STATISTICS

A Guide to the Use of Statistical Methods in the Physical Sciences

Roger Barlow

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Editors' Preface to the Manchester Physics Series

The first book in the Manchester Physics Series was published in 1970, and other titles have been added since, with total sales world-wide of more than a quarter of a million copies in English language editions and in translation. We have been extremely encouraged by the response of readers, both colleagues and students. The books have been reprinted many times, and some of our titles have been rewritten as new editions in order to take into account feedback received from readers and to reflect the changing style and needs of undergraduate courses.

The Manchester Physics Series is a series of textbooks at undergraduate level. It grew out of our experience at Manchester University Physics Department, widely shared elsewhere, that many textbooks contain much more material than can be accommodated in a typical undergraduate course and that this material is only rarely so arranged as to allow the definition of a shorter self-contained course. In planning these books, we have had two objects. One was to produce short books: so that lecturers should find them attractive for undergraduate courses; so that students should not be frightened off by their encyclopaedic size or their price. To achieve this, we have been very selective in the choice of topics, with the emphasis on the basic physics together with some instructive, stimulating and useful applications. Our second aim was to produce books which allow courses of different length and difficulty to be selected, with emphasis on different applications. To achieve such flexibility we have encouraged authors to use flow diagrams showing the logical connections between different chapters and to put some topics in starred sections. These cover more advanced and alternative material which is not required for the understanding of later parts of each volume. Although these books were conceived as a series, each of them is self-contained and can be used independently of the others. Several of them are suitable for wider use in other sciences. Each author's preface gives details about the level, prerequisites, etc., of his volume.

We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, whose helpful criticisms and stimulating comments have led to many improvements. Our particular thanks go to the authors for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and often accepting, our many suggestions and requests. We would also like to thank the publishers, John Wiley & Sons, who have been most helpful.

F. Mandl
R. J. Ellison
January, 1987 D. J. Sandiford

The generall end therefore of all the book is to fashion
a noble person in vertuous and gentle discipline

—*Edmund Spencer*

Author's Preface

Many science students acquire a distinctly negative attitude towards the subject of statistics. The reasons for this are clear. The traditional first year concentrated statistics course of derivations and exhortations makes little impact on the young undergraduates, who want to get to grips with the basic truths of their chosen subject and have no interest in sordid details like error bars. The hapless students then go to laboratory classes, in which their enjoyment of the experiments is marred by the awful chore of the 'error analysis' at the end, where, whatever they do, they inevitably get harshly criticised for doing it wrong. Under such circumstances, 'statistics' can soon become a collection of meaningless ritual, to be gone through correctly if harsh words and bad marks are to be avoided.

As a student I was no different from any other in this respect. But later, in the real world, doing real experiments, statistics began to matter. Over the years I got to grips with the subject, by talking to colleagues and digging in reference books, and was agreeably surprised to discover that it had an internal logic and structure. Once one really got into it, it made sense. Eventually the time came when people started asking me questions, and I somehow acquired a reputation as the local statistics expert. On this basis I devised a course, which was given as a set of lectures to students at

Manchester University. This has convinced me that statistics can be taught to students in such a way as to make it interesting for them, and give them a real grasp of the subject.

This book has grown out of the lecture notes given out with the course. Despite the shelves full of books on 'statistics' in any library or university bookshop, there is a desperate lack of any suitable textbook for the physical sciences beyond the very elementary level. The books available are mainly aimed at the biological and social sciences; for those of us in other fields they are inappropriate, both in content and treatment. They deal largely with samples and surveys, and the problems of hypothesis testing, whereas we are more concerned with the theory of measurements and errors, and with the problem of estimation. Furthermore they assume, usually correctly, that those for whom they are intended (geographers, psychologists, and suchlike) will fear and loathe anything at all mathematical. They therefore avoid anything beyond (or even, in some cases, including) the most elementary algebra. Now, although physicists and chemists may fight shy of high-powered abstract mathematics, they can happily differentiate and integrate simple functions and follow basic algebra. They are thus entitled to a reasonable explanation of the mathematics involved in statistical calculations, and able to benefit from it. This book thus assumes a reasonable degree of numeracy from the reader, but nothing outstanding—any real mathematician will find it hopelessly naive and unrigorous.

This book is thus the textbook I would like to have had available, both as a student and when teaching students, and for my own use with real problems. I hope that others will find it useful and interesting, and that it will eventually lead them not only to use and understand statistics, but to enjoy it.

