

DIGITAL INTEGRATED ELECTRONICS



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Toronto

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Digital Integrated Electronics This book was set in Times Roman.
The editors were Peter D. Nalle and Madelaine Eichberg;
the cover was designed by Scott Chelius;
the production supervisor was Charles Hess.
The drawings were done by J & R Services, Inc.
Fairfield Graphics was printer and binder.

Library of Congress Cataloging in Publication Data

Taub. Herbert, date
Digital integrated electronics.

(McGraw-Hill electrical and electronic engineering series) Includes index.

1. Digital electronics. 2. Integrated circuits

I. Schilling, Donald L., joint author. II. Title.

TK7868.D5T37 621.381 76-4585

ISBN 0-07-062921-8

DIGITAL INTEGRATED ELECTRONICS

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34567890FGRFGR78321098

In 1956 the McGraw-Hill Book company published the text "Pulse and Digital Circuits" coauthored by J. Millman and H. Taub. That book, which undertook to present a rather complete account of the state of the art of digital electronics dealt almost exclusively with vacuum-tube circuits. Semiconductor devices and circuits, which had not long before been introduced, appeared in a single final chapter, added at the last moment, while the book was in production. In the decade that followed semiconductor devices completely supplanted tubes in digital circuitry. In response to this development the same authors prepared a replacement volume "Pulse Digital and Switching Waveforms" which appeared in 1965. In the newer volume the overwhelming importance of the semiconductor was appropriately emphasized and vacuum-tube circuits were presented only incidentally. Now, again after about a decade, the advances in integrated circuitry have prompted this present volume. However, this book is intended as a continuation of the 1965 work rather than as a replacement. Here the present authors have undertaken to describe and analyze all the basic integrated-circuit building blocks from which digital circuits and systems are assembled. As reasonably as is feasible in a textbook, the material presented is up to date. As was the case in the earlier volume, the present authors have taken great pain with the style of

pedagogy. We have striven to make the explanations clear and easily understood without sacrificing depth and completeness of presentation. For this reason, we hope that this work will find a place not only in the classroom but also in a program of self-study for a reader who may want to keep informed about current developments.

The material in the text has been used at the City College of New York in a two-semester course offered to junior and senior electrical engineering students and has been used as well as the basis of two graduate courses. This material has also been presented in a two-semester course offered to technical staff members of the Bell Laboratories, to engineering personnel at NASA and at Lockheed, and in short courses offered in the continuing-education program at the George Washington University.

It is assumed that the reader already has a background in semiconductor devices and circuits. Nevertheless we find it useful to provide in Chapter 1 a review of certain special matters pertaining to the operation of semiconductor devices in a *switching* mode. Semiconductors have rather involved and highly nonlinear volt-ampere characteristics. An exact analysis of semiconductor circuits results in considerable mathematical complexity. In Chapter 1 we present some convenient simplifications which lead to quite good and useful approximations.

The first part of Chapter 2 discusses operational amplifiers. Such amplifiers, intended to be operated linearly rather than in a switching mode, are not our proper concern. Still, in a number of cases we find that operational amplifiers appear as components in what are otherwise digital circuits. Furthermore, by a rather natural extension, operational amplifiers lead to the discussion, in the second part of the chapter, of comparators which are indeed important switching devices.

Chapter 3 introduces the concept of logical variables, Boolean algebra, and methods of analyzing circuits composed of logical gates. Karnaugh maps and their various applications are presented. This chapter is complete in the sense that it presumes no prior acquaintance with the subject and explains all the principles of design and analysis of logical circuits required for an understanding of the entire text. On the other hand, the content of this chapter is inevitably included in a course in logic design and, hence, may be bypassed by readers who have already been exposed to this material.

The electronics of logical gates is begun in Chapter 4. The first part of this chapter deals with resistor-transistor logic (RTL) while the second part is concerned with integrated-injection logic (IIL). RTL is not presently used in new design. Yet there are a number of reasons on account of which it is valuable to consider this family of logic. Being the first widely used family of IC logic available, there are in operation many installations in which it is incorporated. Then, again on account of its elegant simplicity, it is an ideal vehicle through which to present many of the basic concepts and principles universally important in the electronics of logical gates. Finally, it bears an interesting topological

relationship to IIL which is one of the most recently developed families of logic. Chapter 5 considers diode-transistor logic (DTL). In the family of DTL we find high-threshold logic (HTL), which finds extensive application in highly noisy environments.

