Chaotic Vibrations

An Introduction for Applied Scientists and Engineers

FRANCIS C. MOON

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Preface

Had anyone predicted that new discoveries would be made in dynamics three hundred years after publication of Newton's *Principia*, they would have been thought naive or foolish. Yet in the last decade new phenomena have been observed in all areas of nonlinear dynamics, principal among these being chaotic vibrations. Chaotic oscillations are the emergence of randomlike motions from completely deterministic systems. Such motions had been known in fluid mechanics, but they have recently been observed in low-order mechanical and electrical systems and even in simple one-degree-of-freedom problems. Along with these discoveries has come the recognition that nonlinear difference and differential equations can admit bounded, nonperiodic solutions that behave in a random way even though no random quantities appear in the equations. This has prompted the development of new mathematical ideas, new ways of looking at dynamical solutions, which are now making their way into the laboratory.

It is the purpose of this book to help translate these mathematical ideas and techniques into language that engineers and applied scientists can use to study chaotic vibrations. Although I am an experimenter in dynamics, I have had to acquire a certain level of mathematical understanding of these new ideas, such as strange attractor, Poincaré map, or fractal dimension, in order to study chaotic phenomena in the laboratory. A number of excellent mathematical treatises on chaotic dynamics have appeared recently. I have attempted to read and distill these new concepts with the help of my more theoretical colleagues at Cornell University and attempt in this book to explain the relevance of this new language of dynamics to engineers, especially those who must study and measure vibrations. I believe that these new geometric and topological concepts in dynamics will become part of the laboratory tools in vibration analysis in the same way that Fourier analysis has become a permanent part of engineering experimental technique.

Besides the infusion of new ideas, the study of chaotic vibrations is important to engineering vibrations for several reasons. First, in mechanical systems a chaotic or noisy system makes life prediction or fatigue analysis difficult because the precise stress history in the solid is not known. Second, the recognition that simple nonlinearities can lead to chaotic solutions raises the question of predictability in classical physics and the usefulness of numerical simulation of nonlinear systems. It is part of the conventional wisdom that larger and faster supercomputers will allow one to make more precise predictions of a system's behavior. However, for nonlinear problems with chaotic dynamics, the time history is sensitive to initial conditions and precise knowledge of the future may not be possible even when the motion is periodic.

Many new books on chaotic dynamics assume that the reader has had some exposure to advanced dynamics, nonlinear vibrations, and advanced mathematical techniques. In this book I have tried to work from a background that a B.S. engineering graduate would have; namely, ordinary differential equations, some intermediate-level dynamics, and vibration or systems dynamics courses. I have also tried to give examples of systems with chaotic behavior and to offer engineers the tools to measure, predict, and quantify chaotic vibrations in physical systems.

In Chapter 2 I describe some of the characteristics of chaotic vibrations and how to recognize and test for them in physical experiments. The types of physical models and experimental systems in which chaotic behavior has been observed are given in Chapter 3. In Chapter 4 some experimental techniques are presented to measure chaotic phenomena, including Poincaré maps. This is a "how to do it" chapter and can be skipped by those looking for an overview of the field. Chapters 5 and 6 are more mathematical and explore what criteria now exist to predict when chaotic vibrations will occur and the new concepts in fractal mathematics. Fractal concepts are at the center of many of the new ideas in nonlinear dynamics. Beautiful pictures of fractal geometric objects have appeared in the popular press and have added an aesthetic dimension to the study of dynamics. In Chapter 6 I attempt to relate fractal ideas to specific applied problems in nonlinear dynamics.

One might ask: Why write this book now while the field of nonlinear vibrations is undergoing such rapid change? First, it was an opportune time since I was asked to prepare and deliver eight lectures on chaotic vibrations at the Institute for Fundamental Technical Problems in Warsaw, Poland, in August 1984. This book is an outgrowth of those lectures. Second, during

1984–1985 I was invited to give lectures in chaotic vibrations at nearly thirty universities and research laboratories. Many colleagues expressed a desire for a book on chaos, aimed at those in the applied sciences. Also, I felt that many engineers in the field of vibrations were unaware of the exciting new things happening in dynamics. Engineering researchers, armed with new tools of dynamical systems, will I am sure make further advances into this new area by exploring new applications and developing more practical tools for measuring and describing these new phenomena.

