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Spectral, Spatial, and Temporal Properties of Lasers

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А. М. Ратнер

СПЕКТРАЛЬНЫЕ, ПРОСТРАНСТВЕННЫЕ И ВРЕМЕННЫЕ ХАРАКТЕРИСТИКИ ЛАЗЕРА

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TRANSLATION EDITOR'S PREFACE

During the decade there were many developments in laser research and numerous applications of the laser were made in fields of science and engineering. Many theoretical and experimental advances were made in the Soviet Union; often they paralleled those taking place in the United States and elsewhere but started from different points, proceeded along different paths, and yielded different insights into the physical processes taking place in the laser.

The present book offers a unified theory of lasers by which the operating characteristics of the laser are described and related to the details of the radiation emitted. Extensive emendations to the original, Soviet edition were supplied by the author and incorporated into the text, and references to the English literature were added to the translation to permit the reader to readily explore topics in greater detail.

Since the Soviet edition's appearance in 1968 one very important area has developed – that of mode locking and picosecond pulse production. This area is so important that no comprehensive work on the laser published in 1972 could neglect it. Accordingly, Chapter XII was added to round off the treatment. I am indebted to my colleague, Professor A. J. Carruthers of the University of Minnesota for many illuminating discussions on mode locking.

Richard A. Philips

FOREWORD TO THE AMERICAN EDITION

The purpose of the present monograph is to examine the physical essence of certain interrelated problems in the theory of solid-state lasers. In striving for the maximum physical clarity of presentation, the author has attempted to use a mathematical device which is adequate for the physics of the problems investigated. This has been expressed in the following way.

In the most general case the electromagnetic field in a resonator is described by a system of two equations for the electric field E and the electric induction D. One of these equations is the Maxwell equation, while the second is the equation for the polarization of the material, written using the density matrix. However, this rather complex mathematical device is inexpedient for use in investigating the fields of a solid-state laser. In fact, the optical bands of condensed media are usually characterized by a width which substantially exceeds the reciprocal of the optical-transition time. In this case it is not difficult to show that the equation for the polarization of the material takes the trivial form $D = \varepsilon E$, where ε is the complex dielectric constant whose imaginary part is proportional to the effective absorption coefficient. Thus, the field is described by the conventional wave equation with complex ε. This wave equation is used in the present monograph to investigate the spatial and spectral distributions of the fields in the resonator. In particular, the radiation spectrum is determined by competition between modes, which is conveniently investigated using the wave equation in the mode-amplitude representation.

The natural resonator oscillations, which are generated simultaneously in fairly large numbers, behave as a unified whole (i.e., competition between them becomes negligible). In this case the wave equation reduces to the elementary kinetic equation for the radiative energy. This kinetic equation is used in the monograph to investigate regular oscillations of multimode radiation. The energy approach is similarly justified in other cases in which the competition between modes is negligible (for example, in the investigation of various threshold phenomena).

Thus, in investigating each problem we use the simplest adequate mathematical device; the author hopes that because of this the monograph has sufficient physical clarity.

In this edition the inaccuracies and typographical errors appearing in the Soviet edition have been corrected to the extent possible. However, it proved to be practically impossible to supplement the book with the results of later research. The presentation of these results is contained in another monograph by the author [108].

A. Ratner

FOREWORD

Many papers on quantum electronics are currently being published, and they are distinguished by a variety of terminology and approach to the problems investigated.

In our monograph we present several closely related problems in the theory of solid-state lasers from a unified point of view. Chapter I presents certain information from luminescence theory; Chap. II presents basic concepts. In Chaps. III–VI we examine the spectral properties and spatial structure of the electromagnetic field in the laser: the third chapter presents linear resonator theory without considering the active medium, while the fourth and fifth chapters develop the nonlinear theory of a practical resonator containing an active medium and analyze the fundamental differences between this case and the linear case, which are expressed, in particular, in the angular and spectral spread of the radiation. Chapter VI examines a resonator with concave mirrors of arbitrary shape from a similar point of view. Chapter VII transforms the wave equation to the "balance-equation" form on the assumption of spatial uniformity of the active medium, and this equation is then used to consider relaxational oscillations of the radiation. The conditions are analyzed under which proper oscillations can actually be observed. Simple expressions are derived for the period, shape, amplitude, and damping of the oscillations. The behavior of the laser in modulated-Q operating modes is closely connected with the characteristics of free oscillations (Chaps. X and XI). The spectral composition of laser radiation is considered in Chap. VIII while taking account of the most essential physical causes of spectral broadening. Chapter IX is devoted to an investigation of threshold phenomena connected with the microinhomogeneity of the active medium.

Quantities having a clear physical meaning are used as the basic variables. A portion of the material presented in the monograph had to be rephrased somewhat in order to retain a unified terminology.

Special attention is devoted to a detailed qualitative description of the physical picture of the phenomena investigated. Brief conclusions are given at the end of each chapter for the convenience of the reader.

This monograph investigates only solid-state lasers with optical pumping; §§ 7, 8, and 18, which are devoted to linear theory and apply equally to gas lasers, are an exception. In view of its limited size, the monograph cannot pretend to exhaustive completeness in the presentation of the theoretical, let alone the experimental, material.

The author is deeply indebted to Professor G. E. Zil'berman and Doctor of Physicomathematical Sciences A. N. Oraevskii for reviewing the manuscript and their valuable comments, as well as to Candidates of Physicomathematical Sciences B. L. Livshits and V. N. Tsikunov for very helpful discussions.

CONTENTS

BASIC NOTATION

a = radius of the generating region of the sample cross section.

a = subscript with which the parameters of the passive shutter are labeled.

c = velocity of light in a vacuum.

