STEWART

Calculus

EARLY VECTORS

Preliminary Edition

CALCULUS
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Preliminary Edition

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CALCULUS

EARLY VECTORS

PREFACE

The idea for this book came from the Mathematics Department of Texas A&M University. A large number of their students take Engineering and Physics courses concurrently with calculus and these courses require the concept of vectors to explain velocity, force, and so on. The traditional calculus curriculum postpones the study of vectors until the third semester; consequently, many students learn vector concepts from Physics or Engineering departments during their freshman year without any reinforcement from Mathematics departments.

Al Boggess and his colleagues at Texas A&M University asked me to write a version of my calculus text that would introduce vectors at a very early stage and integrate them throughout. I thought that idea made a lot of sense, but I was involved in another writing project at the time and so I suggested that they write the first draft themselves. They drew upon material from my book *Calculus, Early Transcendentals, Third Edition* (henceforth abbreviated as ET) and rearranged it to fit their curriculum, providing new exposition and new exercises where needed. They class-tested the new version for two years. I then took part in the writing, revising the manuscript in collaboration with them. The result is this Preliminary Edition of *Calculus: Early Vectors.*

Although the impetus for this book came from Texas A&M, a number of other universities have expressed interest in it as well. Before we publish the first edition of this book, I would like to have input from other potential users with respect to both broad structure and fine details. If you have any comments or suggestions, particularly on the best way of integrating vectors into first-semester calculus, please send them to me in care of my publisher, Brooks/Cole Publishing Company, 511 Forest Lodge Road, Pacific Grove CA 93950, or e-mail to info@brookscole.com.

VECTORS

The goal of *Calculus: Early Vectors* is to introduce vectors early in the first semester, in a manner that does not dramatically change the rest of the calculus curriculum. Vectors are introduced in two stages. Chapter 1 introduces two-dimensional vectors along with basic operations, including the dot product, and two-dimensional vector functions. Three-dimensional vectors (along with three-dimensional geometry and cross products) are treated in Chapter 11 (covered at the end of the second semester or beginning of the third). Our experience has shown that a full treatment of three-dimensional vectors typically overwhelms the average first-semester calculus student. A thorough treatment in Chapter 1 would also postpone discussion of other calculus topics so that the resulting course would no longer resemble the traditional first-semester course.

■ Chapter 1 on vectors is relatively short. It introduces two-dimensional vectors along with basic vector operations, including sums, scalar multiplication, dot product, and projection. Applications to problems involving force and work are discussed in detail. In fact, the definition of the dot product is motivated by the need to compute the work done by a force that does not point in the direction of motion.

■ Vector functions and parametric curves are introduced in Chapter 1 in the context of projectiles. Limits and rates of change of such functions are discussed in Chapter 2, immediately following the development of the scalar case.

■ Derivatives of vector functions, tangent vectors, and tangents to parametric curves are treated in Chapter 3. (See Sections 3.7 and 3.9.)

■ Chapter 4 (on exponential functions and logarithms) is almost identical to the corresponding chapter of ET.

■ Chapter 5 (applications of differentiation) contains the streamlined approach to graphing that I have used in *Calculus: Concepts and Contexts.* In addition to a more reformed approach that takes advantage of technology (where appropriate), this streamlined approach allows the typical first-semester calculus course to cover topics through Chapter 6 (integration), corresponding roughly to the topics of most traditional first-semester courses.

■ Chapters 7–9 (techniques of integration and their applications) are largely unchanged from ET.

■ Chapter 10 (series) uses the more streamlined approach contained in *Calculus: Concepts and Contexts.*

■ Three-dimensional vectors and geometry are treated in Chapter 11. Here the basic concepts developed in Chapter 1 are reviewed quickly and generalized to three dimensions. We view this repetition as giving the important reinforcement this concept needs because it is so heavily used in Physics and Engineering. Cross products are then introduced with the notion of torque serving as the motivation. A student who has transferred from a more traditional first-semester calculus course that does not contain vectors will be introduced to all the vector concepts from Chapter 1 in the three-dimensional setting covered in Chapter 11.

