ELEMENTS OF STATISTICAL REASONING

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PREFACE

Elements of Statistical Reasoning is intended primarily for students in psychology, social science, and education. It stresses conceptual development and the logic of statistics and was written with a careful eye to the needs of students. Each chapter (except the first) begins with an introduction and concludes with a summary. Required mathematics consists of common arithmetic, an elementary understanding of simple equations, and the ability to plug numbers into an equation and boil them down to a single numerical answer. There is also a math review (Appendix A), intended as a brush-up for those who need it. Although the primary focus is on conceptual development, how-to-do-it has been treated on a step-by-step basis, which should give instructors more time to help with difficult concepts.

Most examples in the text (and almost all problems and exercises) use just a few simple numbers, in line with our objective of stressing comprehension rather than mathematical stamina. Still, we assume that every student will have a pocket calculator. A basic one will be fine, but one with a memory will make life even easier, and a square root key would be a usable luxury.

One of our objectives was to produce a book with clear exposition and uncluttered development, characterized by accuracy of statistical statements and yet avoiding oversimplification. If we have succeeded, you will find a treatment of fundamentals that still manages to stress conceptual development. We hope that this book will be clear and helpful for those who come to the first course with apprehension and that it will be interesting and informative for the very able student.

We have worked to provide a large number of questions and problems and a substantial range of illustrative examples (taking both text and problems into account). Some problems carry knowledge beyond the actual textual development; they are identified with a large color dot (●). The problems and questions are placed at the end of each chapter, but are arranged according to the progression of chapter sections and are so identified. This allows the student to practice after completing a section (or group of sections), and permits the instructor to make assignments directly relevant to the sections covered. Answers to problems and questions appear in Appendix D.

The book also considers the important concerns of modern statistical workers. Issues of experimental design appear from time to time, the problem of effect size is often discussed, interval estimation receives more attention than is customary, and there is a chapter on elementary statistical power analysis.

Those familiar with Minium's *Statistical Reasoning in Psychology and*

Education, Second Edition, will find that this book has much in common with it but that *Elements* is a different book. *Elements* is not intended to replace the earlier work. It is shorter, math requirements are a half-step lower, and the emphasis is on development of fundamentals. It is definitely a one-semester book. Even so, there is more material than can be covered in most one-quarter or one-semester courses. The first fourteen chapters are the basic ones. Beyond that, the instructor must make the selection that seems most appropriate.

A *Student Workbook* and an *Instructor's Manual* are available. The former is designed to serve as a study guide as well as a workbook. Among other things, it contains supplementary homework problems, answers to which are supplied (only) in the Instructor's Manual. Hundreds of multiple choice examination questions will also be found in the instructor's manual.

ACKNOWLEDGMENTS

In the course of the sabbatical rambles of one of the authors (EWM), over two dozen teachers of statistics in the East and West took time from their busy schedules to discuss things they believed important to teach and to describe how they went about it. The insights they revealed have helped to make this a better book, and we thank them for sharing their thoughts and feelings.

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We are also indebted to the several authors and publishers who granted permission to use tables and figures so necessary to this work. Among others, we are grateful to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Longman Group Ltd., London, for permission to reprint Tables III and VI from their book *Statistical Tables for Biological, Agricultural and Medical Research*, (6th edition, 1974).

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CHAPTER 1

Introduction

1.1 DESCRIPTIVE STATISTICS

Among freshmen who declare a Psychology major, what proportion are male? Female? Do those proportions differ among graduating seniors? Among those who graduate with an A.B. in psychology, how many go on to graduate work in the same field? Ten years after graduation, what proportion are employed as professionals in psychology? What proportion are working in allied fields, such as juvenile probation or psychiatric social work? And what proportion are working in positions unrelated to psychology? These are examples of questions for which *descriptive statistics* can help to provide a meaningful and convenient way of describing features of the data that are of interest. In the examples above, *frequencies* and *proportions* will help to do the job.

What is the average gasoline mileage of new American-made automobiles for each of the past 10 years? Has it been increasing? How much? What about the Scholastic Aptitude Test (SAT) scores of entering freshmen over the past decade? Has it been changing? How? One way to show the change is to make a graph showing the average performance in each of the past 10 years. These questions illustrate the use of *averages* and *graphs*, additional tools that can help describe data.

