CLASSICAL FIELDS: GENERAL RELATIVITY AND GAUGE THEORY

MOSHE CARMELI

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175 YEARS OF PUBLISHING
1807 1982

A Wiley-Interscience Publication
JOHN WILEY & SONS,
New York Chichester Brisbane Toronto Singapore

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Library of Congress Cataloging in Publication Data:

Carmeli, Moshe, 1933–
Classical fields.

"A Wiley-Interscience publication."
Includes bibliographies and index.
1. General relativity (Physics) 2. Gauge fields (Physics) 3. Gravitational fields. I. Title.

QC173.6.C37 530.1'4 82-2704
ISBN 0-471-86437-4 AACR2

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

In the last decade classical fields have become of great importance in theoretical physics. The reason for that is the realization by both physicists and mathematicians that gauge fields are just the right mathematical tool for describing particle physics as well as other branches of physics. As a consequence, general relativity theory has become a center of attention from the point of view of gauge fields.

The classical theory of fields is no longer a theory of electrodynamics and gravitation as two separate topics which can be formally and technically put together in one text. Rather, classical fields should include electrodynamics, gauge fields, and gravitation, and the three fields should be presented with a common physical and mathematical foundation. This book is the first text that undertakes such a task in presenting classical fields.

The book is based on lectures given by the author in two graduate courses at the Institute for Theoretical Physics at Stony Brook, New York, where he was a Visiting Professor in 1977–1978, and at the Ben Gurion University thereafter. Approximately half of the material is on gravitation, and the other half deals with classical gauge fields. More than half of the content is based on material that has not yet appeared in other books.

The emphasis here is on the classical field theory aspect of the topic. Also, only those topics of gauge field theory that blend naturally with gravitation are included. These topics of gauge fields include the spinor formulation and the classification of SU(2) gauge fields, as well as the null tetrad formulation of the Yang–Mills field in the presence of gravitation (and, of course, in its absence). Material found in the many available books on *quantum* field theory is not included.

The book consists of ten chapters, which are divided into sections, usually ending with problems, many of which are completely or partially solved. Chapters 1 and 2 are devoted to the physical foundations of the theory of gravitation and to the mathematical theory of the geometry of curved spacetimes needed to describe the general theory of relativity and the other topics in the remainder of the book.

The gravitational field equations, their properties and generalizations, are presented in Chapter 3. Here, the concepts of the Lie derivative, Killing equation, null tetrad formulation of the Einstein field equations along with the Newman–Penrose equations, and perturbation on gravitational background are introduced. In Chapter 4 the Einstein field equations are solved for mass systems. These include, in addition to the standard metrics, the Vaidya radiating metric, the Tolman metric, and the Einstein–Rosen metric describing cylindrical gravitational waves, which is of importance in constructing cosmological models. Chapter 5 is devoted to the general properties of the gravitational field, including such topics as the weak gravitational field, experimental verification of gravitational theory, gravitational radiation, the energy-momentum pseudotensor, and gravitational bremsstrahlung.

Chapter 6 is devoted to the derivation of the equations of motion of material bodies—including spinning particles—within the framework of general relativity. This includes geodesic motion, the Einstein–Infeld–Hoffmann post-Newtonian equation of motion and its Lagrangian formulation, and the Papapetrou equations for a spinning particle and their applications to motion in the Schwarzschild and Vaidya fields. In Chapter 7 the theory of axisymmetric exact solutions of the Einstein equations is given and, using the Ernst potential method, the metrics of Kerr, Tomimatsu–Sato, NUT–Taub, Demianski–Newman, and variable-mass Kerr are presented.

In Chapter 8 the spinor formulation of both the gravitational and the gauge fields is given. Here we introduce two-component spinors, the electromagnetic and the gravitational spinors. The SU(2) gauge field theory is subsequently given. This is then followed by the gauge field spinors and their transformation rules, the geometry of gauge fields, and the Euclidean gauge field spinors.

Chapter 9 is devoted to the classification of gauge fields. This problem is of great importance in connection with the finding of exact solutions to the Yang–Mills field equations, as experience has shown in general relativity theory with respect to the Petrov classification.

In Chapter 10 the Einstein field equations are written in relation to other gauge fields. Also, the Yang–Mills theory is formulated in null coordinates in both the cases of the presence and the absence of gravitation. As is well known, these methods have brought great insight into the theory of gravitation. The chapter also includes the theory of differential geometrical analysis, fiber bundles and their application to gauge fields and general relativity, magnetic monopoles, null tetrad formulation of the Yang–Mills theory, and monopole solution of the Yang–Mills equations.