■ Chapters 12–14 (partial derivatives, multiple integrals, and vector calculus) are largely unchanged from ET with the following exception. Polar and spherical coordinates are now introduced in Chapter 13 (multiple integrals) immediately before they are needed for integrals in polar coordinates and spherical coordinates.

TECHNOLOGY

For many years I have experimented with calculus laboratories for my own students, first with graphing software for computers, then with graphing calculators, and finally with computer algebra systems. Those of us who have watched our students use these machines know how enlivening such experiences can be. We have seen from the expressions on their faces how these devices can engage our students' attention and make them active learners.

Despite my enthusiasm for technology, I think there are potential dangers for misusing it. When I first started using technology, I tended to use it too much, but then I started to see where it is appropriate and where it is not. Many topics in calculus can be explained with chalk and blackboard (and reinforced with pencil and paper exercises) more simply, more quickly, and more clearly than with technology. Other topics cry out for the use of machines. What is important is the *appropriate* use of technology, which can be characterized as involving the *interaction* between technology and calculus. In short, technology is not a panacea, but, when used appropriately, it can be a powerful stimulus to learning.

This textbook can be used either with or without technology and I use two special symbols to indicate clearly when a particular type of machine is required. The icon indicates an example or exercise that requires the use of either a graphing calculator or a computer with graphing software. (Section 3 in Review and Preview discusses the use of these graphing devices and some of the pitfalls that can arise. Section 5.4 is a good example of what I mean by the interaction between technology and calculus.) The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, or Mathematica) are required. In all cases we assume that the student knows how to use the machine—we rarely give explicit commands.

Some of the exercises designated by or CAS require considerable time for their completion. Instructors should therefore consult the solutions manual to determine the complexity of a problem before assigning it. Some of those problems explore the shape of a family of curves depending on one or more parameters. Other such projects involve technology in very different ways. See, for instance, pages 651 (logistic sequences) and 524.

VISUALIZATION

One of the themes of the calculus reform movement is the Rule of Four: Topics should be presented verbally, numerically, graphically, and symbolically, wherever possible. See pages 150 and 588 for examples of how the Rule of Four comes into play. You will also see that I include substantial work with tabular functions and numerical estimates of sums of series.

Many examples and exercises promote visual thinking. Given the graph of a function, I think it is important for a student to be able to sketch the graph of its derivative (page 152) and also to sketch the graph of an antiderivative (page 348) in a qualitative manner. See pages 194, 208, 232, 344, 419, 428, 507, 657, 696, 747, 872, and 891 for other examples of exercises that test students' visual understanding.

There are hundreds of new computer-generated figures that illustrate examples. These are not just pretty pictures—they constantly remind students of the geometric meaning behind the result of a calculation. I have also tried to provide more visual insight into formulas and their proofs (see, for instance, pages 162 and 290).

PROBLEM SOLVING

My educational philosophy was strongly influenced by attending the lectures of George Polya and Gabor Szego when I was a student at Stanford University. Both Polya and Szego consistently introduced a topic by relating it to something concrete or familiar. Wherever practical, I have introduced topics with an intuitive geometrical or physical description and attempted to tie mathematical concepts to the students' experience.

I found Polya's lectures on problem solving very inspirational and his books *How to Solve It, Mathematical Discovery,* and *Mathematics and Plausible Reasoning* have become the core text material for a mathematical problem-solving course that I have instituted and taught at McMaster University. I have adapted these problem-solving strategies to the study of calculus both explicitly, by outlining strategies, and implicitly, by illustration and example.

Students usually have difficulties in situations that involve no single well-defined procedure for obtaining the answer. I think nobody has improved very much on Polya's four-stage problem-solving strategy and, accordingly, I have included in this edition a version of Polya's strategy in Section 4 of Review and Preview, together with several examples and exercises involving precalculus material. I have also rewritten the solutions to certain examples in a more patient manner to make the problem-solving principles more apparent. (See, for instance, Example 1 on page 215.)