Chapters 6 and 7 discuss transistor-transistor logic (TTL) and emitter-coupled logic (ECL) respectively. At the present time these are the most widely used saturating and nonsaturating logic families. Hence the analysis of these families is rather extensive. In ECL particularly, it turns out that some appreciation of the nature of signal transmission over transmission lines is required. Readers who are unfamiliar with transmission line propagation will find an adequate introductory presentation in Appendix A. A more complete discussion appears in Chapter 3 of "Pulse Digital and Switching Waveforms" referred to above. Metal-oxide semiconductor (MOS) logic and complementary-symmetry (CMOS) logic is presented in Chapter 8.

The various families of logic having been considered (Chapters 4 through 8), we begin in Chapter 9 to consider the basic digital structures which are assembled from these gates. Chapter 9 explores in considerable detail the principles of operation of various types of flip-flops and, in addition, analyzes the electronics of the circuitry of a number of representative commercial units. We take considerable pains to make clear how flip-flops are adapted to circumvent timing problems that would otherwise develop in synchronous systems. Registers and counters are discussed in Chapter 10. Procedures for the design of both synchronous and ripple counters of arbitrary modulo are explained, and the use of registers to generate pseudorandom and other specified sequences is also presented.

Logic circuits for performing arithmetic operations are considered in Chapter 11. Emphasis is placed on the operations of addition (and subtraction) since generally multiplication and division are performed by algorithms involving the operation of addition (or subtraction). We have taken rather more care than is usual to explain clearly how negative numbers are expressed and how subtraction is effected in one's-complement and two's-complement notation through the use of logic circuitry which actually performs addition. The use of saturation logic for overflow correction in addition is presented as is the operation of the arithmetic logic unit which is the heart of every microprocessor. Semiconductor memories are examined in Chapter 12. We have omitted core memories since it appears that such core memories are in the process of being supplanted by semiconductor systems. This chapter includes sequential memories, read-only memories and random-access dynamic and static memories. The electronics of memories involving field-effect transistors, the CCD and bipolar junction transistors are also described.

In Chapters 13 and 14 we consider the matter of the interface between digital and analog signals. Chapter 13 presents analog gates, analog multiplexers, sample-and-hold circuits, integrate-and-dump circuits, etc. Chapter 14 examines digital-to-analog and analog-to-digital systems. The various analog-to-digital

systems considered are reasonably representative of the systems which are in wide use. Finally, in Chapter 15, timing circuits—the integrated-circuit equivalents of monostable and astable multivibrators—are discussed.

The circuits presented in this text are typical of those encountered in the field. More than 400 homework problems are provided, ranging from routine exercises to rather sophisticated design problems. A solutions manual is available which instructors can obtain from the publisher. An answer book is also available. The authors will be happy to furnish a set of laboratory experiments currently used at CCNY in conjunction with this text.

We acknowledge gratefully the encouragement given by our colleagues and students. In particular we thank Mr. T. Apelewicz who prepared the solutions manual, Dr. J. Garodnick to whom we are indebted for a critical review and criticism of much of the text material and Mr. Edward Tynan and Dr. Ronald Schilling through whose kindness we were able to receive a great deal of the very useful technical literature published by the Motorola company. We express our particular appreciation to Mrs. Joy Rubin for her skillful service in typing the manuscript.

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CONTENTS

Pretace	XVII
Electronic Devices	1
The Ideal Semiconductor Diode	1
Temperature Dependence of Diode Characteristics	5
Diode Transition Capacitance	6
The Zener Diode	7
Diodes for Integrated Circuits	9
The Transistor as a Switch	10
Analytic Expressions for Transistor Characteristics	12
The Transistor at Cutoff	16
The Transistor Switch in Saturation	17
Application of Ebers-Moll Equations to Saturation	20
The Field-Effect Transistor	26
The Metal-Oxide-Semiconductor FET (MOSFET)	31
The MOS Switch	35
Input-Output Characteristics of a MOSFET Switch	37
Complementary-Symmetry MOSFETs (CMOS)	38
Input Protection	42
Switching Speed of a Diode	42
Storage and Transition Time	44
The Schottky Diode	46
Switching Speed of a Bipolar Transistor	47
	Electronic Devices The Ideal Semiconductor Diode Temperature Dependence of Diode Characteristics Diode Transition Capacitance The Zener Diode Diodes for Integrated Circuits The Transistor as a Switch Analytic Expressions for Transistor Characteristics The Transistor at Cutoff The Transistor Switch in Saturation Application of Ebers-Moll Equations to Saturation The Field-Effect Transistor The Metal-Oxide-Semiconductor FET (MOSFET) The MOS Switch Input-Output Characteristics of a MOSFET Switch Complementary-Symmetry MOSFETs (CMOS) Input Protection Switching Speed of a Diode Storage and Transition Time The Schottky Diode

