OPTICS

Second Edition

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Preface

The first edition of this book evolved from lectures given at the University of Illinois during a one-semester course on light. The students were juniors, seniors, and first-year graduate students in physics, electrical engineering, mechanical engineering, and chemistry. The text was intended to serve in a one-semester to one-year course at the advanced undergraduate/first-year graduate level.

The second edition has been significantly revised throughout. Our main objective was to soften the approach and introduce consistency while maintaining a connection to rigorous concepts. The result is a book that is useful as a text as well as a general reference on the fundamentals of optics. Our revisions are based on five years' experience teaching a course in geometrical and physical optics at Rensselaer Polytechnic Institute. This course was taken by undergraduates—primarily sophomores—during the second semester of the academic year.

Optics presents an introduction to classical concepts of geometrical and physical optics. These are discussed with reference to fundamental theories of light: Fermat's principle, Huygen's principle, and Maxwell's equations. Everything from attenuated total reflection and geometrical aberration theory to spatial filtering, Gaussian beam optics, and statistical fluctuations is covered. In addition, we have presented the conventional groundwork for understanding practical optics: image formation, optical instruments, interference, diffraction, and polarization. The reader is given enough of the principles behind practical optical components and systems so that he or she can do effective laboratory work. With the background presented here, the student can take the next step: to enter into advanced treatments and the optics literature. The most significant omission of this book is the quantum theory of the interaction of light with matter. Consequently, there is no detailed discussion of laser action. However, this revised edition presents the most comprehensive elementary treatment of the optical processing of coherent light and Gaussian beams currently available in any textbook.

Some of the topics worked out in *Optics* will be found in no other introductory textbook. In Chapter 3, specific ray-tracing techniques are presented in the rigorous and paraxial limits. This information can serve as the basis for computerized ray-tracing techniques, and it leads to the use of the matrix technique for the presentation and exploitation of the concepts of lens action in the paraxial limit in the remainder of Chapter 3.

Our treatment of lens aberrations in Chapter 4 is the most straightforward and complete treatment available. Specific formulas are provided for the primary aberrations of a thin lens. Our approach to multiple-reflection interference in Chapter 5 is based on the Jones matrix approach but has never before appeared in textbook form. It is a powerful formalism that can be computerized to deal with very complex problems such as the design of an interference filter (a problem presented at the end of the chapter). The treatment of diffraction in Chapters 6 and 7 is based on the transformation concept as found in advanced theory. However, it is presented here in its most simple and consistent form. This information can be directly applied in real problems that a practicing scientist or engineer might encounter.

Several specific modifications and additions in the revised edition are worthy of note.

- The SI system of units is used throughout.
- All introductory theory is condensed into Chapter 1, which is presented from an historical perspective.
- All the material relating to the interaction of light with matter is reorganized into Chapter 2.
- The matrix convention in Chapter 3 has been changed to conform to the most commonly found standard. More examples of optical imaging have been included, and a redundant treatment of image formation theory has been eliminated.
- The section on aberrations in Chapter 4 has been entirely rewritten.
- Chapter 5, which deals with interference, is new. The matrix method of multiple beam interference and many more examples of the application of interference are provided. Grating phenomena have been moved to this chapter as well.
- The details of Fresnel-Kirchhoff theory in Chapter 6 have been removed and placed in an appendix. Fourier mathematics is presented as a separate section within this chapter.
- More advanced topics in diffraction are found in Chapter 7. The notation has been simplified so as to bring out the easily understood transformation characteristics of the theory. This chapter contains new material on Gaussian beam optics.

- All material involving partial coherence is contained in Chapter 8, including incoherent image formation.
- Approximately half of the figures have been redrawn to emphasize clarity.

The revised version contains a significantly enlarged and broadened collection of problems at the ends of the chapters. There are enough different kinds of exercises to supplement instruction in a wide variety of courses.

The assumed prerequisites for a course taught from this text are introductory physics—including exposure to ideas of electricity, magnetism, and wave motion—and introductory calculus. Differential equations are discussed but only as a connection to wave theory. It is not expected that students be able to solve differential equations. Material from this text has been used with little difficulty in the course taught to sophomores at Rensselaer.