The book can be used as a text for a one-year graduate course in theoretical physics, as has been done by the author in the last four years. It can also be used as a supplementary book to other texts in graduate courses in classical field theory or mathematical physics. We hope that it can fill the gap of a needed text on the subject, where classical fields are treated in a modern approach different from available books. The reader will find other aspects of gauge field theory in flat spacetime (Minkowskian and Euclidean) in the

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author's other book *Classical Fields: Electrodynamics and Gauge Theory*, now in preparation.

I am indebted to my colleagues and students at the Institute for Theoretical Physics at Stony Brook and at Ben Gurion University. In particular, I am indebted to Professors Chen Ning Yang and Max Dresden for their kind hospitality and comments on the content of the book. I am also indebted to Professor J. Ehlers for several suggestions, and to Professor S. Malin for reading the manuscript and for the many suggestions he made. Finally, I am indebted to Mrs. H. Schlowsky and Mrs. A. Rouse from SUNY, to Mrs. Deisa Buranello from ICTP, to Mrs. Y. Ahuvia and Miss M. Jameson from BGU for the excellent job of typing the manuscript, and to Mrs. S. Corrogosky for her assistance.

MOSHE CARMELI

Beer Sheva, Israel
June 1982

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1

THE GRAVITATIONAL FIELD

In this chapter the basic and preliminary properties of the gravitational phenomena are given. These are the prerelativistic properties which lay the foundations of the theory of general relativity. The discussion starts with the Newtonian theory of gravitation, along with other related topics, such as Newton's laws of motion. It then proceeds to the concepts of gravitational and inertial forces and their mutual relationship. This is followed by a discussion of the equality of the gravitational mass to the inertial mass, along with the experimental verification of this important fact. The experiment, known as the Eötvös experiment, is subsequently examined in detail. The chapter is concluded by discussing the principle of equivalence and the principle of general covariance. These two principles were the basis for the physical foundations in the original formulation of the theory of general relativity by Einstein.

1.1 NEWTONIAN GRAVITATION

The Galilean Group

In the classical mechanics of Newton we assume that the laws of motion do not depend on the choice of a particular fixed system of coordinates with respect to which the distances, velocities, accelerations, forces, and so on, are being measured. Furthermore, we assume that the laws of motion do not change their forms by transferring from one system of coordinates into another. These systems of coordinates are all assumed to have uniform, rectilinear, translational motions with respect to each other. They are called *inertial systems of coordinates*.

Thus inertial systems of coordinates differ from one another by orthogonal rotations, accompanied by translations of the origins of the systems, and by motion in uniform velocities. We can further add the translation of the time parameter, namely, the possibility of choosing the origin of time, $t = 0$, at will. We may count the number of parameters, or the number of degrees of freedom, which each coordinate system has with respect to any other one. Thus we have four parameters which account for the translations of the three spatial coordinates and time, three parameters describing the orthogonal rotations of the spatial coordinates, and finally three more parameters accounting for the rectilinear motions of the spatial coordinates. Newtonian laws of classical mechanics are therefore invariant under all of these ten-parameter transformations of inertial systems of coordinates.

A transformation of inertial coordinates having ten parameters, as described above, is called a *Galilean transformation*. Newton's classical laws of mechanics are invariant under the ten-parameter Galilean transformations. We say in this case that we have a *Galilean invariance*. The aggregate of all Galilean transformations forms a group. This group is called the *Galilean group* and has ten parameters.

If we choose two inertial coordinate systems so that their corresponding axes are parallel and coincide at $t = 0$, and if $\mathbf{v}$ is the velocity of one inertial coordinate system with respect to the other, the Galilean transformation can then be reduced into a simple transformation as follows:

$$x' = x + v_x t, \qquad y' = y + v_y t, \qquad z' = z + v_z t. \tag{1.1.1}$$

Here v_x, v_y, and v_z are the components of velocity $\mathbf{v}$ along the x axis, y axis, and z axis, respectively.

Newtonian Mechanics

The Newtonian laws of mechanics are based on three fundamental laws. These laws can be stated as follows:

1 A particle acted upon by no force will assume a rectilinear motion with a constant velocity.

2 A particle acted upon by a force $\mathbf{f}$ will move with an acceleration $\mathbf{a}$ which is proportional to the force. We can then write the relation between the force and the acceleration in the form of Newton's familiar law of motion:

$$\mathbf{f} = m\mathbf{a}, \tag{1.1.2}$$

where m is the mass of the particle.

