Table 1.1 Orders received (in 000s)

	Week number											
	1	2	3	4	5	6	7	8	9	10	11	12
(a)	6	6	6	6	6	6	6	6	6	6	6	6
(b)	6	6	6	6	9	6	6					
(c)	6	6	6	6	9	6	6	6	6	9	6	6
(d)	7	6	6	4	7	6	8	6	5	6	7	5
(e)	8	8	9	8	12	12	15	14	14	16	18	17

week, week 8, but no longer with such confidence. Suppose that we wait a few more weeks to see what happens and enquire into the reason for the demand for 9,000 units. We may find that the data in (b) arise from orders by two customers, the one referred to above and also another customer who orders 3,000 items at regular five-weekly periods. The sensible forecast now would be 6,000 items for weeks 13 and 14 and 9,000 items for week 15. We see then that it is sometimes possible to cope with what appears to be an instability in a structure by considering more sophisticated forms of structure. The apparent instability of data (b) becomes the stable structure when more information is obtained, as in data (c).

In practice, data such as those in (a), (b) and (c) do not occur very frequently. We are much more likely to see data such as those shown in line (d) of Table 1.1. These data do not show the nice simple mathematical structure of (a), but nonetheless they do show a structure, as is seen from Figure 1.1. This gives a simple bar chart of the data, showing the frequency with which the sizes of orders occur, and it reveals a quite clear structure, but in this case it is a statistical structure rather than a mathematical one. It is apparent that the average order is near to 6,000 units, and, on the evidence of this small sample of data, the orders appear to be distributed fairly evenly about this value with decreasing frequencies of occurrence away from the average.

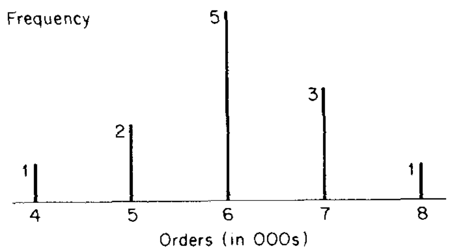


Figure 1.1 Bar chart of the data of Table 1.1(d)