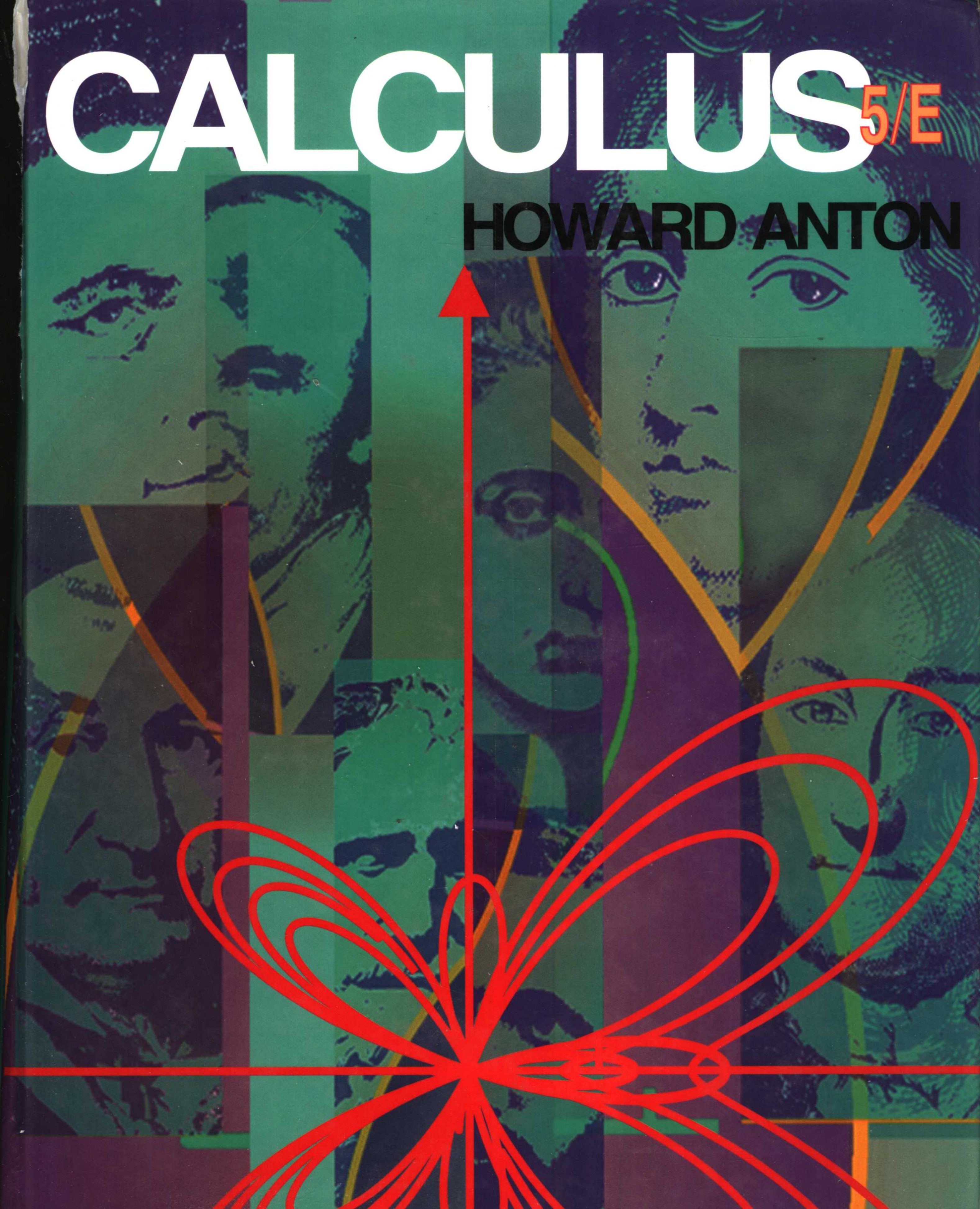
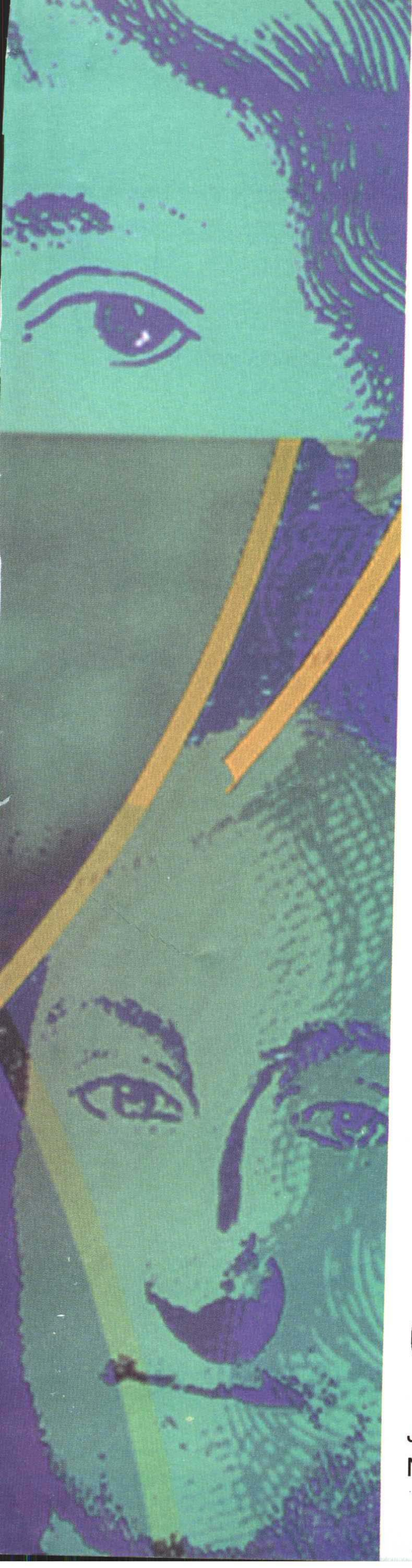
CALCULUS 5/E