3 For each action there is a reaction which is equal to the action, but is directed in the opposite direction of the action.

We conclude these brief remarks on Newtonian mechanics by mentioning the concept of *action-at-a-distance* which the Newtonian theory assumes. Roughly speaking, action-at-a-distance means that interactions between particles take place instantly. This is in contrast to modern physics concepts where we assume that interactions are mediated through intermediate particles, thus leading to the concept of fields.

Newton's Theory of Gravitation

Newton's theory of gravitation is actually a three-dimensional field theory. The gravitational field is assumed to be described by a scalar field $\phi(x, y, z)$, which is a function of the spatial coordinates. The function $\phi(x, y, z)$ satisfies a second-order partial differential equation of the form

$$\nabla^2\phi(x, y, z) = 4\pi G\rho(x, y, z). \qquad (1.1.3)$$

Such an equation is called the *Poisson equation*. Here G is Newton's gravitational constant, whose value is equal to $6.67 \times 10^{-8}\ \text{cm}^3 \cdot \text{g}^{-1} \cdot \text{s}^{-2}$ in CGS units, and $\rho(x, y, z)$ is the mass density of the matter in space producing the gravitational field. The differential operator ∇^2 is given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (1.1.4)$$

and is called the *Laplacian operator*.

A solution of the Poisson equation gives the potential $\phi(x, y, z)$ in terms of the mass distribution $\rho(x, y, z)$ in space. At points where there is no matter, that is, at points of space where $\rho(x, y, z) = 0$, we can solve the equation

$$\nabla^2\phi(x, y, z) = 0. \qquad (1.1.5)$$

The latter equation is called the *Laplace equation*. Its solution then describes the Newtonian potential at points of space where the mass density ρ vanishes.

The Newtonian potential ϕ creates a force field that acts on particles. This gravitational field of forces is proportional to the negative of the gradient of potential ϕ. Hence the force acting on a particle with mass m, located in a Newtonian potential ϕ, is given by

$$\mathbf{F} = -m\nabla\phi, \qquad (1.1.6)$$

where ∇ is the three-dimensional gradient operator,

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right). \qquad (1.1.7)$$

For instance, if the potential ϕ is produced by a single mass M, then the solution of the Poisson equation yields

$$\phi = -\frac{GM}{r}, \tag{1.1.8}$$

and the force acting on another particle with mass m will be

$$\mathbf{F} = GmM\nabla\frac{1}{r} = -\frac{GmM}{r^2}. \tag{1.1.9}$$

Equation (1.1.9) is the familiar inverse-square *law of interaction* of Newton.

The masses m and M appearing in Eq. (1.1.9) are the *gravitational masses*, since they give the gravitational attraction force between the two particles. The mass appearing in Newton's second law, Eq. (1.1.2), on the other hand, is the *inertial mass* of the particle. In Newtonian physics these two concepts are identified. In the sequel we will find that this identification is valid in general relativity theory, too.

Finally the *potential energy* for an arbitrary mass distribution in the Newtonian theory can be found. The potential energy of a particle in a gravitational field is equal to its mass times the potential of the field. Hence we obtain for the potential energy of a general system, with mass density ρ, the following expression:

$$U = \frac{1}{2}\int \rho\phi\, d^3x. \tag{1.1.10}$$

In the next section we discuss more thoroughly the basic properties of the gravitational field. This is done from a more general point of view and not necessarily that of Newtonian physics.

PROBLEMS

1.1.1 Find the Newtonian potential produced by a system of masses at distances that are large compared to the dimensions of the system.

Solution: The Newtonian potential is the solution of the Poisson equation

$$\nabla^2\phi(x) = 4\pi G\rho(x), \tag{1}$$

where $\rho(x)$ is the mass density of the system, and G is Newton's gravitational constant. In Eq. (1) the variable x denotes the three spatial coordinates x, y, z.

The solution of Eq. (1) is given by

$$\phi(x) = -G\int \frac{\rho(x')}{|\mathbf{r} - \mathbf{r}'|} d^3x', \tag{2}$$

where $\mathbf{r} = (x^1, x^2, x^3)$ is the radius vector of the point where the potential is

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