1.21	Switching Speeds in FET Devices	51
1.22	Rise and Fall Times and Delays	51
2	Operational Amplifiers and Comparators	54
2.1	The Operational Amplifier	54
2.2	The Virtual Ground	55
2.3	Operations	56
2.4	Output Impedance	58
2.5	Electronics of Operational Amplifiers	59
2.6	Overall Amplifier	62
2.7	Noninverting Amplification Using an Op-Amp	63
2.8	Impedance of Noninverting Amplifier	64
2.9	A Practical Consideration	66
2.10	Compensation	66
	Common-Mode Rejection Ratio	67
	Characteristics of Op-Amps	67
	The Comparator	69
	An Integrated-Circuit Amplifier Comparator	70
	Calculations for the Integrated-Circuit Amplifier Comparator	72
	Characteristics of Physical Comparators	77
	The Schmitt Trigger Circuit	78
2.18	An Example of a Schmitt Trigger	82
3	Logic Circuits	85
3.1	Introduction	85
3.2	Functions of a Single Binary Variable	86
3.3	Functions of Two Binary Variables	87
3.4	The or Function	89
3.5	The NAND Operation and the NOR Operation	90
3.6	The EXCLUSIVE-OR Operation	92
3.7	Other Functions	94
	Logical Variables	94
	The 0,1 Notation	97
	Necessary and Sufficient Operations	98
	Boolean Algebraic Theorems	100
	An Example	102
	The Binary Number System	104
	The Grey Reflected Binary Code	106
3.15	Standard Forms for Logical Functions: The Standard Sum of	
	Products	107
	The Standard Product of Sums	110
	Minterm and Maxterm Specifications of Logical Functions	111
	The Karnaugh-Map Representation of Logical Functions	113
	Karnaugh-Map Representations for Two, Three, and Four	
	Variables	116

3.20	Simplification of Logical Functions with Karnaugh Maps	117
	Larger Groupings on a K Map	120
3.22	Karnaugh Maps for Five and Six Variables	122
3.23	Use of Karnaugh Maps	124
3.24	Mapping when Function is not Expressed in Minterms	128
	Synthesis Using NAND-OR-NOR Gates	130
3.26	Incompletely Specified Functions	132
4	Resistor-Transistor Logic (RTL) and Integrated-Injection	124
	Logic (IIL)	134
4.1	The Resistor-Transistor-Logic (RTL) Gate	134
4.2	The Direct-Coupled Transistor-Logic (DCTL) Gate	136
4.3	Current Hogging in DCTL Gates	138
4.4	Resistor-Transistor Logic (RTL)	139
4.5	Fan-out	141
4.6	Input-Output Voltage Characteristic of Cascaded RTL Gates	146
4.7	An RTL Buffer	151
4.8	An RTL exclusive-or Gate	154
4.9	Manufacturer's Specifications	156
	Paralleling RTL Gates	157
	Specification of Operating Voltages	159
	Propagation Delay Time	161
	Integrated-Injection Logic (IIL) Physical Layout of IIL	164
	An IIL Decoder	169
	Current and Voltage Levels	172 174
7.10	Current and Voltage Levels	1/4
5	Diode-Transistor Logic	176
5.1	Diode-Transistor Logic (DTL) Gate	176
5.2	Fan-out	179
5.3	Integrated-Circuit DTL Gates	181
5.4	Input-Output Characteristic	184
5.5	Manufacturer's Specifications of DTL Gates	186
5.6	The WIRED-AND Connection	189
	High-Threshold Logic (HTL)	192
5.9	Input-Output Characteristic of the HTL Gate	193
).9	Manufacturer's Specifications	194
	Transistor-Transistor Logic	196
	Transistor-Transistor Logic (TTL)	196
	A Comparison between TTL and DTL	198
	The Input Transistor	198
	The Active Pull-up	200
5.5	Input-Output Characteristic Neglecting the Input Transistor	202

