DICTIONARY OF LOGICAL TERMS AND SYMBOLS

Carol Horn Greenstein

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TO MY PARENTS

PREFACE

The primary objective of the *Dictionary of Logical Terms and Symbols* is to present compactly, concisely, and side by side a variety of alternative notational systems currently used by logicians, computer scientists, and engineers. Familiarity with one or two notational systems, to the exclusion of others, often makes it difficult and time consuming to read through new material. A simple connective that can be expressed symbolically in more than a dozen different ways can, in addition, be translated into English in almost as many. This dictionary is designed to greatly reduce these difficulties by presenting those alternative notational systems most commonly used, in conjunction with many of the ways in which they can be rendered into English. Thus, translating from Polish notation into Peano-Russell or Hilbert notation, or from set theoretic notation into Boolean notation, or from Boolean notation into Peano-Russell notation, or from English into any of these systems, can be done at a glance by using the tables in this dictionary.

In the first chapter, I've represented those systems of connectives, symbolic forms, and expressions that I've found to occur most often in the literature, along with the ways they are most frequently translated into English. If one is familiar with any of the ways in which, for example, the notions of conjunction or material implication can be formalized, then other ways of formalization can be found at once by using the table of contents to find the page on which they are listed. Again, by using the table of contents, these same connectives can be found represented in logical gate notation and in both truth-functional and binary truth tables. More specific systems of alternative symbolic forms are to be found in the separate sections on quantification theory, set theory, Boolean algebra, two-termed relations, and epistemic, doxastic, deontic, and tense logics. Using as a criterion the frequency of occurrence in the literature, I've included valid inferential argument forms and equivalent forms in alternative notation as well. Because punctuation is such an important part of correct symbolic expression, there is a separate section on the alternative methods for punctuating formal expressions.

Often, computer and logical terminology is not expressed in symbolic form, but rather is abbreviated in some way or another and then included

along with symbolic expressions. The chapter on abbreviations contains a list of these abbreviations and acronyms along with their unabbreviated counterparts. Again, using as a criterion the frequency of occurrence in the literature, I've presented those abbreviations and acronyms most often found in current works.

The glossary has two functions. First, it contains a list of technical terms and expressions most likely to be encountered in currently studied works, along with the ways in which they are most commonly characterized, used, or defined. Where there is more than one entry for a given term, the first is not always the main sense of the term. It is not necessarily better than or prior to another entry. Rather, multiple entries reflect alternative or additional senses of that term that occur with some frequency. I have followed current practice for the most part when using some terms interchangeably. *"Traditional logic," "Aristotelian logic,"* and *"syllogistic logic"* are used in this manner. Where the reader is referred to another term or expression, or where there is only a page number after a term or expression, the reference is to another term that can be used in its place, and a definition will be found under that entry. The glossary's second function is to reduce the difficulty in formalizing key English expressions in alternative ways. Those terms and expressions that have a symbolic counterpart are cross-referenced with the first half of the book. The page numbers occurring at the end of an entry in this case refer to the pages on which those terms are expressed in alternative symbolic forms, or to the logical gate or truth table representations of those terms.

The bibliography, although not meant to be exhaustive, includes those works in which all the notational forms I've represented have first been introduced. For a more extensive treatment of the several systems of notation and a more comprehensive account of the terms and expressions listed in the glossary, I refer the reader to the bibliography.

I would like to thank Alberta Gordon of Van Nostrand Reinhold for her help in putting this book into readable form and Professor Joseph Gilbert of State University of New York College at Brockport for his help in reading proof.

I am most grateful to my husband, Professor Harold Greenstein, for his encouragement in preparing this book.

C.H.G.

CONTENTS

ALTERNATIVE NOTATIONAL FORMS

1. 'p' and 'q' represent propositions.

2. 'ϕ' represents any predicate.

3. 'ϕx' represents a propositional function.

4. Alternative notational forms.

a.	**English**	Bar. Curl. Dagger. Denial. It is absurd that. It is not the case that. Negation. Not. Prime. Stroke. Tilde.
	Peano-Russell Notation	$\backsim p$
	Hilbert Notation	$\overline{p}$
	Polish Notation	Np
	Set Theory Notation	$-p$ $\overline{p}$ $\backsim p$
	Boolean Notation	$\overline{p}$ p'
	Variants	$\neg p$ $-p$ $\urcorner p$ $p\|p$ $p \downarrow p$ $p \uparrow p$
b.	**English**	Ampersand. And. Cap. Conjunction. Dagger. Dot. Hump. Intersection. Logical product. Multiplication. Stroke.
	Peano-Russell Notation	$p \cdot q$
	Hilbert Notation	p & q
	Polish Notation	Kpq
	Set Theory Notation	$p \cap q$ pq $p \times q$ $p \cdot q$

Boolean Notation $p \cdot q \quad p \times q \quad p \cap q$

Variants $p \wedge q \quad p \,.\, q \quad p \frown q$
$(p)(q) \quad p|q.|.p|q$
$p \downarrow p. \downarrow .q \downarrow q \quad \backsim (p|q)$

c. **English** Addition. Adjunction. Alternation. Cup. Dagger. Dish. Disjunction. Inclusive disjunction. Join. Logical sum. Stroke. Union. Vee. Vel. Wedge.

