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Outline of Theory and Problems of Introduction to Mathematical Economics

BY
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ALGEBRAIC REVIEW

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- ✔ *Polynomials*
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- ✔ *Functions*
- ✔ *Graphs and Lines*

Exponents

If n is a positive integer, then the expression x^n means that x is multiplied by itself n times. x is the *base*, n is the *exponent*, and the expression x^n is called the *n^{th} power of x*. By definition, $x^0 = 1$ for any nonzero number x. 0^0 is undefined. Other powers can be found by using the following *rules of exponents*:

1. $x^a\left(x^b\right) = x^{a+b}$	4. $(xy)^a = x^a y^a$
2. $\dfrac{x^a}{x^b} = x^{a-b}$	5. $\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$
3. $\left(x^a\right)^b = x^{ab}$	6. $\dfrac{1}{x^a} = x^{-a}$

7. $\sqrt{x} = x^{1/2}$

8. $\sqrt[a]{x} = x^{1/a}$

9. $\sqrt[b]{x^a} = x^{a/b} = \left(x^{1/b}\right)^a$

10. $x^{-(a/b)} = \frac{1}{x^{a/b}}$

Polynomials

For an expression like $5x^2y^3$ we say that 5 is the *coefficient*, x and y are *variables*, and the expression is called a *monomial*. A *polynomial* is made by adding and subtracting monomials, each of which is then called a *term* of the polynomial. Terms with the exact same variables and powers are called *like terms*.

Remember

When adding or subtracting polynomials, you can combine like terms by adding or subtracting their coefficients but you cannot combine unlike terms.

In multiplying two polynomials, each term in the first polynomial must be multiplied by each term in the second. Then, all these products can be collected up and like terms can be combined.

Example 1.1

$$\begin{aligned}(2x+3y)(8x-5y-7z) &= 16x^2-10xy-14xz+24xy-15y^2-21yz \\ &= 16x^2+14xy-14xz-21yz-15y^2\end{aligned}$$

Linear and Quadratic Equations

An *equation* is a mathematical statement equating two algebraic expressions. If the variable x is only raised to the first power, then the equation is a *linear equation in x*. If x is only raised to the first and second powers, then the equation is a *quadratic equation in x*.

A linear equation is solved by moving all terms containing the variable to the left-hand side of the equation, moving all other terms to the right-hand side, then dividing by the coefficient of the variable.

Example 1.2

$$\frac{x}{4}-3=\frac{x}{5}+1 \quad \Rightarrow \quad \frac{x}{4}-\frac{x}{5}=1+3 \quad \Rightarrow \quad .05x=4 \quad \Rightarrow \quad x=\frac{4}{.05}=80$$

A quadratic equation can be arranged to the form $ax^2+bx+c=0$. If a is non-zero, then we obtain the two possible solutions for x by using the *quadratic formula*:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example 1.3

$$5x^2-55x+140=0 \quad \Rightarrow \quad a=5, b=-55, c=140$$

$$\Rightarrow \quad x=\frac{-(-55)\pm\sqrt{(-55)^2-4(5)(140)}}{2(5)}=\frac{55\pm 15}{10}$$

$$\Rightarrow \quad x=\frac{55+15}{10}=7 \quad OR \quad x=\frac{55-15}{10}=4$$

Simultaneous Equations

A *system* of equations is a collection of equations that are supposed to be true simultaneously. If it is impossible for the equations to all be true, then the system is called *inconsistent*. If one of the equations can be made by adding/subtracting some of the other equations together, then we say that the equations are *dependent*. If neither of these conditions arises (so that the system is *consistent* and *independent*) and if the number of equations is equal to the number of variables, then the system will have exactly one solution.

The *substitution method* for solving a system has four steps: 1. Solve one of the equations for any of its variables; 2. Substitute that value of that variable for every occurrence of that variable in the remaining equations; 3. Repeat the first two steps until you an equation that has only one variable in it, so you obtain a numerical value for that variable; 4. Find the value of the remaining variables by substituting back through your other equations.

Example 1.4

Using the substitution method on the system:

$$\begin{Bmatrix} 8x - 3y = 7 \\ -x + 7y = 19 \end{Bmatrix}$$

1. Solve the second equation for x.

$$-x + 7y = 19 \Rightarrow x = 7y - 19$$

2. Substitute this expression for x into the first equation.

$$8(7y - 19) - 3y = 7 \Rightarrow 53y = 159$$

3. Solve this equation for y.

$$y = 159/53 = 3$$

4. Substitute back through the second equation, and solve.

$$-x + 7(3) = 19 \Rightarrow x = 21 - 19 = 2$$

The solution is (2,3).