6.6	Input-Output Characteristic of the Input Transistor	208
6.7	The Multiemitter Transistor	211
6.8	Input Volt-Ampere Characteristic of the TTL Gate	212
6.9		214
6.1	Manufacturer's Data and Specifications: Temperature	
	Dependence and Noise Immunity	217
6.1	Power-Supply Current Drain	221
6.1.	2 Types of TTL Gates	222
6.1.	3 Schottky TTL	226
6.14	Other Logic with TTL Gates	228
7	Emitter-Coupled Logic	229
7.1	Introduction	229
7.2		231
7.3	ECL-Transistor Voltages	232
7.4	Transfer Characteristics: The or Output	232
7.5	The NOR Output	235
7.6	Manufacturer's Specifications: Transfer Characteristic	236
7.7		238
7.8	•	239
7.9	1 115	241
	Logic Versatility of ECL Gates	243
	The Negative Supply Voltage	244
	Level Translation	246
7.13	ECL-Gate Interconnections	249
8	MOS Gates	259
8.1	Analytic Equations for MOSFETS	259
8.2	Temperature Effects	260
8.3	The MOS Inverter	260
8.4	The CMOS Inverter	262
8.5	Calculation of CMOS-Inverter Transfer Characteristic	263
8.6	MOS Gates	265
8.7	Rise Time in an MOS Gate	268
	The Fall Time	269
	The CMOS Gate	271
	Rise and Fall Times in CMOS Gates	272
	Manufacturer's Specifications	273
8.12	Interfacing BJT and CMOS Gates	275
9	Flip-Flops	278
9.1	Introduction	278
9.2	Terminology	279
9.3	The Flip-Flop as a Memory Element	280
9.4	Flip-Flop using NAND Gates	281

9.5	The Chatterless Switch	282
9.6	Clocked Flip-Flop	283
9.7	Interconnection of Flip-Flops: The Master-Slave Flip-Flop	287
9.8	The AC-Coupled Edge-triggered Flip-Flop	292
9.9	The Clocked AC-Coupled Flip-Flop	295
9.10	A Capacitive Storage Flip-Flop	297
9.11	Propagation Delay Flip-Flops	298
9.12	? The JK Flip-Flop	299
9.13	The Type-D Flip-Flop	301
9.14	An RTL SR Flip-Flop	304
	5 A DTL Flip-Flop	304
	6 An RTL Propagation-Delay Flip-Flop	308
	The ECL Flip-Flop	310
	A JK Ac-Coupled ECL Flip-Flop	311
	Manufacturer's Specifications	315
	TTL JK Flip-Flop	315
9.21	MOS Flip-Flops	318
10	Registers and Counters	322
10.1	The Shift Register	322
10.2	Clocking	325
10.3	Serial-Parallel Data Transfer	325
10.4	End-Around Carry	326
10.5	Shift-Right—Shift-Left Register	326
10.6	Ripple Counters	327
10.7	Methods to Improve Counter Speed	331
10.8	Nonbinary Counters	334
10.9	Mod-3 Counters	334
10.10	Mod-5 Counters	336
	Lockout	339
	Combinations of Modulo Counters	341
	Other Counter Designs	343
	The Up-Down Ripple Counter	345
	The Up-Down Synchronous Counter	346
	Ring Counters	348
10.17	Sequence Generators	349
11	Arithmetic Operations	356
11.1	Addition of Two Binary Numbers	356
11.2	The Full Adder	358
11.3	A Serial Adder	360
11.4	Parallel Addition	362
11.5	Addition of More Than Two Numbers	363
11.6	Fast Adders: Look-Ahead Carry	366
11.7	Subtraction	369