HOWARD ANTON





Calculus
with Analytic Geometry

FIFTH EDITION

HOWARD ANTON
Drexel University

in collaboration with
ALBERT HERR, Drexel University

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About Howard Anton

Howard Anton obtained his B.A. from Lehigh University, his M.A. from the University of Illinois, and his Ph.D. from the Polytechnic Institute of Brooklyn, all in mathematics. In the early 1960's he worked for Burroughs Corporation and Avco Corporation at Cape Canaveral, Florida, where he was involved with missile tracking problems for the manned space program. In 1968 he joined the Mathematics Department at Drexel University, where he taught full time until 1983. Since that time he has been an adjunct professor at Drexel and has devoted the majority of his time to textbook writing and activities for mathematical associations. Dr. Anton was President of the EPADEL Section of the Mathematical Association of America (MAA), served on the Board of Governors of that organization, and guided the creation of the Student Chapters of the MAA.

He has published numerous research papers in Functional Analysis, Approximation Theory, and Topology, as well as pedagogical papers on applications of mathematics. He is best known for his textbooks in mathematics, which are among the most widely used in the world. There are currently more than eighty versions of his books including translations into Spanish, Arabic, Portuguese, Italian, Indonesian, French, Japanese, Chinese, and German.

Dr. Anton has an avid interest in computer technology as it relates to mathematical education and publishing. He has developed pedagogical software for teaching linear algebra as well as various software programs for the publishing industry that automate the production of four-color mathematical text and art. For relaxation he enjoys traveling and photography.

About Albert Herr

It is my sad duty to report that Albert J. Herr, my colleague, collaborator, and friend passed away shortly after the first printing of this book appeared. He will be missed greatly by all of those who had the good fortune to work with him.

Al held degrees in Electrical Engineering and Biomedical Engineering from Drexel University. He joined the Department of Mathematics and Computer Science at Drexel in 1964, eventually becoming Assistant Department Head for Undergraduate Programs until his retirement in 1993. Al's career always had a strong focus on the teaching of calculus. In addition to supervising graduate teaching assistants, he coordinated the calculus program at Drexel for many years, wrote the solutions manuals for this text, and assisted in its various revisions. Starting in 1980, he became actively involved in developing software and fostering the use of computers in mathematics education at both the high school and college levels. In 1984 he designed and incorporated the first computer-related materials into the calculus curriculum at Drexel.

In addition to his work in collegiate mathematics, Al actively participated in a variety of programs to stimulate an interest in mathematics and engineering among high school students. He gave numerous invited talks and workshops on the use of computers in mathematics education, and he served as project director of an NSF grant for creating computer graphics mathematics laboratories.

Al received the Lindback Award for excellence in teaching, as well as three outstanding teaching awards from student organizations at Drexel. He also received the Drexel University College of Science Award for dedication and service to students.

PREFACE

ABOUT THIS EDITION

This is a major revision. The goal for this edition is to create a contemporary text that incorporates the best features of calculus reform, yet preserves the main structure of an established and well-tested calculus course. This book is intended for those who want to move forward with calculus reform but do not want to completely dismantle their current course structure with radical or unproved materials. The most salient changes are as follows:

- **Technology** — Each chapter ends with a set of exercises that are designed to be solved using computer algebra systems or graphing calculators. Many of the exercises involve applications, and almost all of them can be solved in a variety of ways that are limited only by the student's imagination.
- **Streamlining** — The text is more than 200 pages shorter than the previous edition. We achieved this by using a less wasteful text design and by rewriting almost every section with the goal of *greater clarity in less space*. No material was omitted or modified for the sake of brevity at the cost of understandability, and the quality of the exposition was ensured by a team of outstanding reviewers that included a Polya award winner (excellence in exposition) and a Lindback award winner (excellence in teaching).
- **Revision of Multivariate Calculus** — The multivariate calculus material was completely rewritten, incorporating the concept of a vector field and focusing more on the major applications of vector analysis to physics and engineering.
- **New Material** — Material not included in previous editions was added: Jacobians, parametric representations of surfaces, Kepler's laws, conics in polar coordinates, integrals with respect to arc length, vector fields, and an appendix with some basic material on complex variables that can serve as a reference for engineers and students who need this material for other courses.
- **Early Transcendental Option** — The chapter on logarithms was completely rewritten. The exposition is greatly improved, and the material is now structured in such a way that much of it can be covered earlier in the text for those who want an earlier treatment of logarithms and exponentials. A free *Early Transcendental Supplement* is available to help implement this option. That supplement breaks the material in Sections 7.1 and 7.2 into self contained units and suggests where those units might be inserted earlier in the text.
- **More Use of Calculator Computations in the Exposition** — We assume in this edition that the student has a numerical calculator available as he or she reads the text, and numerical computations are used more extensively in developing concepts.