I want to thank my colleagues at Cornell University in theoretical and applied mechanics, especially Philip Holmes and Richard Rand, who have patiently tried to explain these new mathematical ideas to me. I have also had useful conversations with John Guckenheimer, formerly of the University of California at Santa Cruz but now at Cornell. The deliberate lack of rigor in this book in describing some of the new geometric and topological concepts must be blamed on me, however. I have proceeded on the assumption that the book will succeed only if it stimulates interest in this new field. Given this stimulation, I hope the reader will seek out more mathematical references to provide more detailed and precise discussion of these new ideas.

Finally I wish to recognize the contributions of graduate students and research associates who have worked so enthusiastically with me on problems of chaos: Joseph Cusumano, Mohammed Golnaraghi, Guang-Xian Li, Chih-Kung Lee, Bimal Poddar, Gabriel Raggio, and Stephan Shaw (now at Michigan State University). Special mention is also made of the technical help of Stephen King and William Holmes who helped design some of the electronic instrumentation in our experiments on chaotic vibration.

Regarding the references at the end of this book, I did not attempt to include all the historically significant papers in chaotic studies and I apologize to those researchers whose fine contributions to the subject have not been cited. The inclusion of more of my own papers than those of others must be interpreted as an author's vanity and not any measure of their relative importance to the field.

I also want to acknowledge funding from the National Science Foundation through the solid mechanics program under Dr. Clifford Astill, from the Air Force Office of Scientific Research through Dr. Anthony Amos of the Aerospace Section, from the Office of Naval Research through Dr. Michael Shlesinger of the Physics Division, from the Army Research Office through Dr. Gary Anderson of the Engineering Sciences Division and from the IBM Corporation.

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Introduction: A New Age of Dynamics

In the beginning, how the heav'ns and earth rose out of chaos
J. Milton, *Paradise Lost*, 1665

1.1 WHAT IS CHAOTIC DYNAMICS?

For some, the study of dynamics began and ended with Newton's Law of $F = mA$. We were told that if the forces between particles and their initial positions and velocities were given, one could predict the motion or history of a system forever into the future, given a big enough computer. However, the arrival of large and fast computers has not fulfilled the promise of infinite predictability in dynamics. In fact, it has been discovered quite recently that the motion of very simple dynamical systems cannot always be predicted far into the future. Such motions have been labeled *chaotic* and their study has prompted a discussion of some exciting new mathematical ideas in dynamics. With the approaching tricentennial of Newton's *Principia* (1687), in which he introduced the calculus into the study of dynamics, it is appropriate that three centuries later new phenomena have been discovered in dynamics and that new mathematical concepts from topology and geometry have entered this venerable science.