Peano-Russell Notation $p \vee q$

Hilbert Notation $p \vee q$

Polish Notation Apq

Set Theory Notation $p + q \quad p \vee q \quad p \cup q$

Boolean Notation $p + q \quad p \cup q$

Variants $pq \quad p|p.|.q|q \quad \backsim p|\backsim q$
$p \downarrow q. \downarrow .p \downarrow q$

d. **English** Aut. Circle sum. Dagger. Exclusive disjunction. Or. Stroke.

Peano-Russell Notation

Hilbert Notation $\overline{p \backsim q}$

Polish Notation Jpq Rpq

Set Theory Notation

Boolean Notation

Variants $p \wedge q \quad p \equiv q \quad p \backsim q$
$(p \vee q) \cdot \backsim (p \cdot q)$
$p|q \quad p\blacksquare q \quad p/q$
$p \veebar q \quad p \oplus q \quad p + q$

e. **English** Conditional. Dagger. Horseshoe. Hypothetical. If . . . then . . . .

Implication. Material implication. Stroke.

Peano-Russell Notation $p \supset q$

Hilbert Notation $p \rightarrow q$

Polish Notation Cpq

Set Theory Notation $p \rightarrow q$

Boolean Notation

Variants $p \rightharpoonup q$ $p \succ q$ $p|\backsim q$
$p \sqsubset q$ $p.|.q|q$
$p \downarrow p. \downarrow .q: \downarrow :p. \downarrow .p \downarrow q$

f. **English** Biconditional. Equivalence. If and only if. Iff. Material equivalence. Triple bar.

Peano-Russell Notation $p \equiv q$

Hilbert Notation $p \backsim q$ $p \rightleftarrows q$

Polish Notation Epq

Set Theory Notation $p \longleftrightarrow q$

Boolean Notation $p = q$

Variants $p \asymp q$ $p \succ\!\prec q$

g. **English** For any x. Given any x. Universal quantification.

Peano-Russell Notation (x)F(x)

Hilbert Notation (x)F(x)

Polish Notation $\Pi x \phi x$

Set Theory Notation $\forall x F(x)$

Boolean Notation

Variants $\cap x$ $\backsim (\exists x) \backsim x$
$\wedge x F(x)$

ı.	**English**	Existential quantification. There exists an x. There is at least one x.
	Peano-Russell Notation	$(\exists x)F(x)$
	Hilbert Notation	$(Ex)F(x)$
	Polish Notation	$\Sigma x \phi x$
	Set Theory Notation	$\exists xF(x)$
	Boolean Notation	
	Variants	$\cup x \quad \backsim (x) \backsim F(x)$ $\vee xF(x)$
i.	**English**	Box. Diamond. It is possible that. It is not necessary that not. It is not necessary that . . . is false.
	Standard Notation	$\Diamond p \quad \backsim \Box \backsim p$
	Polish Notation	$Mp \quad \Delta p \quad N\Gamma Np$
	Variants	$-\Box -p \quad \neg \Box \neg p$
j.	**English**	Box. Diamond. It is necessarily false that. It is necessary that not. It is impossible that. It is not possible that.
	Standard Notation	$\backsim \Diamond p \quad \Box \backsim p$
	Polish Notation	$NMp \quad N\Delta p \quad \Gamma Np$
	Variants	$-\Diamond p \quad \neg \Diamond p \quad \Phi p$ $\Box -p \quad \Box \neg p$
k.	**English**	Box. Diamond. It is not necessary that. It is possibly false that. It is possible that not.
	Standard Notation	$\Diamond \backsim p \quad \backsim \Box p$

Polish Notation MNp △Np NΓp

Variants ◇−p ◇¬p ◫p
−□p ¬□p

l. **English** Box. Diamond. It is necessary that. It is not possible that not. It is not possible that . . . is false.

Standard Notation □p ∽◇∽p

Polish Notation NMNp N△Np Γp Lp

Variants #p −◇−p ¬◇¬p

m. **English** Inverse material implication. p if q.

Symbolic Form p ⊂ q p ← q Bpq

n. **English** Nand. Nonconjunction. Not both p and q. Stroke.

Symbolic Form p|q −(p × q) −(pq)
$\overline{-(p \wedge q)}$ ∽(p · q) (pq)′
$\overline{(p \times q)}$ Dpq

o. **English** Arrow. Dagger. Neither p nor q. Nor.

Symbolic Form p ↓ q p ⊽ q $\overline{-(p \vee q)}$
∽(p ∨ q) $\overline{(p + q)}$
(p ∨ q)′ Xpq

p. **English** p but not q.

Symbolic Form p ↄ q $\overline{-(p \to q)}$
∽(p ⊃ q) $\overline{(p \to q)}$ Lpq

q. **English** q but not p.

Symbolic Form p ⊄ q $\overline{-(p \leftarrow q)}$
∽(p ⊂ q) $\overline{(p \leftarrow q)}$ Mpq