OTHER FEATURES

- **Rule of Four** — The term "rule of four" has recently been coined to describe exposition that presents ideas from the symbolic, geometric, computational, and verbal viewpoints. Readers familiar with earlier editions of this text will recognize that this has always been an integral part of my writing style. This style continues in this edition.
- **Early Differential Equations Option** — First-order linear and separable differential equations appear in the chapter on logarithmic and exponential functions (Chapter 7). This allows us to give some nice applications of logarithms and

exponentials immediately and also helps meet the needs of those engineering and science students who require this material in courses taken concurrently with calculus. This section can be omitted or deferred with no difficulty.

- **Early Logarithm and Exponential Option** — Sections 7.1 to 7.3 provide a *preliminary* discussion of logarithms and exponentials that does not rely on the integral definition of the logarithm or on the theory of inverse functions. Thus, these sections can be isolated and presented earlier, reserving the integral definition of the logarithm and the more theoretical material in Sections 7.4 and 7.5 for later coverage.
- **Trigonometry Review** — Deficiencies in trigonometry plague many students, so I have included a substantial trigonometry review in Appendix B.
- **Rigor** — The challenge of writing a good calculus book is to strike the right balance between rigor and clarity. My goal is to present precise mathematics to the fullest extent possible for the freshman audience, but where clarity and rigor conflict I choose clarity. However, I believe it to be essential that the student understand the difference between a careful proof and an informal argument, so I try to make it clear to the reader when arguments are informal. Theory involving δ-ϵ arguments appear in separate sections, so they can be bypassed if desired.
- **Historical Notes** — The biographies and historical notes have been a hallmark of this text from its first edition, and new biographies have been added in this edition. All of the biographical material has been distilled from standard sources with the goal of capturing the personalities of the great mathematicians and bringing them to life for the student.
- **Section Exercises** — Section exercise sets begin with routine problems and progress gradually toward problems of greater difficulty. Exercises that require a calculator are listed at the beginning of the exercise set and marked with the icon [C]. Many exercise sets contain so-called "spiral" problems, which revisit earlier problem types using concepts from the current section.