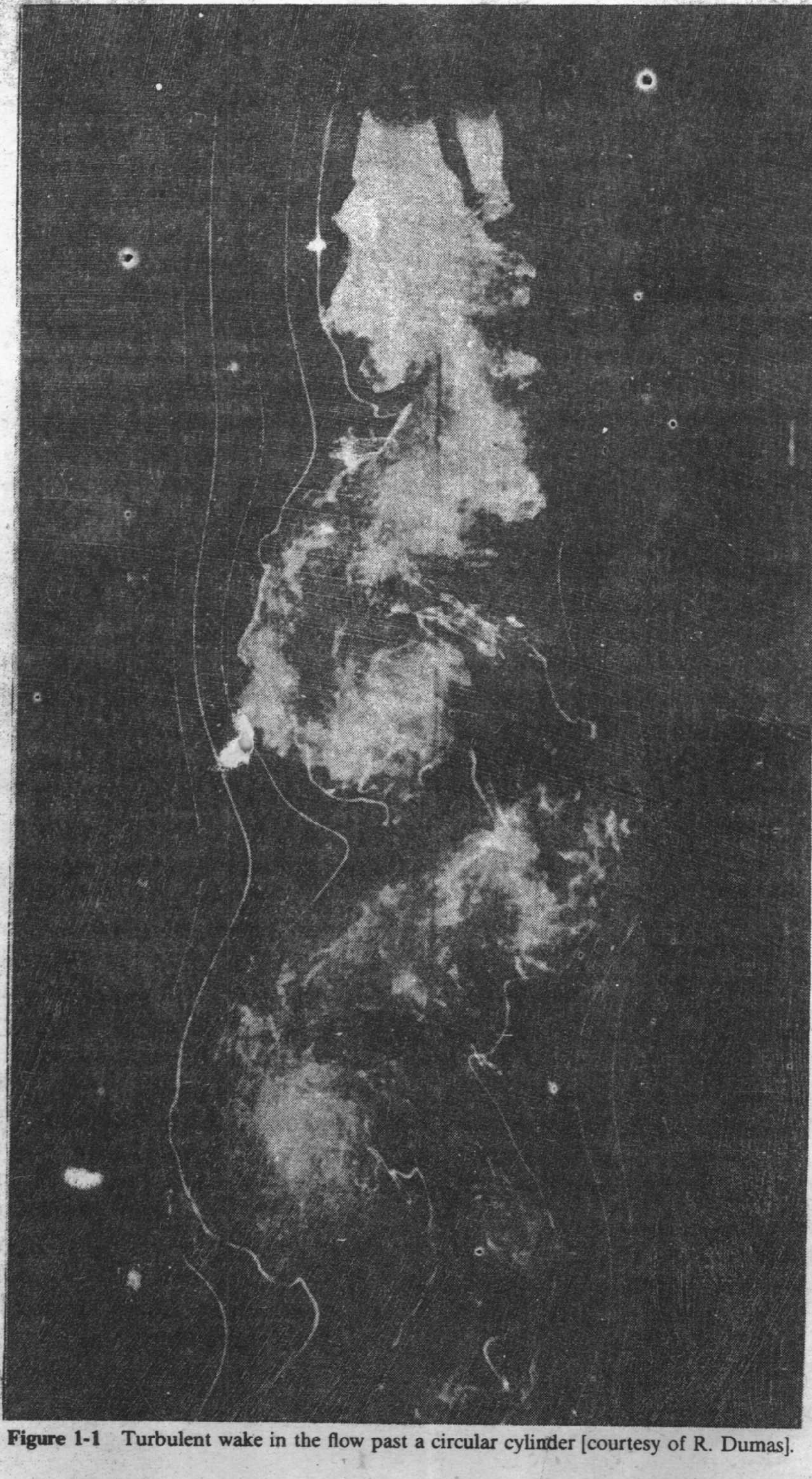

Figure 1-1 Turbulent wake in the flow past a circular cylinder [courtesy of R. Dumas].

The nonscientific concept of chaos[1] is very old and often associated with a physical state or human behavior without pattern and out of control. The term chaos often stirs fear in humankind since it implies that governing laws or traditions no longer have control over events such as prison riots, civil war, or a world war. Yet there is always the hope that some underlying force or reason is behind the chaos or can explain why seemingly random events appear unpredictable.

In the physical sciences, the paragon of chaotic phenomena is turbulence. Thus, a rising column of smoke or the eddies behind a boat or aircraft wing[2] provide graphic examples of chaotic motion (Figure 1-1). The fluid mechanician, however, believes that these events are not random because the governing equations of physics for each fluid element can be written down. Also, at low velocities, the fluid patterns are quite regular and predictable from these equations. Beyond a critical velocity, however, the flow becomes turbulent. A great deal of the excitement in nonlinear dynamics today is centered around the hope that this transition from ordered to disordered flow may be explained or modeled with relatively simple mathematical equations. What we hope to show in this book is that these new ideas about turbulence extend to solid mechanical and electrical continua as well. It is the recognition that chaotic dynamics are inherent in all of nonlinear physical phenomena that has created a sense of revolution in physics today.