$D = \lambda l / a^2$ = small parameter stipulating the diffraction losses.

$\vec{E}$ = electric field vector.

$\mathscr{E}$ = energy of the stimulated emission (in arbitrary units).

$\hbar$ = Planck's constant (divided by 2π).

j = one-half of the number of generated longitudinal modes, § 15.

J = volume energy density of the stimulated emission.

$\vec{k}(k_1, k_2, k_3)$ = wave vector of the light.

$k_\perp = k_1, k_2$ = component of h which is perpendicular to the optic axis.

K = quantum luminescence yield (see (2.3) and (2.4)).

l = resonator length.

l_0 = length of the active rod.

m_3, m_2, m_1 = indices of the longitudinal and transverse modes; $m_\perp = m_1, m_2$.

n = number of excited luminescence centers per unit volume.

n^* = threshold value of n.

$\underline{n}$ and $\bar{n}$ = threshold values of n for an open and closed optical shutter;† $\bar{n}$ coincides with the value of n at the instant generation begins.

$n(\omega)$ = refractive index of the active medium.

N = pump power absorbed per unit volume.

† In the case of a three-level diagram, this value is measured from the value $n_0/2$, where n_0 is the total volume concentration of luminescence centers.

N^* = threshold value of N (in the case of a resonator with modulated Q for a closed optical shutter).

p = ratio between the initial number of excited atoms and n^* (t^*).

$p = \bar{n}/n$ = maximum possible value of p.

$P(\omega) \equiv P_2(\bar{\omega})$ = contour of the luminescence band corresponding to the operating transition, normalized to a unit area.

r = reflectivity of the end mirrors.

R = radius of curvature of a spherical mirror.

s = probability of excitation migration during the time T.

t = time (usually measured from the beginning of generation).

t_0 = characteristic time for the variation of Q.

T = time for a spontaneous optical transition.

t^*, u^* = instant of maximum intensity.

u = the time expressed in $\sqrt{T/v\,\varkappa_1\xi}$ units.

$u(x, y) = z$ = equation for the surface of a concave mirror.

u_{max} = height of a mirror surface or sagitta in the active region of the cross section.

$v = c/n(\omega_0)$ = velocity of light in the material.

$y(t)$ = integral of the effective absorption coefficient along the light path.

Y = oscillation amplitude of the stimulated emission.

Y_1 = initial value of Y.

z = coordinate measured along the optic axis.

γ = additional losses of one of the polarization components.

$\delta = 3/\sqrt{\varkappa_1 k \xi}$ = width of the region in which diffraction is substantial.

δt = width of the intensity peak of the generated light.

$\delta\omega$ = spectral width of the generated radiation.

$\delta\Omega$ = solid angle in which the laser radiation is concentrated.

Δt, Δu = oscillation period of the laser radiation.

$\Delta\omega$ = half-width of the luminescence band.

ε = dielectric constant of the active medium, $\varepsilon_0 = \mathrm{Re}\,\varepsilon$.

ξ = relative amount by which the threshold is exceeded in a four-level diagram $(N - N^*)/N^*$; for a three-level diagram $\xi = [(N - N^*)/N^*] \cdot [(u + \varkappa_1)/\varkappa_1]$.

θ = angular divergence of the laser radiation.

ϑ = angular (admissible) misalignment of the end mirrors.

$\varkappa(\omega, t) =$ effective absorption factor, which allows for light losses and light amplification by the active medium (§ 4).

$\varkappa_1 = \varkappa_0 + (1-r)l =$ losses per unit light path.

$\varkappa_0 =$ absorption coefficient of the basic material.

$\lambda =$ light wavelength.

$\Lambda =$ large parameter of the order of $|\ln \delta\Omega|$.

$\mu(\omega) =$ absorption coefficient in the band corresponding to the working transition for a three-level diagram only.

$\rho(\omega) =$ spectral density of the stimulated radiation.

$\varphi =$ angle between the electric vector and the crystallographic axis.

$\varphi(t) = n^*(t)/n =$ function describing the Q modulation.

$\omega_0 =$ working frequency of the laser.

$\bar{\omega} =$ characteristic frequency interval of the order of the half-width of the luminescence band.

$\omega' = \omega - \omega_0 =$ frequency measured from the working frequency.

Chapter I

CERTAIN INFORMATION FROM LUMINESCENCE THEORY

Nonconducting solids (for example, single crystals) containing a small impurity of luminescent atoms are usually used as the active media for solid-state lasers. Such impurity luminophors can have a relatively narrow luminescence band; this is very essential, since the threshold pump power is proportional to the half-width of the luminescence band. Pure luminescent crystals are impractical active media because of their broad luminescent band (which arises because the transition is between energy bands) and also because of their excessively high absorption. In view of this, we shall restrict our examination to impurity luminescence.

§1. The Absorption and Luminescence Spectra of Impurity Luminophors

Usually the impurity concentration introduced into a nonconducting crystal is so small that the interaction of the impurity atoms with one another can be neglected. Then discrete energy levels, which usually are simply shifted levels of the isolated impurity atom, appear in the interval between the valence band and the conduction band of the dielectric.

The optical transitions between the ground and higher levels usually correspond to rather broad absorption bands (with a width of the order of a thousand cm^{-1}). These impurity absorption bands are shifted toward wavelengths that are long relative to the intrinsic absorption bands of the crystal. Most frequently there are several impurity absorption bands corresponding to transitions to different excited levels.

If the crystal is illuminated by light whose wavelength falls in the impurity absorption band, then the impurity atoms are