INTRODUCTION TO AERONAUTICAL DYNAMICS

MANFRED RAUSCHER
Professor of Aircraft Structures and Design
Eldgenössische Technische Hochschule, Zürich
Formerly of Massachusetts Institute of Technology

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PREFACE

When instruction in aeronautical engineering was begun at the Massachusetts Institute of Technology forty years ago, the phase of the subject developed most fully from a rational viewpoint was the theory of dynamic stability. This theory of the disturbed motion of a rigid body basically in equilibrium between aerodynamic forces and the body's weight proved to be difficult to grasp for the students who had not before encountered a system with so many degrees of freedom subject to such complex influences from its surroundings. A course to bridge the gap between the mathematics and mechanics the students already knew and the problems now awaiting them was therefore added to the curriculum. From a brief review of particle dynamics and vibration theory, the course branched into its two main divisions of fluid dynamics and rigid dynamics, and then joined these subjects in the analysis of airplane motion. Professor E. B. Wilson, who had charge of the course, described it in his *Aeronautics* (John Wiley & Sons, New York, 1920).

The present work retains the fundamental plan of Professor Wilson's. It recognizes, however, that aeronautical engineering today faces dynamical problems of greatly increased variety and complication, and that an *Introduction to Aeronautical Dynamics* thus must seek to give the student a broader and even deeper grounding in dynamical principles than had been needed in the past. As every teacher knows, the very bottom layer of such a grounding, next to the scraped bedrock of basic concepts and elementary formulations of ideas, is the most difficult one to lay properly. A painstaking special attention to this fundamental task will, accordingly, be found throughout the book. And, where the discussion climbs to higher levels of the subject, it is dominated by the view that formulas count less than the understanding gained in their derivation. To stimulate and hold the interest of the practically minded student, illustrative applications of general results established are shown all along in realistic detail. The Problems at the ends of the Chapters add further to the range of these applications and carry them next to the stage of engineering utility.

A difficulty with any comprehensive and at the same time detailed treatise is that its maze of details tends to hide the main lines of the subject. I have tried to help the student to keep these main lines in view by indicating to him the natural logic that leads one on, step by step, in the development of an idea. By thus arraying the beads of detail on the strings of coordinating thought, I hope to have achieved a clearly differentiated picture of the many-sided subject even for the student who is entirely on his own. Ideally, however, the reading of the book as a text ought to be paralleled by classroom discussion in which the teacher sets forth the broad perspectives of the things being studied.

The arrangement of the subject matter in the book corresponds to the sequence in which the various topics are taken up in the classes at the Massachusetts Institute of Technology. There, the course covered by this text now starts at the beginning of the Junior year, as the first subject with an aeronautical title. Working up from particle dynamics through fluid dynamics, the student comes to airfoil theory just at the time he can use that theory to coordinate experimental data as, in the second semester, the theoretical course gets supplemented by a companion course in Applied Aerodynamics. The tie-in thus provided between theory and practice is good for both. By the end of the semester, after the theory has proceeded further through rigid dynamics and vibration analysis, and the applied course has covered general aerodynamic data, including those on the performance of engines, the student is prepared all round for the study of aircraft stability, to which he turns at the beginning of his Senior year. Although many of the things discussed in the introductory courses have no direct connection with the stability problem, this problem, in the end, thus still is the one in which the different branches of theoretical and applied aeronautical dynamics find their crowning connection.

Because the Chapters on Particles, Rigid Bodies, and Vibrations represent about one half of the book, it would readily be possible to use them for a course rounded out in Solid Dynamics alone, and the remaining Chapters for a course of similar length in Fluid Dynamics alone. The coverage of both of the courses by one text would even then provide for helpful cross-references—as between the uses of complex variables in Chapters VII and XIII, or those of Fourier series in Chapters IX and XIII. And though the labels "solid" and "fluid" would tend to accentuate the dissimilarity rather than the similarity of the subjects of the two courses, the uniformity of the subjects' treatment should help to keep some measure of their natural coherence between them.