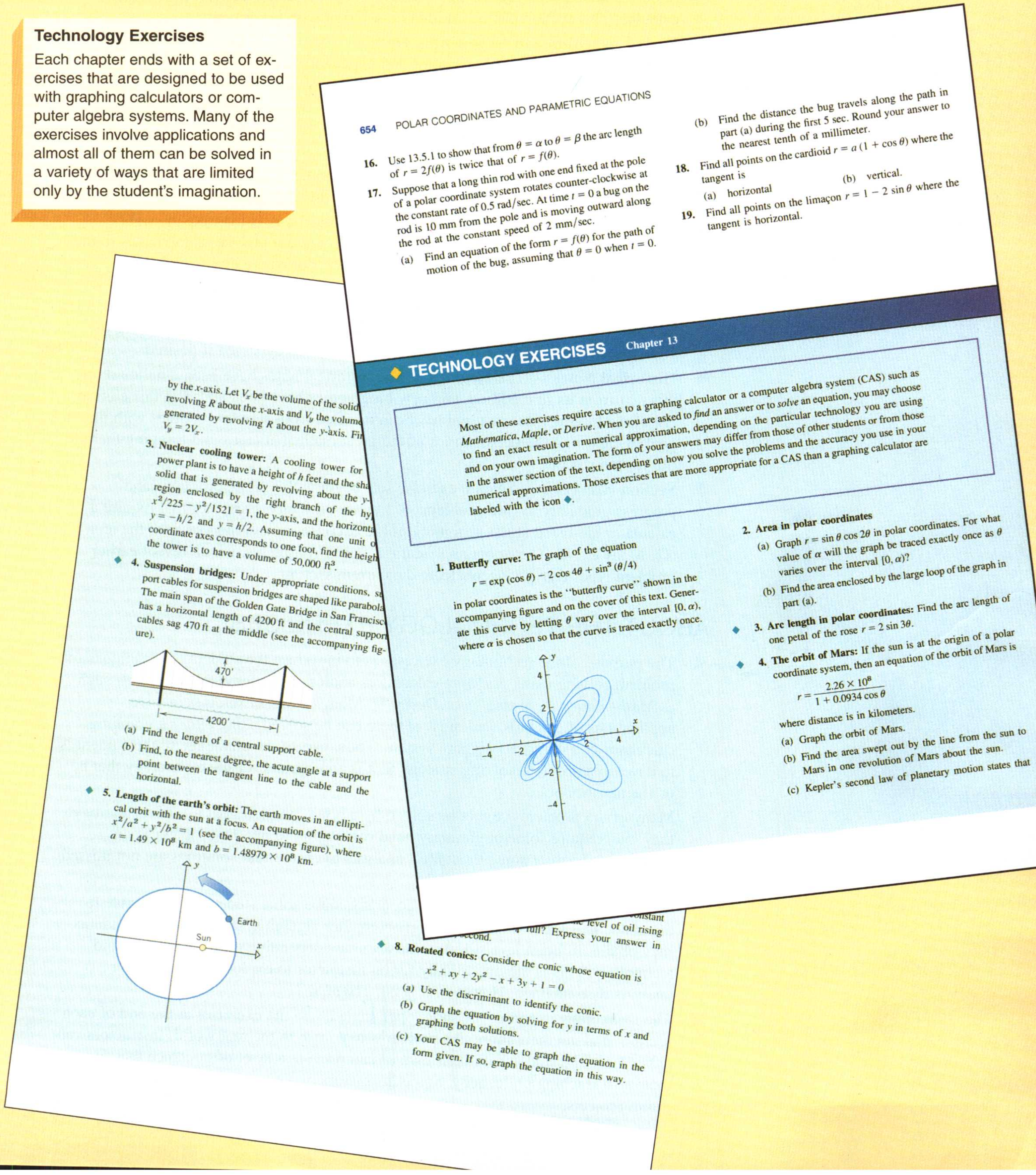
ABOUT THE TECHNOLOGY EXERCISES

- The purpose of the technology exercises is to introduce the student to techniques of problem solving using graphing calculators and/or computer algebra systems such as *Mathematica*™, *Maple*™, or *Derive*™. Many of these exercises involve applications of calculus, and most of them can be solved using *either* a graphing calculator or a computer algebra system. Thus, part of the challenge to the student is to develop a problem-solving strategy that is appropriate for the technology that he or she has available.
- Many of the problems cannot be solved by a blind, unintelligent use of technology; they may require some preliminary hand calculation to put the problem in an appropriate form or some thoughtful analysis to ensure that solutions are not missed when technology is applied.
- Many problems will raise issues of accuracy, since some students may be able to avoid decimal approximations using a computer algebra system and other students may obtain different levels of decimal accuracy depending on their strategy and technology. This is the opportunity for an instructor to explore issues of error analysis if so inclined. However, it is not essential.
- The technology exercises are more open-ended than the exercises at the end of each section, making them more like problems that arise in the real world. Instructors can either leave the students on their own or can provide a level of guidance that fits their own teaching philosophy. Some instructors may want to use these exercises for group projects.

FEATURED IN THIS EDITION

Technology Exercises

Each chapter ends with a set of exercises that are designed to be used with graphing calculators or computer algebra systems. Many of the exercises involve applications and almost all of them can be solved in a variety of ways that are limited only by the student's imagination.



654 POLAR COORDINATES AND PARAMETRIC EQUATIONS

16. Use 13.5.1 to show that from $\theta = \alpha$ to $\theta = \beta$ the arc length of $r = 2f(\theta)$ is twice that of $r = f(\theta)$.

17. Suppose that a long thin rod with one end fixed at the pole of a polar coordinate system rotates counter-clockwise at the constant rate of 0.5 rad/sec. At time $t = 0$ a bug on the rod is 10 mm from the pole and is moving outward along the rod at the constant speed of 2 mm/sec.
(a) Find an equation of the form $r = f(\theta)$ for the path of motion of the bug, assuming that $\theta = 0$ when $t = 0$.
(b) Find the distance the bug travels along the path in part (a) during the first 5 sec. Round your answer to the nearest tenth of a millimeter.

18. Find all points on the cardioid $r = a(1 + \cos\theta)$ where the tangent is
(a) horizontal (b) vertical.

19. Find all points on the limaçon $r = 1 - 2\sin\theta$ where the tangent is horizontal.

◆ **TECHNOLOGY EXERCISES** Chapter 13

Most of these exercises require access to a graphing calculator or a computer algebra system (CAS) such as *Mathematica*, *Maple*, or *Derive*. When you are asked to *find* an answer or to *solve* an equation, you may choose to find an exact result or a numerical approximation, depending on the particular technology you are using and on your own imagination. The form of your answers may differ from those of other students or from those in the answer section of the text, depending on how you solve the problems and the accuracy you use in your numerical approximations. Those exercises that are more appropriate for a CAS than a graphing calculator are labeled with the icon ◆.

1. Butterfly curve: The graph of the equation

$$r = \exp(\cos\theta) - 2\cos 4\theta + \sin^3(\theta/4)$$

in polar coordinates is the "butterfly curve" shown in the accompanying figure and on the cover of this text. Generate this curve by letting θ vary over the interval $[0, \alpha)$, where α is chosen so that the curve is traced exactly once.

2. Area in polar coordinates
(a) Graph $r = \sin\theta\cos 2\theta$ in polar coordinates. For what value of α will the graph be traced exactly once as θ varies over the interval $[0, \alpha)$?
(b) Find the area enclosed by the large loop of the graph in part (a).