The *elimination method* for solving a system is usually faster than the substitution method: 1. We multiply two of our equations by different numbers, in order to make the coefficients for one of the variables match. 2. When we subtract the equations, we obtain a new equation that does not have as many variables. We eventually reduce the number of variables to one, and finish by using steps 3 and 4 from the substitution method.

Example 1.5

Using the elimination method on the system:

$$\begin{Bmatrix} 8x - 3y = 7 \\ -x + 7y = 19 \end{Bmatrix}$$

1. Multiply the first equation by 1 and the second by −8, in order to make the coefficients on x match each other.

$$\begin{Bmatrix} 8x - 3y = 7 \\ 8x - 56y = -152 \end{Bmatrix}$$

2. Subtract the second equation from the first.

$$53y = 159$$

3. Solve this equation for y.

$$y = 159/53 = 3$$

4. Substitute back through the second equation, and solve.

$$-x + 7(3) = 19 \Rightarrow x = 21 - 19 = 2$$

The solution is (2,3).

Functions

A *function* on the variable x is a rule that assigns to each value of x a unique numerical value $f(x)$. x is called the *argument* of the function, and $f(x)$ is called the *value of the function at x*. The *domain of f* refers to the set of all x where the function can meaningfully be applied; the *range* is the set of $f(x)$ values that result. The following types of functions occur frequently in economics.

Linear function:

$$f(x) = mx + b$$

Quadratic function:

$$f(x) = ax^2 + bx + c$$

Polynomial function of degree n:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

(where n is a nonnegative integer, and a_n is not 0)

Rational function:

$$f(x) = \frac{g(x)}{h(x)}$$

(where $g(x)$ and $h(x)$ are both polynomials)

Power function:

$$f(x) = ax^n$$

(where n is any real number)

Graphs and Lines

In graphing a function $y = f(x)$, we usually put the argument x on the horizontal axis and call it the *independent variable*. y is put on the vertical axis and is called the *dependent variable*. (In some contexts economists will put the independent variable on the vertical axis, and it is always best to clarify which variable is regarded as independent.)

The graph of linear equation is a straight line. The *slope* measures the ratio $\Delta y / \Delta x$, where Δy is the *change in y* and Δx is the *change in x*, and indicates the direction and steepness of the line. A positively sloped line moves up from left to right; a negatively sloped line moves down. The greater the absolute value of the slope, the steeper the line. A horizontal line has slope 0; the slope of a vertical line is not defined but we will frequently say that it has *infinite slope*. The *intercept* of a line is the value $f(0)$, which is the value when the graph crosses the y-axis. The *x-intercept* of a line is the value of x that makes $f(x) = 0$ true.

We can graph a linear function by finding any two points on the line and connecting them as in Figure 1-1. If the line is in *slope-intercept form* $y = mx + b$, then we will usually use $(0,b)$ as one of the two points. The second point can be chosen by plugging in any value of x, or by noticing that $(-b/m,0)$ is the location of the x-intercept.

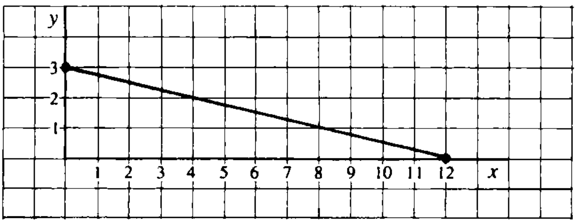


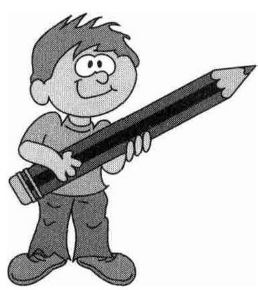
Figure 1-1. The graph of the function $f(x) = -1/4\,x + 3$

Chapter 2
Economic Applications of Graphs and Equations

In This Chapter:

- ✔ *Isocost Lines*
- ✔ *Supply and Demand Analysis*
- ✔ *Income Determination Models*
- ✔ *IS-LM Analysis*
- ✔ *Solved Problems*

Isocost Lines



If the expenditures of a company must be divided between costs for two different components x and y with respective prices p_x and p_y, then we can write $E = p_x x + p_y y$. An *isocost line* (or *budget line*) shows all the choices that the company can make with a particular level of expenditure. E and the individual prices are held constant; only the different combinations of (x,y) inputs are allowed to change. The line can be graphed by plotting and connecting the intercepts $(0,E/p_y)$ and $(E/p_x,0)$. Its slope is $-p_y/p_x$.