I would like to record my acknowledgements to the many people who, by discussions and advice, have helped form my ideas on the subject, to the students on my course for acting as guinea-pigs for the material, to John Ellison for many helpful comments in preparing the manuscript for publication, and finally to my wife Ann for putting up with the trials of a traumatic author with patience and understanding.

ROGER BARLOW
Manchester

4 October 1988

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'It's not the figures themselves,' she said finally, 'it's what you do with them that matters'

—K.A.C. Manderville

CHAPTER 1

Using Statistics

Statistics is a tool. In experimental science you plan and carry out experiments, and then analyse and interpret the results. To do this you use statistical arguments and calculations. Like any other tool—an oscilloscope, for example, or a spectrometer, or even a humble spanner—you can use it delicately or clumsily, skilfully or ineptly. The more you know about it and understand how it works, the better you will be able to use it and the more useful it will be.

The fundamental laws of classical science do not deal with statistics or errors. Newton's law of gravitation, for example, reads

$$F = \frac{GMm}{r^2}$$

in pure and beautiful simplicity. The figure in the denominator is given as 2—exactly 2, not 2.000 ± 0.012 or anything messy like that. This can lead people to the idea that statistics has nothing to do with 'real' scientific knowledge.

But where do the laws come from? Newton's justification came from the many detailed and accurate astronomical observations of Tycho Brahe and others. Likewise Ohm's law

$$V = IR$$

which appears so straightforward and elementary to us today, was based on Ohm's many careful measurements with primitive apparatus. When you are *studying* science you may find no use for statistics—until you meet quantum mechanics, but that is another story—but as soon as you begin *doing* science, and want to know what measurements really mean, it becomes a matter of vital importance.

This is a textbook on statistics for the physical sciences. It treats the subject from the basic level up to a point where it can be usefully applied in analysing real experiments. It aims to cover most situations that are likely to be met with, and also provide a grasp of statistical ideas, terminology, and language, so that more advanced works can be consulted and understood should the need arise. It is thus intended to be usable both as a textbook for students taking a course in the subject, and also as a handbook and reference manual for research workers and others when they need statistical tools to extract their experimental results.

These two modes of use give rise to requirements in the ordering of the material which are not always happily reconcilable. For reference use one wants to group all material on a given topic together, but for teaching purposes this would be like learning a language from a dictionary. The solution adopted is that the unstarred sections cover the material roughly appropriate to a first year undergraduate course. They can sensibly be taken in order, with no anticipation of later material. The starred sections fill in the gaps; they may require knowledge of material in later sections, but when this occurs it is explicitly pointed out. Most of the basic material is in the early chapters, and Chapters 7, 9, and 10 contain entirely higher-level material. First-time-through readers should not be scared or put off by any apparent mathematical complexity they observe in some of the starred sections: these can (and should) be skipped over with a clean conscience, as they are not needed for later unstarred sections of the course.

'Data! Data! Data!', he cried impatiently.
'I can't make bricks without clay'.

—Sir Arthur Conan Doyle

CHAPTER 2

Describing the Data

It all starts with the data. You may call them a *set of results*, or a *sample* or the *events*, but whatever the name, they consist of a set of basic measurements from which you're trying to extract some meaningful information.

To make your data mean something, particularly to an outside audience, you need to display them pictorially, or to extract one or two important numbers. There are many such numbers and ways of presenting the data in graphic form, and this chapter is devoted to methods of describing the data in a useful and meaningful way, without attempting any deeper analysis or inference. This is known as *descriptive statistics.*

2.1 TYPES OF DATA

Data† are called *quantitative* or *numeric* if they can be written down as numbers, and *qualitative* or *non-numeric* if they cannot. Qualitative data are

†Note for pedants: 'data' is a plural noun. Thus one should say 'The data fit...', 'Data were observed...' rather than 'The data fits...', 'Data was observed...'. The singular, never used, is 'datum'.

rather hard to work with as they do not offer much scope for mathematical treatment, so most of the subject of statistics, and likewise most of this book, deals with quantitative, numerical measurements.

Quantitative measurements divide further into two types. Some, by their very nature, have to be integers and these are called *discrete* data. Others are not constrained in this way and their values are real numbers. These are called *continuous* data. Continuous data cannot be recorded exactly, as you cannot write down an infinite number of decimal places. Some sort of *rounding* and loss of precision has to occur.