The classic calculus situations where problem-solving skills are especially important are related rates problems, maximum and minimum problems, and techniques of integration. In these and other situations I have adapted Polya's strategies to the matter at hand. See, in particular, Section 8.6 (Strategy for Integration).

I include what I call *Problems Plus* after odd-numbered chapters. These are problems that go beyond the usual exercises in one way or another and require a higher level of problem-solving ability. The very fact that they do not occur in the context of any particular chapter makes them a little more challenging. For instance, a problem that occurs after Chapter 10 need not have anything to do with Chapter 10. I particularly value problems in which a student has to combine methods from two or three different chapters. The examples in the Problems Plus sections serve not as solutions to imitate (there are no problems like them), but rather as examples of how to tackle a challenging calculus problem. (See the example on page 360.) Many of the problems have a geometric flavor (see Problems 9, 10, 18, 27 after Chapter 3 and Problem 14 after Chapter 10). I have been testing these Problems Plus on my own students by putting them on assignments, tests, and exams. Because of their challenging nature I grade these problems in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant. My aim is to teach my students to be unafraid to tackle a problem the likes of which they have never seen before.

REAL WORLD APPLICATIONS

I have included substantial applied problems that I believe will capture the attention of students. See, for instance, Problem 10 on page 421 (investigating the shape of a can), Problem 7 on page 523 (positioning a shortstop to make the best relay to home plate), and Problem 9 (choosing a seat in a movie theater) and Problem 10 (explaining the formation and location of rainbows) on page 524. These are all extended problems that would make good projects. They happen to be located in the *Applications Plus* sections, which occur after even-numbered chapters (starting with Chapter 4) and are a counterpart to the Problems Plus. (Again the idea is often to combine ideas and techniques from different parts of the book.) But there are many good applied problems in the ordinary sections of the book as well. (See, for instance, Exercise 56 on page 340 and Exercise 32 on page 233).

THE TEACHING PACKAGE

Calculus: Early Vectors, Preliminary Edition, is supported by ancillaries designed to enhance student understanding and to facilitate creative instruction. The *Student Solutions Manual* (by Jeffery A. Cole, *Anoka-Ramsey Community College,* James Stewart, *McMaster University,* Daniel Anderson, *University of Iowa,* and Daniel Drucker, *Wayne State University*) includes detailed solutions to all odd-numbered text exercises. The *Complete Solutions Manual,* by the same team of authors, provides detailed solutions to all text exercises.

In addition, *Test Items* and *Computerized Testing* are available, offering multiple-choice and short-answer questions. Computerized testing allows instructors to insert their own questions and to customize the questions provided, with some algorithmically generated. Full-color *Transparencies* are available of over 100 of the more complex text diagrams.

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Finally, I thank Al Boggess and his colleagues at Texas A&M University, not only for suggesting the idea of a calculus text with early vectors, but also for working with me to make it a reality. My publisher, Gary W. Ostedt, also deserves special thanks for arranging this collaboration and for giving us the benefit of his extensive experience and keen editorial insight.

JAMES STEWART

TO THE STUDENT

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper at hand to make a calculation or sketch a diagram.

Some students start by trying their homework problems and only read the text if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should study the definitions to see the exact meanings of the terms.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected step-by-step fashion with explanatory words and symbols—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix L. There are often several different forms in which to express an answer, so if your answer differs from mine, don't immediately assume you are wrong. There may be an algebraic or trigonometric identity that connects the answers. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you are right and rationalizing the denominator will show that the expressions are equivalent.

The symbol [graphing icon] indicates an example or exercise that requires the use of either a graphing calculator or a computer with graphing software. (Section 3 in Review and Preview discusses the use of these graphing devices and some of the pitfalls that you may encounter.) The icon CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, or Mathematica) are required. You will also encounter the symbol ⊘, which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Calculus is an exciting subject; I hope you find it both useful and interesting in its own right.

