Michio Kaku

# Introduction to Superstrings

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# Introduction to Superstrings

With 48 Illustrations

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*This Book is Dedicated to My Parents*

# Preface

> We are all agreed that your theory is crazy. The question which divides us is whether it is crazy enough.
>
> Niels Bohr

Superstring theory has emerged as the most promising candidate for a quantum theory of all known interactions. Superstrings apparently solve a problem that has defied solution for the past 50 years, namely the unification of the two great fundamental physical theories of the century, quantum field theory and general relativity. Superstring theory introduces an entirely new physical picture into theoretical physics and a new mathematics that has startled even the mathematicians.

Ironically, although superstring theory is supposed to provide a unified field theory of the universe, the theory itself often seems like a confused jumble of folklore, random rules of thumb, and intuition. This is because the development of superstring theory has been unlike that of any other theory, such as general relativity, which began with a geometry and an action and later evolved into a quantum theory. Superstring theory, by contrast, has been evolving backward for the past 20 years. It has a bizarre history, beginning with the purely accidental discovery of the quantum theory in 1968 by G. Veneziano and M. Suzuki.

Thumbing through old math books, they stumbled by chance on the Beta function, written down in the last century by mathematician Leonhard Euler. To their amazement, they discovered that the Beta function satisfied almost all the stringent requirements of the scattering matrix describing particle interactions. Never in the history of physics has an important scientific discovery been made in quite this random fashion.

Because of this accident of history, physicists have ever since been trying to work backward to fathom the physical principles and symmetries that underlie the theory. Unlike Einstein's theory of general relativity, which began with a geometric principle, the equivalence principle, from which the action could be derived, the fundamental physical and geometric principles that lie at the foundation of superstring theory are still unknown.

To reduce the amount of hand-waving and confusion this has caused, two themes have been stressed throughout this book. To provide the student with a solid foundation in superstring theory, we have first stressed the method of *Feynman path integrals*, which provides by far the most powerful formalism in which to discuss the model. Path integrals have become an indispensable tool for theoretical physicists, especially when quantizing gauge theories. Therefore, we have devoted Chapter 1 of this book to introducing the student to the methods of path integrals for point particles.

The second theme of this book is the method of *second quantization*. Although traditionally field theory is formulated as a second quantized theory, the bulk of superstring theory is formulated as a first quantized theory, presenting numerous conceptual problems for the beginner. Unlike the method of second quantization, where all the rules can be derived from a single action, the method of first quantization must be supplemented with numerous other rules and coventions. The hope is that the second quantized theory will reveal the underlying geometry on which the entire model is based. Thus, we have tried to stress the importance of second quantization and string field theory throughout this book. This is not just for pedagogical reasons. Ultimately, the second quantized formulation may solve the outstanding problem of superstring theory: dynamically breaking a 10-dimensional theory down to 4 dimensions.

In addition to providing the student with a firm foundation in path integrals and field theory, the other purpose of this book is to introduce students to the latest developments in superstring theory, that is, to acquaint them with the fast-paced areas that are currently the most active in theoretical research, such as:

String field theory
Conformal field theory
Kac–Moody algebras
Multiloop amplitudes and Teichmüller spaces
Calabi–Yau phenomenology
Orbifolds and four-dimensional superstrings

The goal of this book is to provide students with an overview by which to evaluate the research areas of string theory and perhaps even engage in original research. The only prerequisite for this book is a familiarity with advanced quantum mechanics. However, the mathematics of superstring theory has soared to dizzying heights. In order to provide an introduction to more advanced mathematical concepts, such as Lie groups, general relativity, supersymmetry, and supergravity, we have included a short introduction to them in the Appendix, which we hope will fill the gaps that may exist in the students' preparation. Finally, terms that may be unfamiliar to graduate students are included in the Glossary of Terms, also in the Appendix.

For the student, we should mention how to approach this book. Chapters 1–5 represent Part I, the results of first quantization. They form an essential foundation for the next chapters and cannot be skipped. Chapter 1, however, may be skipped by one who is relatively fluent in the methods of ordinary quantum field theory, such as gauge invariance and Faddeev–Popov quantization. (But we emphasize that the method of path integrals forms the foundation for this book, and hence even an advanced student may profit from reviewing Chapter 1.)

Chapters 2 and 3 form the heart of an elementary introduction to string theory. Chapter 4, however, can be omitted by one who only wants an overview of string theory. With the exception of the fermion vertex function and ghosts, most of the results of string theory can be developed using Chapters 2 and 3 without conformal field theory, and hence a beginner may overlook this chapter. (However, we emphasize that most modern approaches to first quantized string theory use the results of conformal field theory because it is the most versatile. A serious student of string theory, therefore, should be thoroughly familiar with the results of Chapter 4.)

Chapter 5 is essential to understand the miraculous cancellation of divergences of the theory, which separates string theory from all other field theories. Because the theory of automorphic functions gets increasingly difficult as one describes multiloop amplitudes, the beginner may skip the discussion of higher loops. The serious student, though, will find that multiloop amplitudes form an area of active research.

Part II begins a discussion of the field theory of strings, and Part III examines phenomenology. The order of these two parts can be interchanged without difficulty. Each part was written to be relatively independent of the other, so the more phenomenologically inclined student may skip directly to Part III without suffering any loss.