Instructors wishing to use this book for a one-semester course might follow these guidelines: Introductory theory (sections 1.1–1.5); light-matter interactions (sections 2.1.C, 2.2.B–2.2.E, 2.3); image formation and optical instruments (sections 3.1.A.1, 3.2.A, 3.3–3.5); stops (section 4.1.A); interference (5.1–5.6); far-field diffraction (sections 6.1, 6.2); near-field diffraction (sections 7.1, 7.2), and polarization (sections 9.1, 9.2).

We acknowledge the help of many colleagues and students for suggestions and remarks, especially R. D. Sard and H. Macksey. We are grateful to Nila Meredith, Nancy Fowler, Darcy Sorocco, and Geri Frank for their careful typing assistance, and to Marc de Peo for making some of the diffraction photographs. But most of all, we thank our families for their understanding and support during this project.

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Contents

1 The Nature of Light

The study of optics covers those phenomena involving the production and propagation of light and its interaction with matter. Throughout history, philosophers and scientists have tried to explain what light is; in so doing, they have tested their evolving knowledge of our physical world. Although many early ideas have been proven false, others have been repeatedly verified by experimental tests. Among these are the concept of the finiteness of the speed of light; the principle of least time, which applies to the path of propagation; and the idea that light behaves like a wave.

1.1 Early Ideas and Observations

It is difficult for us to appreciate the mystery surrounding the nature of light and vision in the ancient world. Not only was the mechanism of the eye unknown, but fundamental optical principles that we take for granted were also obscure. In spite of this, motivated by interests in geometry, art, and deception (magic), the Greeks developed some relatively sophisticated notions.

A. Rectilinear Propagation

The earliest surviving optics record, Euclid's *Optics* (280 B.C.), recognized that *in homogeneous media, light travels in straight lines.* However, following the teaching of Plato, Euclid thought that "rays" of light originate in the eye and intercept those objects which end up being seen by the observer (Fig. 1.1). To the ancient philosopher, light was synonymous with vision. The speed with which the rays were thought to emerge from the eye was known to be very high, if not infinite. An observer with eyes closed could open them and immediately see the distance stars.

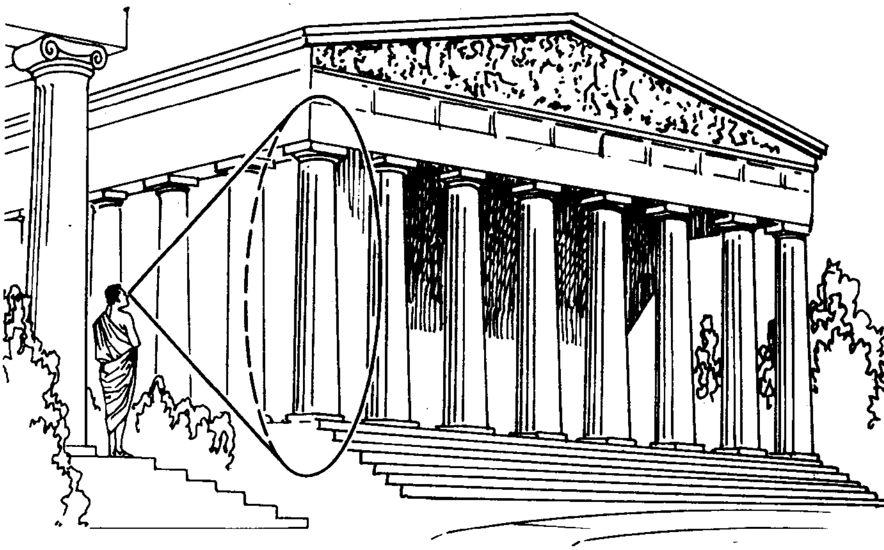


Fig. 1.1 The Greek impression of light was geometrical. All within the cone of vision was seen. Outside the cone, light had no meaning.

Hero of Alexandria in his *Catoptrics* (during the first century B.C.) also rationalized that, because light travels with infinite speed and therefore constant velocity, it must move in straight lines. This conclusion was based on analogy with mechanical events in which the concept that we now recognize as inertia plays an important role.

In the geometrical tradition of the Greeks, Hero identified the shortest point-to-point path that the light, by nature of its large velocity, was required to follow—the straight line. This shortest path concept is the earliest of the fundamental ideas concerning light that remain valid today. The underlying reason why light chooses the shortest path (or more rigorously the extremal path) was not understood, of course, until much later. (We will talk about that in due time.) This is a geometrical concept. Therefore, with regard to choosing the proper path, it makes no difference whether the light travels from the eye to the object or from the object to the eye or, for that matter, if the propagation is instantaneous. This is why Hero's idea worked, even though he too thought that the eye was the originating element and that the propagation speed was infinite.