◆ **3. Arc length in polar coordinates:** Find the arc length of one petal of the rose $r = 2\sin 3\theta$.

◆ **4. The orbit of Mars:** If the sun is at the origin of a polar coordinate system, then an equation of the orbit of Mars is

$$r = \frac{2.26 \times 10^8}{1 + 0.0934\cos\theta}$$

where distance is in kilometers.
(a) Graph the orbit of Mars.
(b) Find the area swept out by the line from the sun to Mars in one revolution of Mars about the sun.
(c) Kepler's second law of planetary motion states that

by the *x*-axis. Let V_x be the volume of the solid revolving *R* about the *x*-axis and V_y the volume generated by revolving *R* about the *y*-axis. Fi $V_y = 2V_x$.

3. Nuclear cooling tower: A cooling tower for power plant is to have a height of *h* feet and the sha solid that is generated by revolving about the *y*- region enclosed by the right branch of the hy $x^2/225 - y^2/1521 = 1$, the *y*-axis, and the horizonta $y = -h/2$ and $y = h/2$. Assuming that one unit o coordinate axes corresponds to one foot, find the heigh the tower is to have a volume of 50,000 ft³.

◆ **4. Suspension bridges:** Under appropriate conditions, s port cables for suspension bridges are shaped like parabola The main span of the Golden Gate Bridge in San Francisc has a horizontal length of 4200 ft and the central suppor cables sag 470 ft at the middle (see the accompanying figure).
(a) Find the length of a central support cable.
(b) Find, to the nearest degree, the acute angle at a support point between the tangent line to the cable and the horizontal.

◆ **5. Length of the earth's orbit:** The earth moves in an elliptical orbit with the sun at a focus. An equation of the orbit is $x^2/a^2 + y^2/b^2 = 1$ (see the accompanying figure), where $a = 1.49 \times 10^8$ km and $b = 1.48979 \times 10^8$ km.

...onstant ...level of oil rising ...full? Express your answer in ...cond.

◆ **8. Rotated conics:** Consider the conic whose equation is

$$x^2 + xy + 2y^2 - x + 3y + 1 = 0$$

(a) Use the discriminant to identify the conic.
(b) Graph the equation by solving for *y* in terms of *x* and graphing both solutions.
(c) Your CAS may be able to graph the equation in the form given. If so, graph the equation in this way.

Clarity

Clarity has been the hallmark of the Anton texts. The fifth edition achieves a rare combination of outstanding exposition and sound mathematics to the fullest extent possible for the freshman audience.

Table 17.7.1

DETERMINATION OF LIMITS	INTEGRAL
	$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{\rho_0} f(\rho,\theta,\phi)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

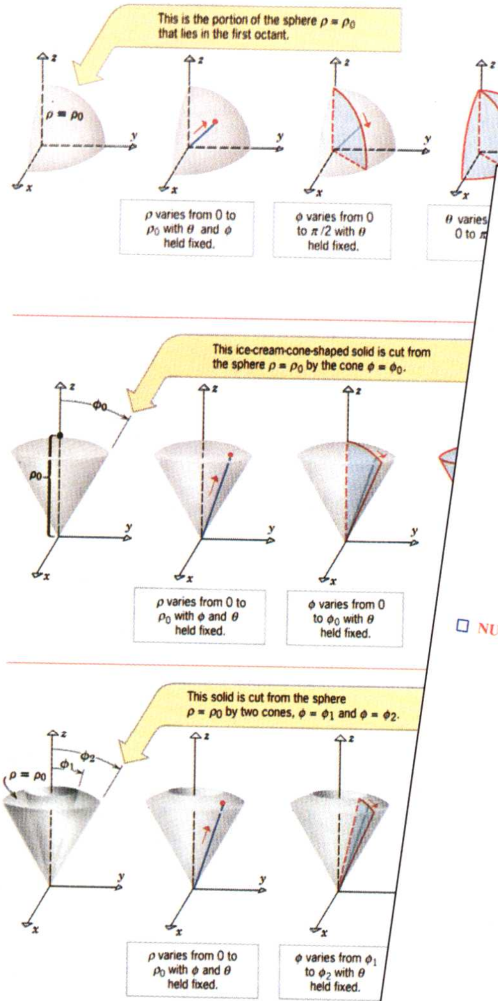


Table 2.4.3

MATHEMATICAL SITUATION	NOTATION	HOW TO READ THE NOTATION
The value of $f(x)$ approaches the number L_1 as x approaches x_0 from the right side.	$\lim_{x\to x_0^+} f(x) = L_1$	The limit of $f(x)$ as x approaches x_0 from the right is equal to L_1.
The value of $f(x)$ approaches the number L_2 as x approaches x_0 from the left side.	$\lim_{x\to x_0^-} f(x) = L_2$	The limit of $f(x)$ as x approaches x_0 from the left is equal to L_2.
The value of $f(x)$ approaches the number L as x approaches x_0 from either the left or right side; that is, $\lim_{x\to x_0^+} f(x) = \lim_{x\to x_0^-} f(x) = L$	$\lim_{x\to x_0} f(x) = L$	The limit of $f(x)$ as x approaches x_0 is equal to L.