If any of expenditure or price parameters are changed, the isocost line may shift or change slope. An increase in the expenditure level E will cause the line to shift to the right; it will be parallel to the old line because the slope $-p_y/p_x$ is unchanged. A change in the cost of component y only will affect the y-intercept, but not the x-intercept.

Supply and Demand Analysis

Market curves show the production and consumption responses to the price level P of a commodity—the *supply curve* expresses the suppliers' production level Q_s as a function of P, and the *demand curve* expresses the consumers' desired consumption level Q_d as a function of P. *Market equilibrium* (or *market clearing*) occurs where the market curves cross, at a price where production equals consumption. This price can often be found algebraically, by equating the supply and demand functions.

Income Determination Models

The equation for a *four-sector economy* equates national income Y to the sum of *consumption* C, *investment* I, *government expenditures* G, and *trade surplus* $(X - Z)$ where X = exports and Z = imports: $Y = C + I + G + (X - Z)$. Typically, at least one of the components on the right-hand side (usually C) is given as a function of Y; the others may not be present in the problem, or may be given as constants. By *aggregating* (adding up) the formulas for the four components, the right-hand side can be graphed as a function of Y.

Economic equilibrium occurs where this aggregate equals Y and can be seen graphically as the point where the aggregate curve crosses the 45° line drawn from the origin. Algebraically, this equilibrium income can be found by setting the aggregate equal to Y, and then solving for the value of Y.

IS-LM Analysis

IS-LM analysis extends the income determination model to incorporate money markets and the level of interest rates (represented by a new variable "i"). The equation $Y = C + I + G + (X - Z)$ is still used for the commodities market, but now each component is allowed to be a function of Y and/or i. Additionally, a second equation $M_s = M_t + M_z$ represents the money markets, by equating the supply of money M_s with the sum of the *transaction-precautionary demand* for money M_t and the *speculative demand* for money M_z. In this new equation M_s is usually given as a constant, while M_t and M_z are allowed to be functions of Y and/or I.

The *IS-schedule* of the economy is the set of (Y,i) combinations leading to equilibrium in the commodities market. The *LM-schedule* is the set of (Y,i) combinations leading to equilibrium in the money market. *Economic equilibrium* occurs when both markets are in equilibrium simultaneously and can be seen graphically as the intersection of the IS and LM curves. Algebraically, the equilibrium income and interest level can be found by simultaneously solving the system formed by the IS equation and the LM equation.

Solved Problems

Solved Problem 2.1 A company with a \$120 budget can produce two different goods x and y, with manufacture prices \$3 and \$5. Show (by drawing two isocost lines on a single graph) the effect of (a) a 25% reduction in the budget, (b) a doubling in the price of x, (c) a 20% reduction in the price of y.

Solution: The original isocost line is $3x + 5y = 120$, and can be graphed by plotting the intercepts (40,0) and (0,24). This graph is shown as the solid line in each graph of Figure 2-1. The new isocost lines are (a) $3x + 5y = 90$, (b) $6x + 5y = 120$, (c) $3x + 4y = 120$; these are plotted as dashed lines in Figure 2-1.

Solved Problem 2.2 Find the equilibrium price and quantity for the one-commodity market $Q_s = -45 + 8P$, $Q_d = 125 - 2P$.

Solution: At equilibrium, $Q_s = Q_d$. $-45 + 8P = 125 - 2P \Rightarrow 10P = 170$. So $P = 17$, and the equilibrium quantity is $Q_e = Q_s(17) = -45 + 8(17) = 91$.

Solved Problem 2.3 Find the equilibrium conditions for the following two-commodity market for beef B and chicken C: $Q_{dB} = 82 - 3P_B + P_C$, $Q_{dC} = 92 + 2P_B - 4P_C$, $Q_{sB} = -5 + 15P_B$, $Q_{sC} = -6 + 32P_C$.