We now understand that rectilinear propagation is not rigorously true because, through diffraction, light can bend around corners and, as explained by general relativity, light can be deflected by a strong gravitational field. This was not known to the ancient Greeks because the effects of diffraction are small and those described by relativity are observable only with sophisticated techniques and advanced instrumentation.

Today the *law of rectilinear propagation* has become one of the three principles of what we call "geometrical optics." The other two are the *law of reflection* and the *law of refraction.* The geometrical treatment is a phenomenological nonrelativistic approach wherein light is characterized by static rays and wherein the objects with which the light interacts are relatively large.

B. Reflection

At an interface between two different homogeneous optical media, incident light is, in general, partially transmitted and partially reflected. The interface might be a plane or a curved surface. In either case, the *surface normal* (the line perpendicular to the interface) at the point where an incident ray meets the interface is uniquely defined (see Fig. 1.2). The surface normal and the incident ray define a plane, the *plane of incidence.* The *law of reflection* states: *The reflected ray lies in the plane of incidence, and the angle of reflection, θ'' equals the angle of incidence, θ.*

The quantitative nature of the law of reflection was known in Aristotle's time and is documented in Euclid's book. Hero applied his "shortest path" principle to reflection and was able to geometrically prove the equality of the angles. Figure 1.3 reproduces the steps in Hero's proof. The initial conditions are illustrated in (*a*), where the interface *AB* and the surface normal *OC* are identified. The plane of incidence is the plane of the diagram. The incident ray forms angle θ with the normal. In (*b*), the reflected ray is constructed such that $\theta'' = \theta$ and two equal right triangles are formed. The path $\overline{POP''}$ must be demonstrated to be the shortest of all possible optical paths involving reflection at the interface. To achieve this, Hero drew the extension of $P''O$ to S as shown in (*c*), thus creating four congruent right triangles. Note that the marked lines *PO* and *SO* are equal in length. Thus $\overline{SCP''} = \overline{POP''}$.

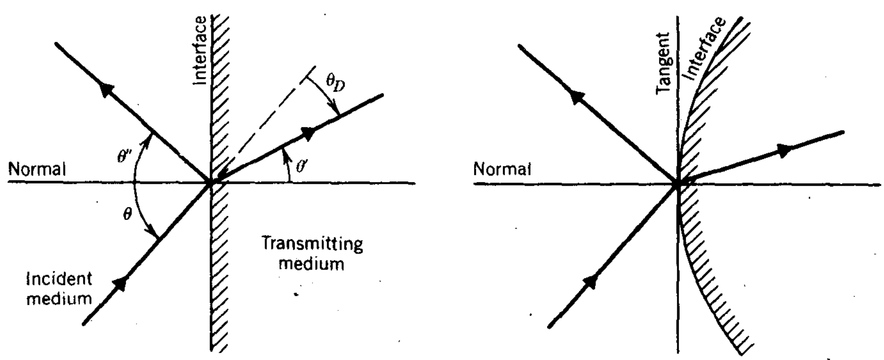
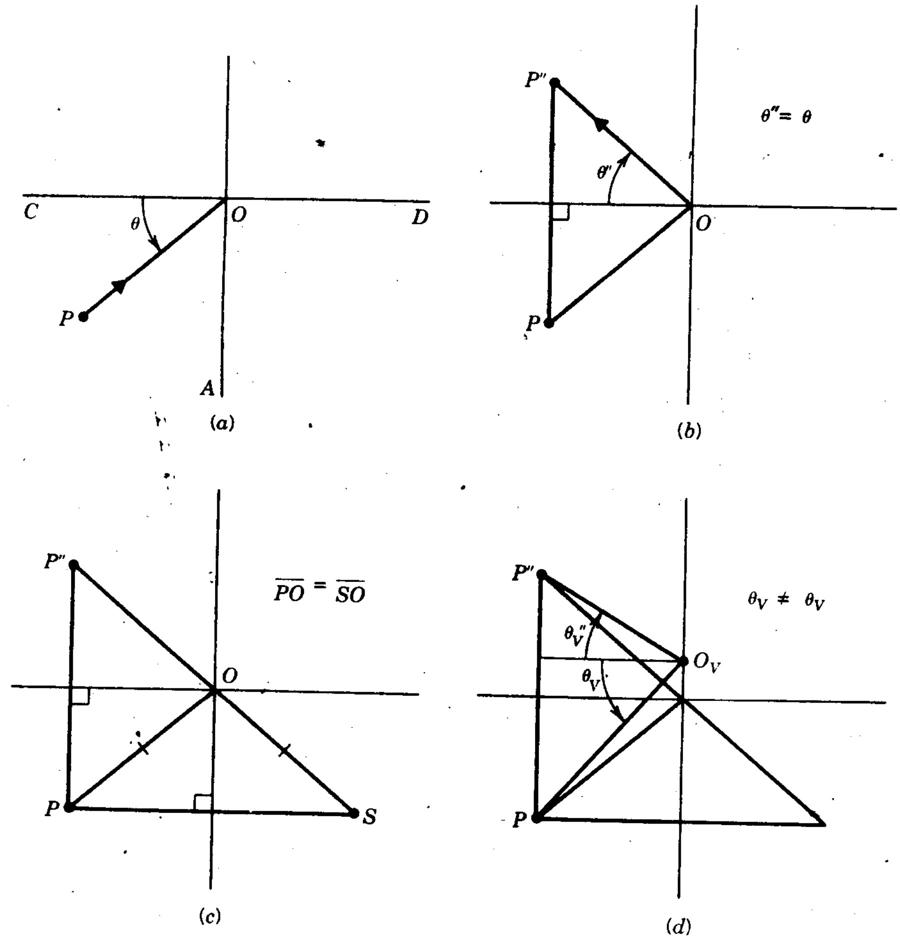