For example, if you were to examine a sample of motor cars and record their colours, these would be qualitative data. The number of seats in each car has to be an integer, and would be discrete numeric data, as would the number of wheels. The lengths and the weights of the cars would be continuous numeric data.

Usually one of the first things to do in making sense of the data (which is just a pile of raw results) is to divide them into *bins* (also called *groups* or *classes* or *blocks*). For example, the results of tossing 20 coins, each of which comes down either heads (H) or tails (T)

$$\{H, T, H, H, T, H, T, H, H, H, T, T, H, T, T, H, T, H, H, T\}$$

can be written as $\{11H, 9T\}$. This conveys the same information much more clearly and concisely.

For continuous numeric data it is not quite so simple, as your values are (almost certainly) all different, if you use enough decimal places. You have to group together adjacent numbers, using a range of values to define each bin. This means further rounding of values and throwing away precision information, which is the price you pay for rendering the data comprehensible. Usually the bins are chosen to be all the same uniform size, but in some cases it makes sense to use non-uniform bins of different sizes.

For discrete numeric data this grouping together of adjacent values is not compulsory, but it may be desirable when the numbers of data points with any particular value are small.

2.2 BAR CHARTS AND HISTOGRAMS

The numbers of events in the bins can be used to draw bar charts (see Figure 2.1) and histograms.

There is a technical difference between a bar chart and a histogram in that the number represented is proportional to the *length* of bar in the former and the *area* in the latter. This matters if non-uniform binning is used. Bar charts can be used for qualitative or quantitative data, whereas histograms can only be used for quantitative data, as no meaning can be attached to the width of the bins if the data are qualitative.

For quantitative, numeric, data, you have to choose the width of the bins

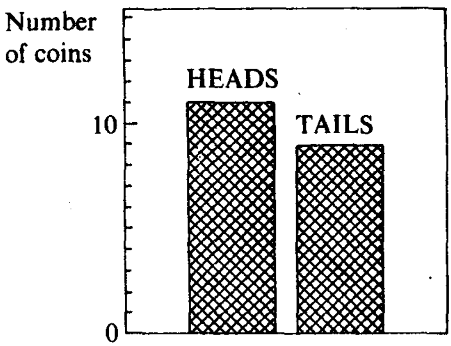

Fig. 2.1. A bar chart displaying the data in the previous section.

to be used in the display (see Figures 2.2). This requires thought. If the bins chosen are too narrow, then there are very few events in each bin, and the numbers are dominated by fluctuations. Ideally there should be at least ten events in each bin, and the more the better. If they are too wide, then real detail can be obscured if the bin stretches over genuine variations in the distribution. Ideally, the difference between contents of adjacent bins should be small. The choice is yours—it is a matter of personal judgement. It may well be, particularly if the number of events is small, that there is no way of satisfying both ideal requirements. In this case you just have to do the best you can with the data available.

There are other ways of representing the data using pictures: ideographs, frequency polygons, pie charts, prismograms, scatter plots, and many more. However, it is not necessary to give you all the details. They are designed to be straightforward to understand, and are therefore straightforward to use. Some people become very excited about 'right' and 'wrong' ways of doing things, and come almost to blows over whether gaps between bars in a bar

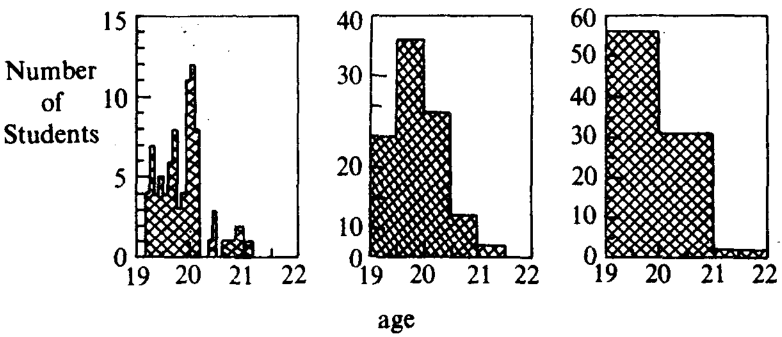

Fig. 2.2 The ages (in years) of a group of second year students, showing the effects of choosing different bin sizes for the same data.