Chapters 6–8 in Part II present the evolution of three approaches to string field theory. Chapter 6 discusses the original light cone theory and how to quantize multiloop theories based on strings. However, Chapter 7 was written in a relatively self-contained fashion, so the serious student may skip Chapter 6 and delve directly into the covariant theory.

Ultimately, all of string field theory will be written geometrically, and one promising geometric formalism is presented in Chapter 8. The beginning student may find Chapter 8 a bit difficult, because it is highly mathematical. Therefore, the beginner may omit this chapter.

In Part III, the beginner may skip Chapter 9. The discussion of anomalies is rather technical and mainly based on point particles, and overlaps the discussions found in other books. Chapter 10 cannot be omitted, as it represents one of the most promising of the various superstring theories. Likewise, Chapter 11 forms an essential part of our understanding of how the superstring theory may eventually make contact with experimental data.

The author hopes that this will help both the beginner and the more advanced student to decide how to approach this book.

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New York, New York MICHIO KAKU

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## Part II. Second Quantization and the Search for Geometry



## Part III. Phenomenology and Model Building

# Part I

# First Quantization and Path Integrals

CHAPTER 1

# Path Integrals and Point Particles

## §1.1. Why Strings?

One of the greatest scientific challenges of our time is the struggle to unite the two fundamental theories of modern physics, quantum field theory and general relativity, into one theoretical framework. Remarkably, these two theories together embody the sum total of all human knowledge concerning the most fundamental forces of nature. Quantum field theory, for example, has had phenomenal success in explaining the physics of the microcosm, down to distances less than $10^{-15}$ cm. General relativity, on the other hand, is unrivaled in explaining the large-scale behavior of the cosmos, providing a fascinating and compelling description of the origin of the universe itself. The astonishing success of these two theories is that together they can explain the behavior of matter and energy over a staggering 40 orders of magnitude, from the subnuclear to the cosmic domain.

The great mystery of the past five decades, however, has been the total incompatibility of these two theories. It's as if nature had two minds, each working independently of the other in its own particular domain, operating in total isolation of the other. Why should nature, at its deepest and most fundamental level, require two totally distinct frameworks, with two sets of mathematics, two sets of assumptions, and two sets of physical principles?

Ideally, we would want a unified field theory to unite these two fundamental theories:

$$\left.\begin{array}{l}\text{Quantum field theory}\\ \text{General relativity}\end{array}\right\}\text{Unified field theory}$$

However, the history of attempts over the past decades to unite these two theories has been dismal. They have inevitably been riddled with infinities or have violated some of the cherished principles of physics, such as causality. The powerful techniques of renormalization theory developed in quantum field theory over the past decades have failed to eliminate the infinities of quantum gravity. Apparently, a fundamental piece of the jigsaw puzzle is still missing.

Although quantum field theory and general relativity seem totally incompatible, the past two decades of intense theoretical research have made it increasingly clear that the secret to this mystery most likely lies in the power of *gauge symmetry*. One of the most remarkable features of nature is that its basic laws have great unity and symmetry when expressed in terms of group theory. Unification through gauge symmetry, apparently, is one of the great lessons of physics. In particular, the use of local symmetries in Yang–Mills theories has had enormous success in banishing the infinities of quantum field theory and in unifying the laws of elementary particle physics into an elegant and comprehensive framework. Nature, it seems, does not simply incorporate symmetry into physical laws for aesthetic reasons. Nature *demands* symmetry.

The problem has been, however, that even the powerful gauge symmetries of Yang–Mills theory and the general covariance of Einstein's equations are insufficient to yield a finite quantum theory of gravity.

At present, the most promising hope for a truly unified and finite description of these two fundamental theories is the superstring theory [1–12]. Superstrings possess by far the largest set of gauge symmetries ever found in physics, perhaps even large enough to eliminate all divergences of quantum gravity. Not only does the superstring's symmetry include that of Einstein's theory of general relativity and the Yang–Mills theory, it also includes supergravity and the Grand Unified Theories (GUTs) [13] as subsets.

Roughly speaking the way in which superstring theory solves the riddle of infinities can be visualized as in Fig. 1.1, where we calculate the scattering of two point particles by summing over an infinite set of Feynman diagrams with loops. These diagrams, in general, have singularities that correspond to "pinching" one of the internal lines until the topology of the graph is altered. By contrast, in Fig. 1.2 we have the single-loop contribution to the scattering

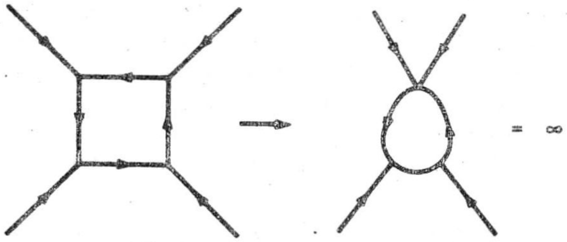


Figure 1.1. Single-loop Feynman diagram for four-particle scattering. The ultraviolet divergence of this diagram corresponds to the pinching of one internal leg, i.e., when one internal line shrinks to a point.