Fig. 1.2 Geometry of reflection and refraction at (*a*) a planar interface and (*b*) a curved interface.



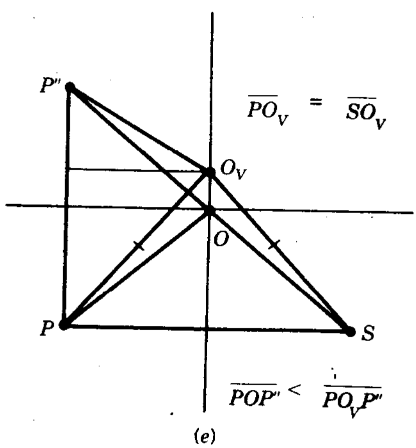


Fig. 1.5 Hero's proof of the law of reflection based on the requirement that light follow the shortest path.

If the law of reflection were not true, then other points, O_V (subscript V for "virtual" path), could be found and the resulting ray PO_VP'' would identify $\theta_V'' \neq \theta_V$, as shown in (*d*). In (*e*), $\overline{SO_V}$, which must be equal to $\overline{PO_V}$, is drawn. Thus $\overline{SO_VP''} = \overline{PO_VP''}$. However, it is clear that $\overline{SOP}$ (or $\overline{POP''}$) is less than $\overline{SO_VP}$ (or $\overline{PO_VP''}$), independent of the location of O_V on the interface, including points out of the plane of the diagram. Therefore, the ray that satisfies the law of reflection is also the shortest possible reflecting path.

The relative *distribution* of light *intensity* between the reflected component and the transmitted component was not properly explained until the 19th century. However, the more modern theory verifies that Hero's intuitive concept of light propagation along the "shortest path," and its consequences for geometrical optics, are correct.

C. Refraction

The geometry of refraction was experimentally studied by Claudius Ptolemy (100–170) and is reported in book V of his *Optics*. He recognized that the angle of deviation, θ_D in Fig. 1.2a, depended on the difference in density between the media forming the interface. He documented the quantitative relationship between θ' and θ, arriving at the empirical result that $\theta' = a\theta - b\theta^2$, where a and b are constants that depend on the two media. This expression approaches the correct result when θ is very small. However, for larger angles of incidence, this is, of course, not correct. In spite of its inaccuracy, Ptolemy's picture of refraction persisted for nearly 1500 years! People were simply not motivated to seek the correct answer.