11.8	Complementary Numbers	371
	Representation of Signed Numbers in Registers	374
	Subtraction Through Complementation and Addition	376
	Twos-complement Addition and Subtraction	377
	Ones-complement Addition and Subtraction	378
	Addition and Subtraction of a Sequence of Signed Numbers	380
	A Saturating Adder	381
	Scaling	383
	Multiplication	386
	Division	387
11.18	The Arithmetic Logic Unit (ALU)	389
12	Semiconductor Memories	392
12.1	Type of Memories	392
12.2	Shift-Register Sequential Memories	394
12.3	MOS Register Stages	396
12.4	Two-Phase Ratioless Shift Register	400
12.5	Four-Phase Ratioless Register Stage	402
12.6	CMOS Register Stages	403
12.7		405
12.8	A Three-Phase Static Register Stage	407
12.9	The Read-Only Memory	409
12.10	Implementation of ROMs	412
	Programmable and Eraseable ROMs	413
	Applications of ROMs	413
12.13	Bipolar-Junction-Transistor Random-Access-Memory Cells	416
	Other Bipolar-Transistor Memory Cells	418
12.15	MOS RAMs	420
	Organization of a RAM	425
12.17	Paralleling of Semiconductor Memory Integrated-Circuit Chips	427
	The Charged-Coupled Device (CCD)	431
	Storage of Charge	433
	Transfer of Charge	434
12.21	Input and Output Arrangement	437
13	Analog Switches	440
13.1	Basic Operating Principles of Analog Gates	441
13.2	Applications of Switching Circuits	442
13.3	Diode Transmission Gates	451
13.4	Bipolar Junction Transistor Gates	463
	FET Gates	466
	Operational Amplifiers	467
13.7	An FET Gate with an Op-Amp Load	468
	A Sample-and-Hold Circuit	470
	FET Gate Drivers	476
13.10	CMOS Gates	479

13.1.	Application of Analog Switches	48.
13.12	? Manufacturer's Specification of S/H Amplifiers	483
14	Analog-to-Digital Conversion	486
14.1	Introduction	486
14.2	The Sampling Theorem	487
14.3	Time-Division Multiplexing	489
14.4	Quantization	491
14.5	The Weighted-Resistor D/A Converter	494
14.6	The R - 2R Ladder D/A Converter	497
14.7	/	501
14.8	A Current-Driven D/A Converter	503
14.9	The Inverted-Ladder D/A Converter	507
	Input and Output Formats of a D/A Converter	510
	Specifications for D/A Converters	513
14.12	A/D Converters: A Parallel-Comparator Type	516
	Successive-Approximation Converter	522
	The Counting Converter	526
	The Dual-Slope Converter	529
	A Comparison of Converter Types	531
14.17	A Converter Using Voltage-to-Frequency Conversion	532
14.18	A Converter Using Voltage-to-Time Conversion	534
	A/D Converter Specifications	536
	Interconnecting the S/H and the A/D Converter	536
	Delta Modulation	538
14.24	Adaptive Delta Modulation	541
15	Timing Circuits	544
15.1	CMOS Multivibrators	545
15.2	The CMOS Astable Multivibrator	551
15.3	Monostable Multivibrators Using ECL Gates	552
15.4	Multivibrators for Short Timing Intervals	557
15.5	An Integrated-Circuit TTL Monostable Multivibrator	559
15.6	An Integrated-Circuit Timer	562
A	Appendix Transmission Lines	567
<i>A.1</i> 1	ntroduction	567
4.2	The Characteristic Impedance	569
	Reflections	570
	Multiple Reflections	571
4.5 I	Effect of Waveform Rise Time	572
F	roblems	575
R	epresentative Manufacturers' Specifications	631
I	ndex	641

ELECTRONIC DEVICES

As with analog circuits, the electronic devices used in digital processing circuits include the diode, the bipolar transistor, and the field-effect transistor. We assume that the reader is familiar with these devices but principally in applications involving analog circuitry, where they are used as linear elements. In digital circuits, these devices are used principally in a nonlinear manner, i.e., in a switching mode, where they are abruptly driven between the extremes of nonconduction and conduction. In this chapter we shall review some matters of interest in connection with these devices with special emphasis on their behavior when used as switches.

1.1 THE IDEAL SEMICONDUCTOR DIODE

For an ideal pn junction diode the current I is related to the voltage V by the equation

$$I = I_0(\epsilon^{V/V_T} - 1) \tag{1.1-1}$$

As indicated in Fig. 1.1-1a, the current I is positive when the current flows

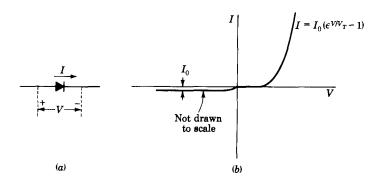


FIGURE 1.1-1 (a) The symbols I and V used in the diode equation (1.1-1), defined. (b) The voltampere characteristic of an ideal diode.