■ A NOTE ON LOGIC

In understanding the theorems it is important to know the meaning of certain logical terms and symbols. If P and Q are mathematical statements, then $P \Rightarrow Q$ is read as "P implies Q" and means the same as "If P is true, then Q is true." The *converse* of a theorem of the form $P \Rightarrow Q$ is the statement $Q \Rightarrow P$. (The converse of a theorem may or may not be true. For example, the converse of the statement "If it rains, then I take my umbrella" is "If I take my umbrella, then it rains.") The symbol $\Longleftrightarrow$ indicates that two statements are equivalent. Thus $P \Longleftrightarrow Q$ means that both $P \Rightarrow Q$ and $Q \Rightarrow P$. The phrase "if and only if" is also used in this situation. Thus "P is true if and only if Q is true" means the same as $P \Longleftrightarrow Q$. The *contrapositive* of a theorem $P \Rightarrow Q$ is the statement that $\sim Q \Rightarrow \sim P$, where $\sim P$ means *not* P. So the contrapositive says "If Q is false, then P is false." Unlike converses, the contrapositive of a theorem is always true.

ALGEBRA

ARITHMETIC OPERATIONS

$$a(b+c) = ab + ac \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b} \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

EXPONENTS AND RADICALS

$$x^m x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn} \qquad x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y} \qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$$

FACTORING SPECIAL POLYNOMIALS

$$x^2 - y^2 = (x+y)(x-y)$$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

BINOMIAL THEOREM

$$(x+y)^2 = x^2 + 2xy + y^2 \qquad (x-y)^2 = x^2 - 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

where $\binom{n}{k} = \frac{k(k-1)\cdots(k-n+1)}{1\cdot 2\cdot 3\cdot\cdots\cdot n}$

QUADRATIC FORMULA

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

INEQUALITIES AND ABSOLUTE VALUE

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$

$|x| < a$ means $-a < x < a$

$|x| > a$ means $x > a$ or $x < -a$

GEOMETRY

GEOMETRIC FORMULAS

Formulas for area A, circumference C, and volume V:

Triangle

$A = \frac{1}{2}bh$

$= \frac{1}{2}ab\sin\theta$

Circle

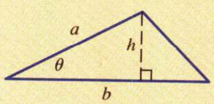
$A = \pi r^2$

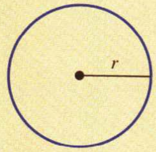
$C = 2\pi r$

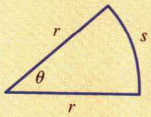
Sector of Circle

$A = \frac{1}{2}r^2\theta$

$s = r\theta$ (θ in radians)







Sphere

$V = \frac{4}{3}\pi r^3$

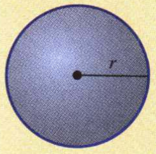
$A = 4\pi r^2$

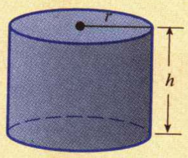
Cylinder

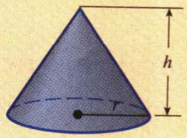
$V = \pi r^2 h$

Cone

$V = \frac{1}{3}\pi r^2 h$







DISTANCE AND MIDPOINT FORMULAS

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

LINES

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

CIRCLES

Equation of the circle with center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

TRIGONOMETRY

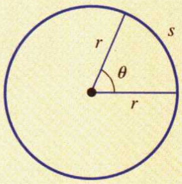
ANGLE MEASUREMENT

π radians $= 180°$

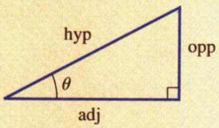
$1° = \dfrac{\pi}{180}$ rad　　$1 \text{ rad} = \dfrac{180°}{\pi}$

$s = r\theta$

(θ in radians)



RIGHT ANGLE TRIGONOMETRY



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$　　$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$　　$\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

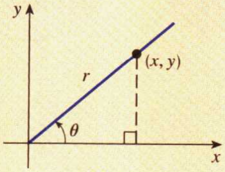
$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$　　$\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