Things began changing after A.D. 1280, when Italian artisans accidently discovered the spectacle lens. This development was still regarded as a curiosity by the educated community until Galileo Galilei (1564–1642), the famous Florentine mathematician, began experimenting with combinations of lenses that he ground for himself around 1609. Although the telescope was known before that time, Galileo was the first intellectual to take it seriously. Using his own instrument, which consisted of really good lenses, he discovered the moons of Jupiter and a host of other heavenly wonders.

Johannes Kepler (1571–1630), the great mathematician, optician, and astronomer at Cologne, Germany, for the first time summarized much of the known work with his *Dioptrice* (1609). This work was written after he had verified the discoveries of Galileo. It contains the theory of lenses and lens combinations. Kepler recognized that, provided the angles were small, the phenomenon of refraction followed the relation $\theta' = N\theta$ (where N is a constant depending on the two media). The resulting formalism is similar to that which we use today under the same limitation.

The true *law of refraction* ensures that: *The transmitted ray is in the plane of incidence, and the appropriate angles are related by* $\sin\theta' = N\sin\theta$.

Hero of Alexandria was able to derive the law of reflection using the shortest path principle in the first century B.C. The application of this principle in refraction is more complicated than in reflection. It requires knowledge about the finiteness of the speed of propagation of light, and how the speed depends on the propagation medium. This was one of the most confusing issues surrounding the theory of light. It was not satisfactorily worked out until 14 years after Kepler's death.

D. Theory of Light

Before the 17th century, knowledge about light was truly in the "Dark Ages." Although the Greeks had made significant progress with geometrical models in their time, this information, along with their other contributions, was suppressed in the years following the decline of Greek influence.

While the Western world struggled with barbarism, intellectual activity continued in the East along somewhat independent lines. Abu Ali Mohamed Ibn Al Hasan Ign Al Haytham (965–1039), or Alhazen for short, wrote a collection of seven books on optics in Baghdad around the year 1000. These are noted for their insightful comments concerning several key concepts.

Alhazen recognized that light sources illuminate objects, after which the light from the object is detected by the eye. He had a very good idea of how the optics of the eye worked. He described the operation of a "camera obscura." This was 500 years earlier than Leonardo da Vinci (1452–1519), who is usually credited with the discovery of the *pinhole camera* and its demonstration of the rectilinear propagation of light. In addition to these observations, Alhazen correctly hypothesized that light travels with a finite speed and that the speed is smaller in more dense media.

His physical picture was not correct, however, as it depended too much on

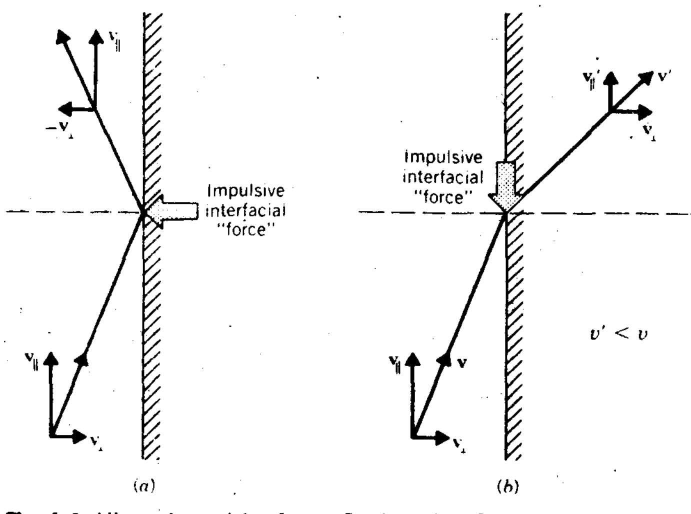


Fig. 1.4 Alhazen's models of (*a*) reflection, (*b*) refraction. This, as well as any other classical particle model, is conceptually incorrect, although Alhazen understood finite propagation and speed decrease in dense media.

mechanical analogies. Alhazen had the idea of light as a stream of particles that were subjected to surface forces on reflection and refraction. Reflected light particles were thought to be influenced by the forces that were only parallel to the surface, as shown in Fig. 1.4. He had many of the right answers, but rationalized them with incorrect reasons. The maturity of Alhazen's arguments influenced creative thought for more than 500 years, although the evolution of later ideas was not always direct.