from the p side to the n side of the diode. The voltage V is the voltage drop from the p side to the n side. When V is positive, the diode is forwardbiased. The symbol V_T stands for the electron volt equivalent of the temperature and is given by

$$V_T \equiv \frac{kT}{\rho} \tag{1.1-2}$$

where $k = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J/K}$

 $e = \text{electronic charge} = 1.602 \times 10^{-19} \text{ C}$

T = absolute temperature, kelvins

Substituting, we find that $V_T = T/11,600$ V and that at room temperature $(T \approx 300 \text{ K}) V_T \approx 25 \text{ mV}.$

The form, in principle, of the diode volt-ampere characteristic is shown in Fig. 1.1-1b. When the voltage V is positive and several times V_T , the exponential term in Eq. (1.1-1) greatly exceeds unity and the -1 term in the parentheses may be neglected. Consequently, except for a small range in the neighborhood of the origin, the current increases exponentially with voltage. When the diode is reversed-biased and |V| is several times larger than V_T , $|I| \approx I_0$. The reverse current is therefore constant, independently of the applied reverse bias. Accordingly, Io is referred to as the reverse saturation current. This current is shown in Fig. 1.1-1b using a greatly enlarged scale since the value of I_0 is orders of magnitude less than typical values of I.

As noted, we shall be interested in the operation of diodes (and other elements) as switches. The diode is an open switch when back-biased and a closed switch when forward-biased. We shall generally find, in circuits of interest to us, that when a diode is called upon to make its presence felt in a circuit as a closed switch, it may typically carry a current of the order of a milliampere,

i.e., in the range 0.1 to 10 mA. How large a voltage must be impressed across the diode to produce this nominal forward current depends, of course, on the diode cross section. If a diode yielded a forward current of 1 μ A at an applied voltage V, a second diode of cross section 1,000 times larger would yield a current of 1 mA.

When a diode is manufactured, whether as a discrete component or an element in an integrated circuit, it is economical to use a cross section no larger than necessary. Such is particularly the case in integrated circuits (IC). For here, since many circuit elements are included on a single chip, a small increase in the cross section of one element is multiplied many-fold. This may result in an appreciable increase in the size of the silicon chip, or, equivalently, the same size chip will contain fewer diodes. The cross section of a diode will then be selected in part on the basis that with a reasonable margin of safety the diode should be able to dissipate the heat generated within it without an unacceptable increase in temperature. Additionally, the cross-sectional area must be large enough to reduce the ohmic resistance of the diode to an acceptable value.

A diode model When we examine the volt-ampere characteristics of commercial silicon diodes intended for application in low-power electronic circuits, we find that currents of the order of a milliampere correspond to a forward voltage of about 0.75 V. Diodes incorporated into integrated circuits appear to have comparable characteristics, again requiring about 0.75 V for forward currents in the range of a milliampere. Since we shall frequently have occasion to refer to this voltage, we assign to it a symbol $V_{\sigma} = 0.75$ V. When, then, the forward diode voltage is V_{σ} , the diode, used as a switch, is in the closed position.

If the diode switch is to be in the open position, it is really not necessary, as a matter of practicality, that the diode be reverse-biased. It is only necessary that the voltage across the diode correspond to a forward current which is negligibly small in comparison with the current corresponding to V_{σ} . Let us consider that the diode current is negligible when it has been reduced to 1 percent of the current corresponding to V_{σ} . The diode voltage, corresponding to this reduced current, we call V_{ν} .

If currents I_{σ} and I_{γ} correspond to voltages V_{σ} and V_{γ} , then from Eq. (1.1-1), we have

$$I_{\sigma} = I_0(\epsilon^{V_{\sigma}/V_T} - 1) \tag{1.1-3a}$$

and

$$I_{\gamma} = I_0(\epsilon^{V_{\gamma}/V_T} - 1) \tag{1.1-3b}$$

Since ϵ^{V_o/V_T} and ϵ^{V_o/V_T} are each much greater than unity, we have

$$\frac{I_{\sigma}}{I_{\gamma}} = 100 = \epsilon^{(V_{\sigma} - V_{\gamma})/V_{T}} \tag{1.1-4}$$

Hence

$$V_{\sigma} - V_{\gamma} = V_T \ln 100 \approx 120 \text{ mV}$$
 (1.1-5)

Thus, since $V_{\sigma} \approx 0.75 \text{ V}$, $V_{\gamma} \approx 0.63 \text{ V}$.