□ NUMERICAL PITFALLS

It is important to keep in mind that the limits in (1) and (2) are really guesses about the behavior of $f(x)$ based on numerical evidence obtained by evaluating $f(x)$ at selected values of x. It is conceivable that different choices of x might have produced different conclusions about the limit. For example, consider the function

$$f(x) = \sin\frac{\pi}{x}$$

The values of $f(x)$ in Table 2.4.4 would lead us to believe that

$$\lim_{x\to 0^+} \sin\frac{\pi}{x} = \lim_{x\to 0^-} \sin\frac{\pi}{x} = 0$$

Table 2.4.4

x (RADIANS)	$f(x) = \sin\frac{\pi}{x}$	x (RADIANS)	$f(x) = \sin\frac{\pi}{x}$
$x = 1$	$\sin \pi = 0$	$x = -1$	$\sin(-\pi) = 0$
$x = 0.1$	$\sin 10\pi = 0$	$x = -0.1$	$\sin(-10\pi) = 0$
$x = 0.01$	$\sin 100\pi = 0$	$x = -0.01$	$\sin(-100\pi) = 0$
$x = 0.001$	$\sin 1000\pi = 0$	$x = -0.001$	$\sin(-1000\pi) = 0$
$x = 0.0001$	$\sin 10{,}000\pi = 0$	$x = -0.0001$	$\sin(-10{,}000\pi) = 0$
⋮	⋮	⋮	⋮

However, this is not correct; the values of $f(x)$ actually oscillate between -1 and 1 with increasing rapidity as x approaches 0 from either the left or the right. This is illustrated in Figure 2.4.9, which shows an artistically enhanced computer-generated graph of f. For example, if $x > 0$, then values of 1 occur when $x = \frac{2}{5}, \frac{2}{9}, \frac{2}{13}, \ldots$ and values of -1 occur when $x = \frac{2}{3}, \frac{2}{7}, \frac{2}{11}, \ldots$ (verify). Thus, the values of $f(x)$ do not approach any limiting value as

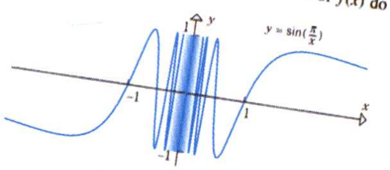


Figure 2.4.9

Streamlined Exposition

Almost every section in the fifth edition has been rewritten with the goal of greater clarity in less space. No material was omitted or modified for the sake of brevity at the cost of understandability, and the quality of the exposition was ensured by a team of outstanding, award-winning instructors and expositors.

Revision of Multivariable Calculus

The multivariable calculus material was completely rewritten, incorporating the concept of a vector field and focusing more on the major applications of vector analysis to physics and engineering.

18.1 VECTOR FIELDS

In this section we consider functions that associate vectors with points in 2-space or 3-space. We shall see that such functions play an important role in the study of fluid flow, gravitational force fields, electromagnetic force fields, and a wide range of other applied problems.

VECTOR FIELDS

To motivate the mathematical ideas in this section, consider a unit point mass located at any point in the universe. According to Newton's Universal Law of Gravitation, the earth exerts an attractive force on the mass that is directed toward the earth's center and has a magnitude that is inversely proportional to the square of the distance from the mass to the earth's center (Figure 18.1.1). This association of force vectors with points in space is called the earth's *gravitational field*. A similar idea arises in fluid flow. Imagine a stream in which the water flows horizontally at every level, and consider the layer of water at a

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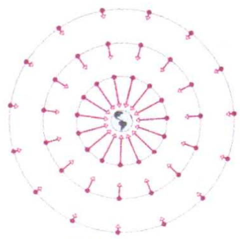


Figure 18.1.1

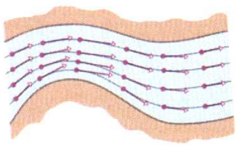


Figure 18.1.2

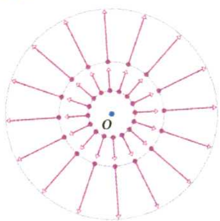


Figure 18.1.3

specific depth. At each point of the layer, the water has a certain ... velocity ... represent by a vector at that point (Figure 18.1.2). This association of velocity ... points in the two-dimensional layer is called the *flow field* at that layer. These ideas are captured in the following definition.

18.1.1 DEFINITION. A ***vector field*** is a function that associates a unique vector $\mathbf{F}(P)$ with each point P in a region of 2-space or 3-space.