The issues involving the nature of light were then and are now: (1) its speed of propagation, (2) the cause of the sensation we call color, (3) its tendency to travel in straight lines (rectilinear propagation), (4) the law of reflection, and (5) the phenomenon of refraction, whose "law" was not discovered until later. Any theory of light must deal with all of these characteristics. In addition, a comprehensive theory must deal with the phenomenon of interference and diffraction, which were not known at the time. It must also be able to explain the subtleties of relativistic effects and the details of light/matter interactions, which were not revealed until the 19th and 20th centuries.

1.2 The Particle Models

Further development of optical theory took place by virtue of the ideas of three prominent individuals: René du Perron Descartes (1596–1650, France), Pierre de Fermat (1601–1665, France), and Isaac Newton (1642–1727, England). There are fundamental differences in the philosophies of natural phenomena espoused by these early physicists. Most notable among the constrasts are the Cartesian (geometrically oriented) versus the Newtonian (force-oriented) points of view. These three early physicists are grouped together here because particle dynamics played an important role in their explanations of light. We have already mentioned that mechanical models are inadequate; however, Descartes and Newton had such dominating influence in their day that any study of optics is incomplete without some appreciation of their ideas. Among their lasting contributions—Descartes was the first to publish the correct form of the law of refraction, and Newton first explained refraction's chromatic character. Fermat developed the *principle of least time*, which was similar to Hero's shortest path principle. This, we have said, is a fundamental concept that is reinforced by modern theory.

A. Descartes

René Descartes' notions about light were consistent with his impressions of the physical world. To him, all things were related to geometry. Motion was the one fundamental "power" in nature. The only type of motion was that whereby a body passed from one geometrical state to another by successive steps. Motion could be communicated from one body to another only by impact. Cartesian matter was infinitely divisible and incompressible (because a void was thought to be impossible). These were, to Descartes, a priori truths.

With this background we can understand the Cartesian theory of light. According to Descartes light was a *tendency* toward movement that was transmitted through the all pervading medium, the "ether." This is similar to the way pressure is transmitted through a stick. To Descartes, this "tendency" followed the same laws that movement itself would follow. The "tendency," which Descartes equated with light, was propagated instantaneously, but mechanical movement analogies that required a finite time to evolve were used in discussing how the light would behave.

In 1637 Descartes published *La Dioptrique* in which the laws of optics were derived from his a priori truths. To discuss the action of light on encountering an interface, Descartes compared light to a mechanical particle. As an example he chose a tennis ball. In both reflection and refraction, the component of the velocity of his mechanical analog parallel to the interface was assumed to remain constant (see Fig. 1.5). The light, as a "tendency" toward motion, in reflection would have to follow the same path as that of a perfectly elastic rebounding ball.

In the mechanical analogy for refraction, the interface was a frail canvas. The speed after the encounter with the interface was assumed to be directly proportional to the initial speed $Nv' = v$. Conservation of the parallel component required $v' \sin \theta' = v \sin \theta$. Together these relations led Descartes to the law of refraction, which turns out to be experimentally correct: $\sin \theta' = N \sin \theta$. However, if the parallel component of the velocity were to remain constant, this required that the perpendicular component of the velocity be increased after refraction if the tennis

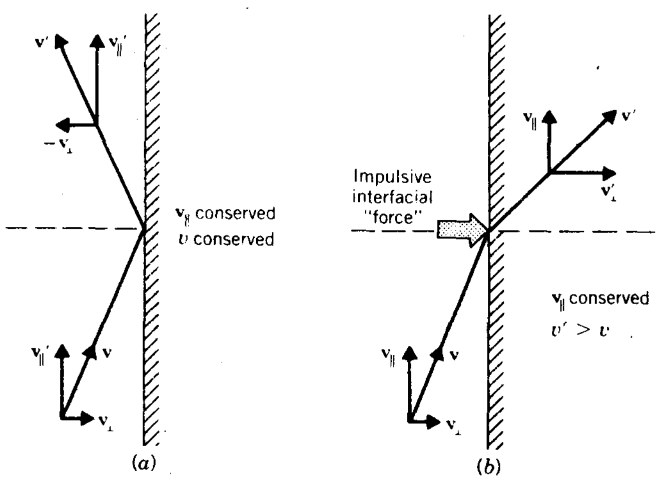


Fig. 1.5 Descartes' models of (*a*) reflection, (*b*) refraction. This shows the incorrect mechanical analogy that was required to explain refraction if $\mathbf{v}_{\parallel}$ was conserved. Light, as a "tendency" toward motion, was thought to follow the path of the analogy, but with infinite speed.