Professor Goodfellow, your charming, kindly statistics teacher, has given a test of elementary math on the first day of class. You, Doris Diligent, are pleased to learn that your performance is better than 90% of beginning statistics students. Professor G. arranges the test scores in order of magnitude, and notes that the distance between scores of the top and bottom performers is not very great and that the class average is rather higher than expected from past records. He is pleased because he finds that the general level of preparation seems to be good and that the group is not exceedingly diverse in their skills. It will make his teaching job easier. This scenario illustrates the use of more tools of descriptive statistics: the *frequency distribution,* which shows the scores in ordered arrangement, the *centile* (percentile), a way of describing location of a person's performance among that of a group, and the *range,* a measure of variability of performance.

The purpose of descriptive statistics is to organize and to summarize data so that they are more readily comprehended.

What we've just said about descriptive statistics is a key idea concerning *all* applied statistics. When the question "Should I use statistics?" comes up, next ask "Would the story the data have to tell be clearer if I did?" *Don't "statisticate" unless the answer is "Yes."*

1.2 INFERENTIAL STATISTICS

What is the attitude of the voting public toward the elimination of capital punishment? Pollsters find it impossible to put this question to all members of this group. Instead, they study the responses of a portion of it, and from that knowledge they estimate the attitudes of the whole. The outcome, like any estimate, is subject to error. But, if the sample of voters selected for study has been chosen according to statistical principles, it is possible to determine what margin of error is involved.

A second branch of statistical practice, known as *inferential statistics,* provides the basis for answering questions of this kind. These procedures allow us to account for chance error in drawing inferences about conditions that exist in the larger set of observations from study of a part of that set. This branch of statistics is also known as *sampling statistics.*

Another application of inferential statistics is particularly suited to evaluation of the outcome of an experiment. Is it possible that a certain drug has an effect on the speed of learning? Let us suppose that an investigator decides on the kind of subjects she wishes to study, selects at random two groups of 25 subjects each, and administers the drug to one of the groups. Both groups are given a learning task and are in all ways treated alike except for the drug. From the outcome of the study, she finds that the average learning score of the two groups differs by five points.

Now some difference between the groups would be expected even if they were treated alike, because of chance factors involved in the random selection of groups. The question faced by the experimenter is whether the observed difference is within the limits of expected variation. If certain preconditions have been met, statistical theory can provide the basis for an answer. If the experimenter finds that the obtained difference of five points is larger than can be accounted for by chance variation, she will infer that other factors must be at work. If examination of her experimental procedures reveals no reasonable cause for the difference other than the deliberate difference in experimental treatment, she may conclude that the drug is the responsible factor.

The purpose of inferential statistics is to take into account the chance factors associated with sample selection when drawing conclusions from our results.

1.3 RELATIONSHIP AND PREDICTION

Suppose we are developing a test of dogmatism (sample question: "Do you agree that to change your mind is a sign of weakness?"). We give this test to 100 subjects and determine their "dogmatism score." Some of these people score high on the scale, some score low, and many are in between. What if we test the same persons four months later; will they maintain the same position among the group? (Test constructors call this a question of "test reliability.") How can we express the degree to which those tested maintain the same position in the group that they obtained in the first administration of the test?

The personnel office of an industrial concern gives an aptitude test to its prospective clerical employees. Is there any relationship between the score on this test and subsequent level of job proficiency? Will high scorers do well on the job? How well? Will low scorers do poorly? How poorly? What percent of applicants may be expected to succeed on the job if only those who score above a certain point on the test are accepted for employment? How does this compare with what would happen if the test were eliminated from the company's procedures?

These are examples of questions that probe the existence and extent of *relationship* between two (or more) characteristics and explore the possibility of *predicting* standing in one characteristic from knowledge of standing in the other. This kind of analysis is so frequently of interest that a considerable body of statistical techniques has been developed to deal with it. Because of their importance and distinctiveness, these questions of relationship and prediction have been separately identified here. Nevertheless, both are problems in description and inference, the two major categories of statistical endeavor.