Example 1 Let O be a fixed point in 2-space, and for each point P in 2-space define the vector field $\mathbf{F}(P)$ by $\mathbf{F}(P) = \overrightarrow{OP}$. Some typical vectors in this vector field are shown in Figure 18.1.3. In that figure we have followed the standard convention of positioning the vector $\mathbf{F}(P)$ with its initial point at P. ◀

Observe that the concept of a vector field has been defined without reference to a coordinate system; it is said to be a ***coordinate-free*** definition. However, for computational purposes it is often desirable to work with vector fields in coordinate systems. If $\mathbf{F}(P)$ is a vector field in 2-space with an xy-coordinate system, then the point P has coordinates (x, y), and the components of the vector $\mathbf{F}(P)$ are functions of x and y. Thus, $\mathbf{F}(P)$ can be expressed as

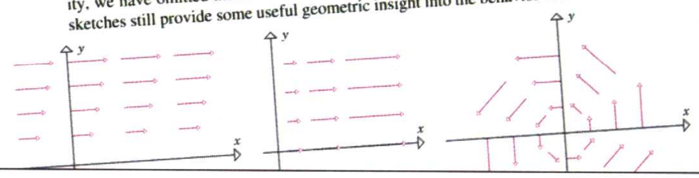
$$\mathbf{F}(x, y) = f(x, y)\mathbf{i} + g(x, y)\mathbf{j}$$

Similarly, in 3-space with an xyz-coordinate system, a vector field $\mathbf{F}(P)$ can be expressed as

$$\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$$

Just as it is impossible to describe a curve completely by plotting finitely many points, so it is impossible to describe a vector field completely by drawing finitely many vectors. Nevertheless, it is often possible to get a useful picture of a vector field by sketching a finite number of vectors that are well chosen.

Example 2 Figure 18.1.4 shows sketches of three vector fields in 2-space. For simplicity, we have omitted the scales and selected vectors that do not overlap; nevertheless, the sketches still provide some useful geometric insight into the behavior of the fields. ◀



PARAMETRIC REPRESENTATION OF SURFACES

We have seen that curves in 3-space can be represented parametrically by three equations involving one parameter. Similarly, surfaces in 3-space can be represented by three equations involving two parameters, say u and v, as

$$x = f(u, v), \quad y = g(u, v), \quad z = h(u, v)$$

or by a single vector-valued function

$$\mathbf{r}(u, v) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = f(u, v)\mathbf{i} + g(u, v)\mathbf{j} + h(u, v)\mathbf{k}$$

We can view $\mathbf{r}(u, v) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ as a ***radius vector*** from the origin to a point (x, y, z) that moves over the surface as u and v vary (Figure 16.1.17).

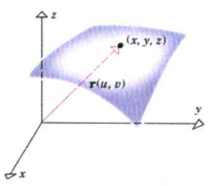


Figure 16.1.17

Example 11 Consider the portion of the paraboloid $z = 4 - x^2 - y^2$ that lies in the first octant (Figure 16.1.18). We can obtain a parametric representation of this surface by letting $x = u$ and $y = v$, from which it follows that $z = 4 - u^2 - v^2$. Thus, the paraboloid can be represented parametrically as

$$x = u, \quad y = v, \quad z = 4 - u^2 - v^2$$

or in vector form as

$$\mathbf{r}(u, v) = u\mathbf{i} + v\mathbf{j} + (4 - u^2 - v^2)\mathbf{k}$$

New Material

Material not included in the previous edition has been added: parametric representation of surfaces, Jacobians, conics in polar coordinates, integrals with respect to arc length, vector fields, Kepler's laws, and an appendix with basic material on complex variables.

Calculators and Computers in the Exposition

The student is assumed to have a numerical calculator available as he or she reads the text, and numerical computations are used extensively in developing concepts.

which implies that the graph of e^x is concave up on $(-\infty, +\infty)$.

Similarly, we can verify that $\ln x$ is increasing and concave down from its first and second derivatives. For all x in $(0, +\infty)$ we have

$$\frac{d}{dx}[\ln x] = \frac{1}{x} > 0$$

which implies that $\ln x$ is increasing on $(0, +\infty)$, and

$$\frac{d^2}{dx^2}[\ln x] = \frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2} < 0$$

which implies that $\ln x$ is concave down on $(0, +\infty)$.

☐ LIMITS INVOLVING $\ln x$ AND e^x

The following limits, which are consistent with Figure 7.3.1, will be proved later.

$$\lim_{x\to+\infty} e^x = +\infty \quad \text{and} \quad \lim_{x\to-\infty} e^x = 0 \qquad (1\text{–}2)$$

$$\lim_{x\to+\infty} \ln x = +\infty \quad \text{and} \quad \lim_{x\to 0^+} \ln x = -\infty \qquad (3\text{–}4)$$

The graph of $y = \ln x$ rises so slowly that Figure 7.3.1 does not adequately convey that

$$\lim_{x\to+\infty} \ln x = +\infty$$

even though we shall prove this to be so later. Moreover, since the graph of e^x is the reflection of the graph of $\ln x$ about the line $y = x$, the slow growth of $\ln x$ corresponds to a rapid growth of e^x. Table 7.3.2, which was generated with a calculator, illustrates the slow growth of $\ln x$ and the rapid growth of e^x.