During the twenty-some years it has been in the writing, the book was issued piecemeal, in the form of mimeographed notes, to students taking the subject. Its various parts thus have, in a sense, been published for some time. The Sections dealing with the fundamental mechanics of variable mass, for example, date back to the earliest thirties; and the material on air-fed rockets was prepared, still with an element of prophecy, in the middle forties. In the final editing of the book, I have endeavored to smooth out the unevennesses in the treatments of the different parts, and to remove such marks of time as I could see here and there. Still, I am aware that in some respects the book is not as complete as it might be. Particularly noticeable is the absence of discussions of compressible flows in more than one dimension, of thin airfoils as surfaces of discontinuity, under unsteady as well as under steady conditions, of three-dimensional flow problems, and of servomechanisms. My failure to include these topics is due to a lack of time so far to write them up. Perhaps, if the book in the form it now has finds a sufficiently favorable reception, there will be an opportunity to fill the gaps in a second edition a few years hence. I shall be grateful to colleagues and students for all suggestions of possible improvements in the work.

A question I expect to be asked more than once is: How can a work like the present one afford to pass up the use of formal vector analysis? I agree that vector notation, like schemes of shorthand generally, will simplify, and perhaps clarify, a discussion for someone really familiar with the notation's meaning. Few of the Engineering Juniors for whom primarily this book is written, however, possess such a real familiarity with vector operations. To remain on dependably firm ground, I have therefore drawn on nothing but the most elementary geometrical sense to examine one after another of the vector problems encountered in the different parts of the book. When he is through with the book, the student has had a close view of vector changes illuminated from a variety of angles; he has seen scalar and vector products of pairs of vectors worked out on the basis of one set of pictures in Particle Dynamics (Work, Moment), and of another set of pictures in Fluid Mechanics (Stream Function, Velocity Potential); and he has been led to Gauss' and Stokes' Theorems by way of the concepts of Yield (divergence) and Emanation, here newly named, and of Vorticity (curl or rotation) and Circulation. Thus strengthened in his acquaintance with basic vector manipulations, he should then turn to his general vector analysis and discover in retrospect how much of dynamics can actually be simplified by its use.

Some criticism may be directed at the omission of Lagrange's Equations from the discussion of general dynamics, and of the Navier-Stokes Equations from the treatment of viscous fluids. These Equations had actually been considered for inclusion. They were finally left out because they would have represented little more than bald bridgeheads pointing into fields outside the scope of the present work. I see their logical place at the beginning of higher-level texts in which they are then also put to extensive illustrative use.

An oddity I recognize as such is Chapter V. In contrast to the rest of the book, which labors to bring out as vividly and straightforwardly as possible the concrete sense of every idea, this Chapter, in its presentation of the Stream Function, deliberately disregards the possibility of a direct physical approach to the Function, and leads to it in a roundabout mathematical way instead. This procedure has grown out of the experience that the Stream Function, with its associated concepts entering into Gauss' Theorem, is quickly understood and accepted by the student, whereas the companion function, the Velocity Potential, and the related concepts underlying Stokes' Theorem, lack all natural appeal and tend to remain unfamiliar and unappreciated. The plan adopted is to arrange the development of the Stream Function in steps corresponding exactly to those in the subsequent development of the Velocity Potential, so as to provide a continual parallelism between two evolutions of which the first, supported by solid commonsense at every turn, then helps to pull through the second, which hangs precariously on abstract logic. In short, the self-propelling group of ideas about the Stream Function is harnessed as a tractor for the dragging group about the Velocity Potential. Anyone who doubts the worth of the scheme should try it before he laughs it off.

As I have already stated, the inspiration for the book goes back to Professor E. B. Wilson. My indebtedness to other authors, and to my own professors, colleagues, and students, is great, but difficult to detail. Among the authors, H. Glauert and E. H. Barton, both listed as references at the end of the book, deserve to be singled out for mention: the reader familiar with Glauert will