We must distinguish here between so-called random and chaotic motions. The former is reserved for problems in which we truly do not know the input forces or we only know some statistical measures of the parameters. The term chaotic is reserved for those deterministic problems for which there are no random or unpredictable inputs or parameters. The existence of chaotic or unpredictable motions from the classical equations of physics was known by Poincaré.[3] Consider the following excerpt from

[1] The origin of the word *chaos* is a Greek verb which means *to gape open* and which was often used to refer to the primeval emptiness of the universe before things came into being (*Encyclopaedia Britannica*, Vol. 5, p. 276). To the stoics, chaos was identified with water and the watery state which follows the periodic destruction of the earth by fire. Ovid in *Metamorphises* used the term to denote the raw and formless mass in which all is disorder and from which the ordered universe is created. A modern dictionary definition of chaos (Funk and Wagnalls) provides two meanings: (i) utter disorder and confusion and (ii) the unformed original state of the universe.

[2] The reader should look at the beautiful collection of photos of fluid turbulent phenomena compiled by Van Dyke (1982).

[3] Henri Poincaré (1854–1912) was a French mathematician, physicist, and philosopher whose career spanned the grand age of classical mechanics and the revolutionary ideas of relativity and quantum mechanics. His work on problems of celestial mechanics led him to questions of dynamic stability and the problem of finding precise mathematical formulas for the dynamic

this essay on *Science and Method*:

> It may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible.

In the current literature, *chaotic* is a term assigned to that class of motions in deterministic physical and mathematical systems whose time history has a *sensitive dependence on initial conditions*.

Two examples of mechanical systems that exhibit chaotic dynamics are shown in Figure 1-2. The first is a thought experiment of an idealized billiard ball (rigid body rotation is neglected) which bounces off the sides of an elliptical billiard table. When elastic impact is assumed, the energy remains conserved, but the ball may wander around the table without exactly repeating a previous motion for certain elliptically shaped tables.

Another example, which the reader with access to a laboratory can see for oneself, is the ball in a two-well potential shown in Figure 1-2*b*. Here the ball has two equilibrium states when the table or base does not vibrate. However, when the table vibrates with periodic motion of large enough amplitude, the ball will jump from one well to the other in an apparently random manner; that is, periodic input of one frequency leads to a randomlike output with a broad spectrum of frequencies. The generation of a continuous spectrum of frequencies below the single input frequency is one of the characteristics of chaotic vibrations (Figure 1-3).

Loss of information about initial conditions is another property of a chaotic system. Suppose one has the ability to measure a position with accuracy Δx and a velocity with accuracy Δv. Then in the position–velocity plane (known as the phase plane) we can divide up the space into areas of size $\Delta x\, \Delta v$ as shown in Figure 1-4. If we are given initial conditions to the stated accuracy, we know the system is somewhere in the shaded box in the phase plane. But if the system is chaotic, this uncertainty grows in time to $N(t)$ boxes as shown in Figure 1-4*b*. The growth in uncertainty given by

$$N \approx N_0 e^{ht} \tag{1-1.1}$$

is another property of chaotic systems. The constant h is related to the concept of *entropy* in information theory (e.g., see Shaw, 1981, 1984) and will also be related to another concept called the *Lyapunov exponent* (see

history of a complex system. In the course of this research he invented the "the method of sections," now known as the Poincaré section or map.

An excellent discussion of uncertainties and determinism and Poincaré's ideas on these subjects may be found in the very readable book by L. Brillouin (1964, Chapter IX).