TRIGONOMETRIC FUNCTIONS

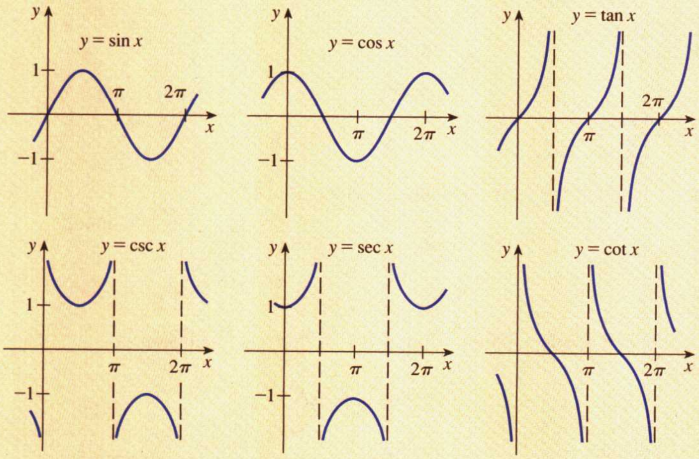
$\sin\theta = \dfrac{y}{r}$　　$\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$　　$\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$　　$\cot\theta = \dfrac{x}{y}$



GRAPHS OF THE TRIGONOMETRIC FUNCTIONS



TRIGONOMETRIC FUNCTIONS OF IMPORTANT ANGLES

θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

FUNDAMENTAL IDENTITIES

$\csc\theta = \dfrac{1}{\sin\theta}$　　$\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$　　$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$　　$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$　　$1 + \cot^2\theta = \csc^2\theta$

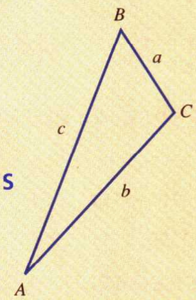
$\sin(-\theta) = -\sin\theta$　　$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$　　$\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$　　$\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

THE LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



THE LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

ADDITION AND SUBTRACTION FORMULAS

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

DOUBLE-ANGLE FORMULAS

$\sin 2x = 2\sin x\cos x$

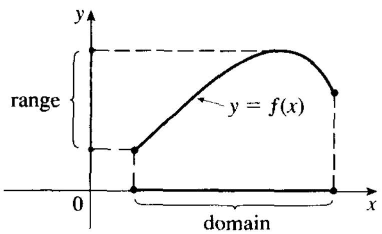
$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

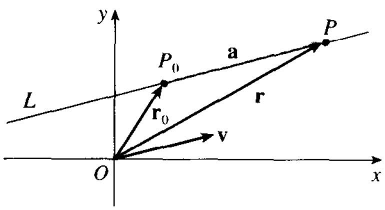
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

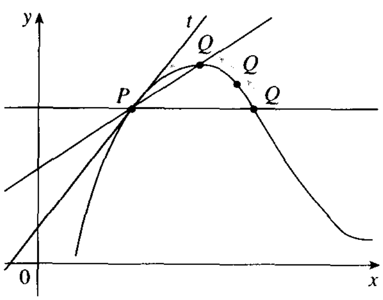
HALF-ANGLE FORMULAS

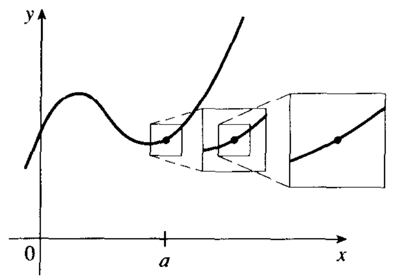
$\sin^2 x = \dfrac{1 - \cos 2x}{2}$　　$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