detect his strong influence in portions of Chapters VI, VIII, and IX, while Barton's lead can be traced clearly through parts of Chapters XI and XII. To Professor A. Betz of Goettingen belongs the credit for the simple approach to Biot-Savart's Law presented in Section 9.3. Professor C. L. E. Moore, who led the recitation course based on the Wilson text when I took it thirty years ago, was the patron of my subsequent teaching of the course, and so ultimately of this book. I acknowledge my debt to him in deepest thankfulness. I want to thank also Professors E. P. Warner, C. F. Tayler, and J. C. Hunsaker, who, as successive Heads of the Department of Aeronautical Engineering at the Massachusetts Institute of Technology in my time, encouraged and supported my work. With gratitude I would further mention the ever benevolent interest of Professor R. H. Smith, for many years my superior as Executive Officer of the Department. My young colleagues and successors, Professors Holt Ashley and R. L. Halfman, both helped to speed the completion of the book toward the end of my stay at the Institute. The relentless, friendly pushing of Professor Ashley in particular was what really got the manuscript finished and put into the hands of the publishers. I cannot thank him enough for his faithful devotion to the cause. The Figures are mostly the work of Mr. Leonard Glancy. I appreciate the love and skill with which he did them. For the typing of the manuscript and the handling of various chores in its assembly, I am indebted especially to Miss Dorothy Howe and Miss Rose Marie Pratt, and to Mr. G. P. Haviland. Many others, not identified here by name, have given me valuable assistance and are assured of my gratitude. Among them, I would especially mention my students, whose eyeopening reactions, both sharp and obtuse, are responsible in large measure for the way in which I am now presenting Aeronautical Dynamics to a wider circle of young engineers.

MANFRED RAUSCHER.

Zürich, July 6, 1953.

CONTENTS

VII TRANSFORMATIONS

VIII AIRFOILS

IX AIRFOILS IN THREE DIMENSIONS

X VISCOSITY

XI KINEMATICS OF RIGID BODIES

XII Dynamics of Rigid Bodies

XIII Oscillations of Systems with One Degree of Freedom

PRELIMINARY

NEWTON'S LAW AND THE CONSERVATION OF MATTER

0·1. Dynamics. *Dynamics* deals with the forces involved in the movements of bodies. The emphasis of the word lies simply on force (Greek: δυναμις). An alternative term for dynamics, emphasizing more the part of movement (Greek: κινησις), is *Kinetics*. Movement dissociated from all considerations of force is the concern of the subsidiary science of *Kinematics*.

0·2. Newton's Law. The basis of dynamics is Newton's Law: "A force produces momentum in the direction of the force, and at a rate proportional to the force."

Momentum, or "quantity of motion", is defined as the product of mass (quantity of matter, m) and velocity (rate of displacement, v). Thus, in mathematical terms, if F signifies the force, t the time, and s the displacement,

$$F \sim \frac{d}{dt}(mv), \tag{0.1}$$

or

$$F = k\frac{d}{dt}(mv) = k\frac{d}{dt}\left(m\frac{ds}{dt}\right),$$

k denoting a constant. The Equation simplifies into

$$F = km\frac{dv}{dt} = km\frac{d^2s}{dt^2}$$

when the mass of the body is constant, and further into

$$F = m\frac{dv}{dt} = m\frac{d^2s}{dt^2} \tag{0.2}$$

if the units of F, m, s, and t are so chosen that a unit force acting on a body of unit mass produces unit acceleration (rate of change of velocity).

0·3. Units. The relation between the various units implied in the last equation may be determined from the acceleration imparted to a given mass by a specific force. The force most conveniently used is the force of gravity—the weight. Thus, if m's acceleration under its weight W is g,

$$W = mg, \quad \text{or} \quad m = \frac{W}{g}.$$

Now, the meaning of W is "number of units of force" making up the weight of the body; and m stands for "number of units of mass" contained in the body. The Equation thus shows that the number of weight units for any body must be g times the number of mass units—so that, numerically, the weight of the body is g times the body's mass, or the mass $1/g$ times the body's weight.

The established unit of weight is the pound (lb); and accelerations are expressed in terms of the standard units of length and time, feet (ft) and second (sec), as feet per second per second. The unit of mass that fits into this "Foot-Pound-Second System" is the slug (sl). With $g = 32.2$ ft/sec^2, a slug is represented by an amount of matter weighing 32.2 lb.