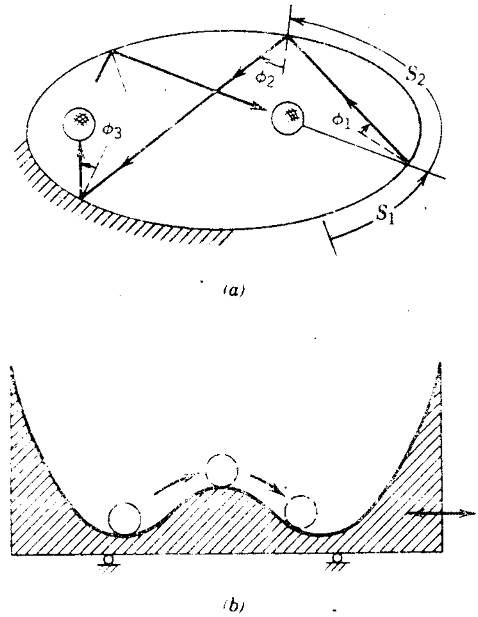

Figure 1-2 (*a*) The motion of a ball after several impacts with an elliptically shaped billiard table. The motion can be described by a set of discrete numbers (s_i, ϕ_i) called a map. (*b*) The motion of a particle in a two-well potential under periodic excitation. Under certain conditions, the particle jumps back and forth in a periodic way, that is, LRLR ···, or LLRLLR ···, and so on, and for other conditions the jumping is chaotic that is, it shows no pattern in the sequence of symbols L and R.

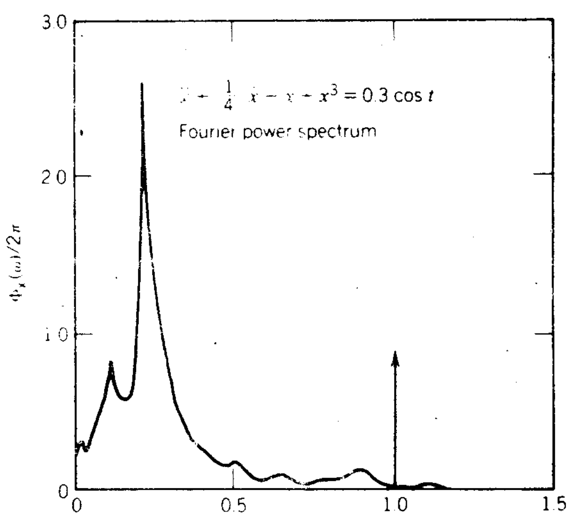

Figure 1-3 The power spectral density (Fourier transform) of chaotic motion in a two-well potential (after Y. Udea, Kyoto University).

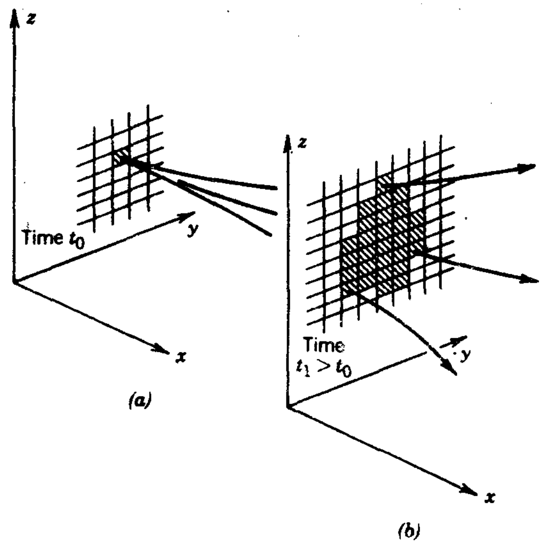

Figure 1-4 An illustration of the growth of uncertainty or loss of information in a dynamical system. The black box at time $t = t_0$ represents the uncertainty in initial conditions.

Chapter 5) which measures the rate at which nearby trajectories of a system in phase space diverge.

The reader may ask: With predictability lost in chaotic systems, is there any order left in the system? For dissipative systems the answer is yes; there is an underlying structure to chaotic dynamics. This structure is not apparent by looking at the dynamics in the conventional way, that is, the output versus time or from frequency spectra. One must search for this order in phase space (position versus velocity). There one will find that chaotic motions exhibit a new geometric property called *fractal* structure. One of the goals of this book is to teach how to discover the fractal structure in chaotic vibrations as well as to measure the loss of information in these randomlike motions.

Why Should Engineers Study Chaotic Dynamics?

Recently, the subject of chaos has become newsworthy—the study of mathematical chaos that is. Many popular science magazines and even *The New York Times* and *Newsweek* have carried articles on the new studies into mathematics of chaotic dynamics. But engineers have always known about chaos—it was called noise or turbulence and fudge factors or factors of safety were used to design around these apparent random unknowns that seem to crop up in every technical device. So what is new about chaos?