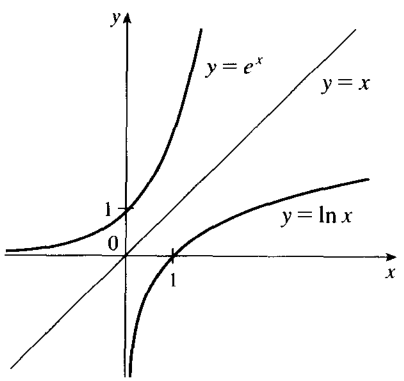
CONTENTS

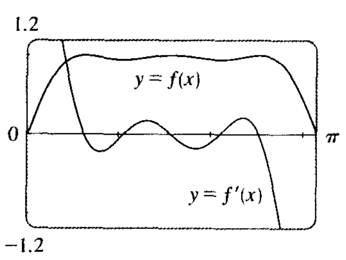


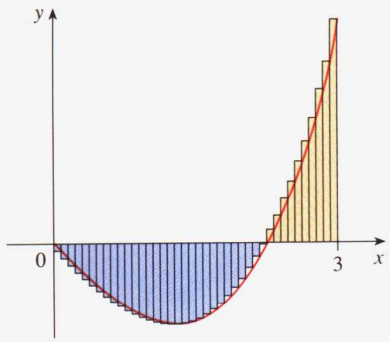


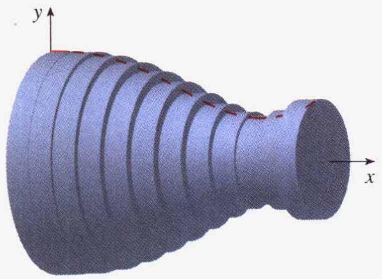


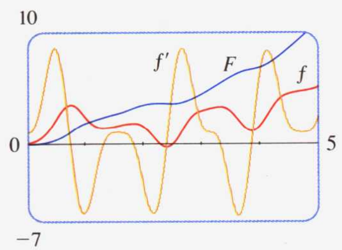

y
0
a
x

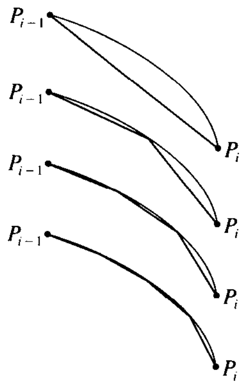

y
$y = e^x$
$y = x$
$y = \ln x$
1
0
1
x


1.2
$y = f(x)$
0
π
$y = f'(x)$
−1.2


y
0
3
x


y
x


10
f′
F
f
0
5
−7



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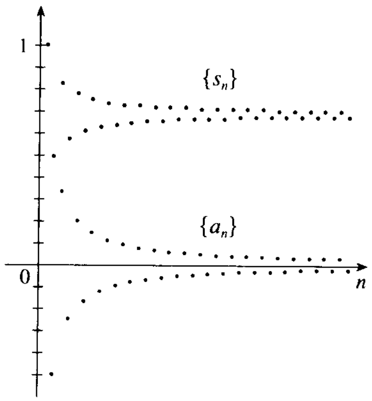

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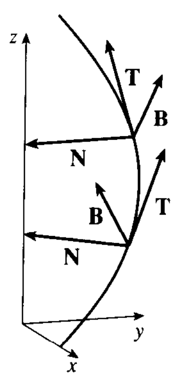

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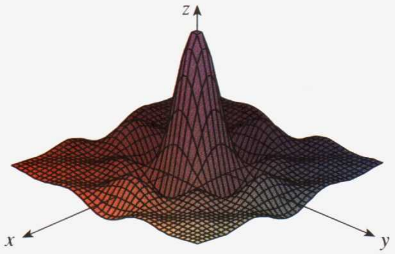

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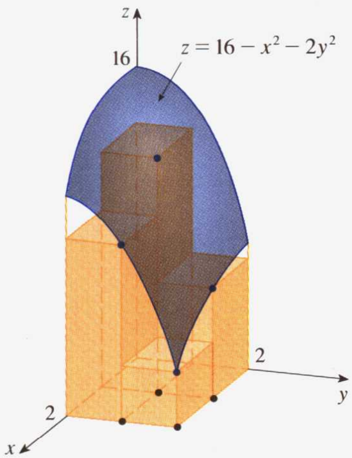

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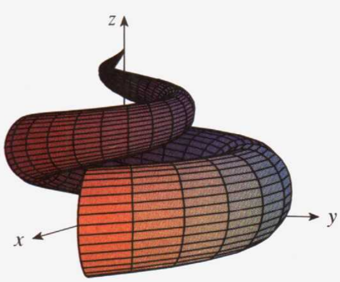

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