0·4. Mass and Force. The definition of mass as "quantity of matter" is the one offered by Newton. Through it, force becomes a derived quantity: namely, that which is capable of changing the velocity of mass. From the point of view of the engineer, force is perhaps nearer to being a fundamental concept than mass, as it corresponds directly to one's sense of muscular exertion—in the stretching of a spring, or the lifting of a weight—while mass is noted but indirectly in the sluggishness, or inertia, with which physical objects respond to the action of forces. Seen from this angle, mass is then simply the factor connecting force and acceleration—its numerical value for any given body being the ratio between the magnitudes of those quantities.

0·5. The Conservation of Matter. Intimately associated with Newton's Law is the fundamental axiom of the Conservation of Matter. This principle roots so directly in elementary physical sense that allowance for it tends to be made unconsciously, and so without explicit mention. Formal recognition of the continued existence of all matter becomes, however, necessary in the application of Newton's Law to bodies of variable mass. Any mass gained or lost by a body must be understood always to be so much matter picked up or dropped—never matter created or destroyed—and the term "quantity of motion" in Newton's Law must accordingly be understood to include not only the momentum of the body proper, but also the momenta of any acquired or discarded masses before and after their attachment to the body. Also, in the study of the flow of fluids, the knowledge that matter cannot appear or disappear is the basic means of correlating the movements of the individual mass elements in an aggregate whose different portions merge into each other without visible demarcation. The Conservation of Matter thus is a principle that continually supplements Newton's Law in the shaping of mechanical events.

CHAPTER I

KINEMATICS OF A POINT

1·1. Displacement, Velocity, and Acceleration. Path and Hodograph. Before it is possible to proceed to the calculation of the forces involved in the motions of bodies, methods must be developed to deal with motion as such. This requires a general study of displacements, velocities, and accelerations.

Displacements in space are characterized by magnitude, direction, and sense. They can be compounded and resolved according to the parallelogram rule, and by this property are defined as vectors.

From the fact that displacements are vectors, it follows that *velocities* are vectors also. To see this, consider two velocities v_1 and v_2, affecting the same point. In a time dt the point is displaced a distance $ds_1 = v_1\,dt$ in the direction of v_1 and a distance $ds_2 = v_2\,dt$ in the direction of v_2. As these displacements may be combined by the parallelogram law, the resultant displacement ds_3 comes out just the same as if v_1 and v_2 had been replaced by their vector sum v_3 in the first place (Fig. 1.1).

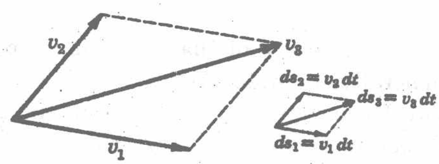


FIG. 1.1. Vector composition of velocities.

Similarly, it can be seen that *accelerations* combine by the parallelogram rule and hence are vectors: the resultant effect of two accelerations a_1 and a_2 in a time dt, being the vector sum of two velocities $a_1\,dt$ and $a_2\,dt$, is exactly that corresponding to a single acceleration a_3 obtained by vectorial addition of a_1 and a_2.

Infinitesimal changes in the displacement and velocity vectors from instant to instant are the basic elements with which the study of an evolving motion is concerned. The successive elementary displacements $v\,dt$ during successive elementary intervals dt, added together in sequence, yield the *path* along which the motion proceeds from an initial displacement s_0. As each arc $v\,dt$ has the direction of the instantaneous velocity v that carries a point along the arc, the velocity vector is everywhere tangent to the path. The velocity is, then, fixed by this direction and by its magnitude, which is the rate at which the point advances along the path, or, in other words, moves along the locus of the heads of the successive displacement vectors s laid off from a common origin.

For a study of the changes in the velocity vectors between successive instants, and hence a determination of the accelerations involved in the motion, it is helpful to construct a supplementary diagram in which the successive velocity vectors are brought back to a common base point for comparison with each other (Fig. 1.2). Each of these vectors evolves from its predecessor through the accretion, during an interval dt, of an elementary vector $a\,dt$ pointing in the direction of the momentary acceleration a. Setting out from the head of the initial velocity vector v_0, the velocity vector head, carried along by the successive changes, thus traces a curve called the *hodograph* (Greek: ὁδός = way) of the motion. The advance of the vector head along this curve is always in the direction of the instantaneous acceleration a, as already noted, and proceeds at the rate $(a\ dt)/dt = a$. The velocity of the velocity vector head along the hodograph thus represents, in both magnitude and direction, the acceleration of the motion from instant to instant.

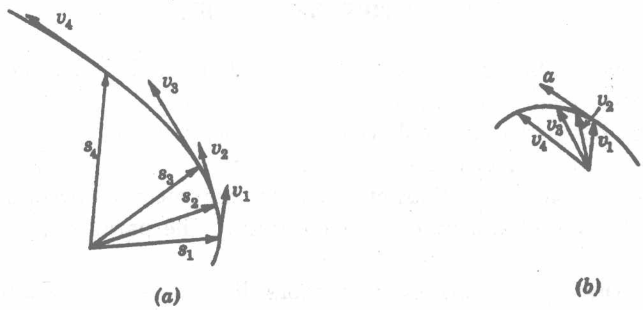


FIG. 1.2. Hodographs showing: (a) Velocities, and (b) Accelerations.

In the study of diagrams like the ones just discussed, and of vector problems generally, it often proves convenient to view a vector change as a combination of two basic types of changes—namely, a stretching or contracting of the vector in the direction in which the vector is pointing at the instant, and a swinging around of the head of the vector with respect to the vector's tail in a direction perpendicular to the vector (Fig. 1.3). When, for example the head of a displacement vector s is moving along at a velocity v, this velocity can be resolved into components v_a and v_b. The first of these components goes into stretching the vector at the rate v_a. The second

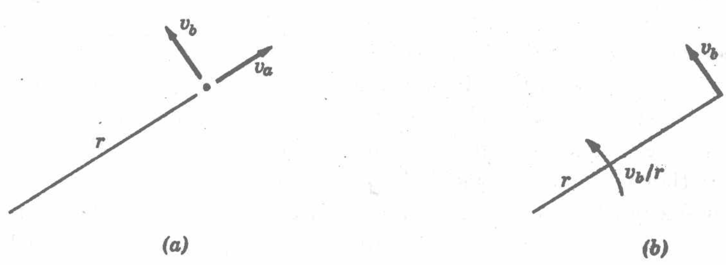
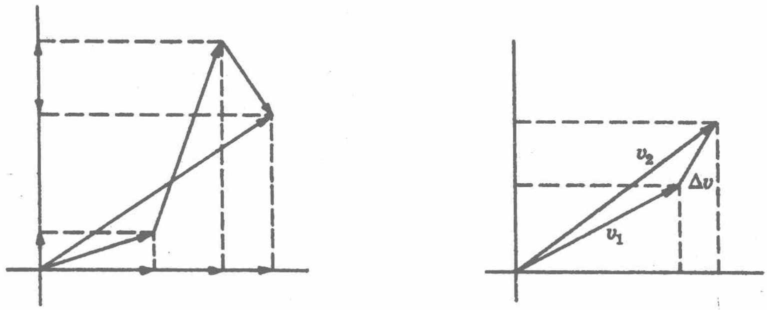


FIG. 1.3. Velocity vector diagram showing: (a) Positions of velocity components v_a and v_b, and (b) Effect of v_b on angular motion of r.

component, carrying the vector head sidewise at the rate v_b, swings the vector around at the angular rate v_b/r, where r is the magnitude of s. The effect of v_a is seen to be independent of v_b, and the effect of v_b independent of v_a; i.e., the stretching or contracting and the swinging around of the vector contribute nothing to each other, and so can be taken into account separately, and their effects simply added. This possibility of considering any vector change as the sum of two simple basic changes enables one to visualize what goes on in many situations that would otherwise be difficult to see through.

1·2. Reference Systems. The motion of a body is completely determined if the positions of all the points of the body are known at every instant. To define these positions, it is necessary to establish a base point, or *origin*, from which displacements can be measured, and a set of reference directions, or *axes*, with respect to which it is possible to judge the displacements' orientations. The analysis is then carried through most conveniently by a resolution of the displacements, velocities, and accelerations



(*a*) (*b*)

FIG. 1.4. Vector Projection: (*a*) Sum of projections of components = Projection of resultant, (*b*) Projection of change = Change in projection.

into components in the coordinate directions at every point. Three such directions will generally be needed.

Vector resolution becomes simple orthogonal projection when the coordinate directions are mutually perpendicular, as they will be taken to be throughout this work. Because the projection of a vector on any axis is equal to the sum of the projections of the components of the vector on that axis (Fig. 1.4*a*), the velocity component in any direction can be found as the sum of the projections of the vector rates of change of the component displacements, and the acceleration component in any direction can be found as the sum of the projections of the vector rates of change of the component velocities. Part of the same observation is that the projection of the change in a vector on any axis is equal to the change in the projection of the vector on that axis (Fig. 1.4*b*). These basic facts will be utilized in the determination of the velocities and accelerations in various reference systems in the following Sections.

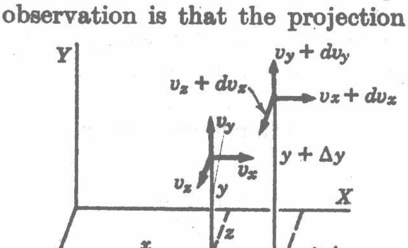


FIG. 1.5. Cartesian coordinates: Velocity components of a point at two instants dt apart.

1·3. Cartesian Coordinates. In the *Cartesian* system of coordinates, a point is determined by three lengths—the components in the coordinate directions of the displacement vector from the origin to the point. The axes of the reference frame are normally labeled X, Y, Z, and the coordinates of any point in the frame are correspondingly denoted by x, y, z (Fig. 1.5). The velocity components in the coordinate directions will be designated v_x, v_y, v_z, and the acceleration components a_x, a_y, a_z.

If, in a time dt, a point moves from a position $P(x, y, z)$ to a position $P'(x + dx, y + dy, z + dz)$ nearby, its velocities parallel to the three axes are

$$\left.\begin{aligned} v_x &= \frac{\text{Change, in time } dt\text{, in projection of resultant displacement vector upon } OX}{dt} \\ &= \frac{(x + dx) - x}{dt} \\ &= \frac{dx}{dt}, \\ \text{and} \quad v_y &= \frac{dy}{dt}, \\ v_z &= \frac{dz}{dt}. \end{aligned}\right\} \quad (1.1)$$

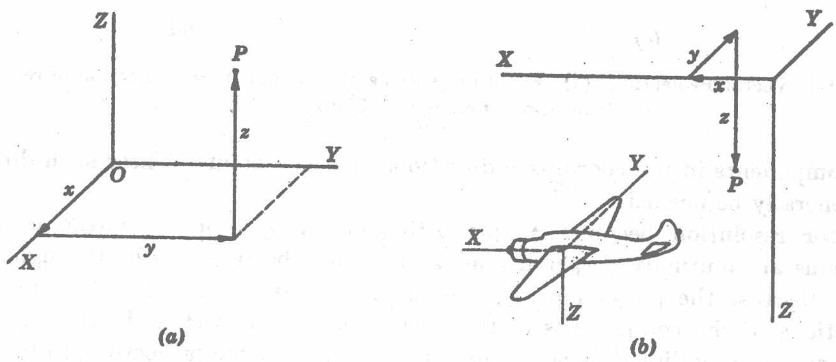


FIG. 1.6. Cartesian coordinates: Right-handed systems used in: (a) Mechanics, and (b) Aeronautics.

Also, with v_x, v_y, v_z changing into $v_x + dv_x$, $v_y + dv_y$, $v_z + dv_z$ between one instant and the next, the accelerations in the three coordinate directions are

$$\left.\begin{aligned} a_x &= \frac{\text{Change, in time } dt\text{, in projection of resultant velocity vector upon } OX}{dt} \\ &= \frac{(v_x + dv_x) - v_x}{dt} \\ &= \frac{dv_x}{dt} = \frac{d^2x}{dt^2}, \\ \text{and} \quad a_y &= \frac{dv_y}{dt} = \frac{d^2y}{dt^2}, \\ a_z &= \frac{dv_z}{dt} = \frac{d^2z}{dt^2}. \end{aligned}\right\} \quad (1.2)$$