Table 7.3.2

x	e^x	$\ln x$
1	2.718282	0
2	7.389056	0.693147
3	20.08554	1.098612
4	54.59815	1.386294
5	148.4132	1.609438
6	403.4288	1.791759
7	1096.633	1.945910
8	2980.958	2.079442
9	8103.084	2.197225
10	22026.47	2.302585

☐ EXPONENTIAL AND LOGARITHMIC GROWTH

Mathematicians often use powers of x as a "measuring stick" for describing how rapidly a function grows. For example, we shall prove later that if n is any positive integer, then

$$\lim_{x\to+\infty} \frac{e^x}{x^n} = +\infty \quad \text{and} \quad \lim_{x\to+\infty} \frac{\ln x}{x^n} = 0$$

Limit (5) tel
n, division b
more rapidl
slowly that d
results in a li
integer powe
The follow

$\lim_{x\to+\infty} \frac{x}{e}$

Furthermore, t

$\lim_{x\to 0^+} x^n$

☐ OBTAINING GRAPHS USING PROPERTIES OF EXPONENTS AND LOGARITHMS

Graphs of equat
making appropri

Example 1 Sk

Thus, from Formula 90

$$\int e^{\pi x}\sin^{-1}(e^{\pi x})\,dx = \frac{1}{\pi}\left[u\sin^{-1}u + \sqrt{1-u^2}\right] + C$$
$$= \frac{1}{\pi}\left[e^{\pi x}\sin^{-1}(e^{\pi x}) + \sqrt{1-e^{2\pi x}}\right] + C \quad ◀$$

☐ USING COMPUTER ALGEBRA SYSTEMS

The form of an answer to an indefinite integration can vary widely between computer algebra systems, depending on the method of integration used by the program and the manner in which it simplifies the result. Indeed, the variation can be so great that it may be difficult to see that results produced by different programs are actually equivalent.

Table 9.1.1 shows how the integrals in Example 1 are evaluated by *Mathematica*, *Maple*, and *Derive*. (Some of these were simplified using the computer algebra system's simplification capability.) In the exercises we ask the reader to verify that the results produced by the three computer algebra systems are equivalent to the results obtained from the endpaper Table of Integrals.

Table 9.1.1

	$\int x^2\sqrt{7+3x}\,dx$
TABLE OF INTEGRALS	$\frac{2}{2835}(135x^2 - 252x + 392)(7+3x)^{3/2}$
Mathematica	$\text{Sqrt}[7+3x]\left(\frac{784}{405} - \frac{56x}{135} + \frac{2x^2}{15} + \frac{2x^3}{7}\right)$
Maple	$\frac{2}{2835}(7+3x)^{3/2}(392 - 252x + 135x^2)$
Derive	$\frac{2(3x+7)^{3/2}(135x^2 - 252x + 392)}{2835}$
	$\int \sqrt{x-4x^2}\,dx$
TABLE OF INTEGRALS	$\frac{8x-1}{16}\sqrt{x-4x^2} + \frac{1}{64}\sin^{-1}(8x-1)$
Mathematica	$\left(-\left(\frac{1}{16}\right) + \frac{x}{2}\right)\text{Sqrt}[x-4x^2] - \frac{\text{ArcSin}[1-8x]}{64}$
Maple	$-\frac{1}{16}(-8x+1)\sqrt{x-4x^2} + \frac{1}{64}\arcsin(8x-1)$
Derive	$\frac{\text{ASIN}(8x-1)}{64} + \frac{(8x-1)\sqrt{x(1-4x)}}{16}$
	$\int e^{\pi x}\sin^{-1}(e^{\pi x})\,dx$
TABLE OF INTEGRALS	$\frac{1}{\pi}\left[e^{\pi x}\sin^{-1}(e^{\pi x}) + \sqrt{1-e^{2\pi x}}\right]$
Mathematica	$\frac{\text{Sqrt}[1 - E^{2\,\text{Pi}\,x}]}{\text{Pi}} + \frac{E^{\text{Pi}\,x}\,\text{ArcSin}[E^{\text{Pi}\,x}]}{\text{Pi}}$
Maple	$\frac{e^{\pi x}\arcsin(e^{\pi x}) + \sqrt{1-(e^{\pi x})^2}}{\pi}$
Derive	$\frac{e^{\pi x}\,\text{ASIN}(e^{\pi x})}{\pi} + \frac{(1-e^{2\pi x})^{3/2}}{3\pi} + \frac{\sqrt{1-e^{2\pi x}}(e^{2\pi x}+2)}{3\pi}$

Historical Perspectives

Historical biographies that focus on the personalities of the great mathematicians bring these people to life and give the student a sense of mathematical history.

The relationship between continuity and differentiability was of great historical significance in the development of calculus. In the early nineteenth century mathematicians believed that the graph of a continuous function could not have too many points of nondifferentiability bunched up. They felt that if a continuous function had many points of nondifferentiability, these points, like the tips of a sawblade, would have to be separated from each other and joined by smooth curve segments (Figure 3.2.12). This misconception was shattered by a series of discoveries beginning in 1834. In that year a Bohemian priest, philosopher, and mathematician named Bernhard Bolzano* discovered a procedure for constructing a continuous function that is not differentiable at any point. Later, in 1860, the great German mathematician, Karl Weierstrass** produced the first formula for such a function. The graphs of such functions are impossible to draw; it is as if the corners are so numerous that any segment of the curve, when suitably enlarged, reveals more corners. The discovery of these pathological functions was important in that it made mathematicians distrustful of their geometric intuition and more reliant on precise mathematical proof. However, they remained only mathematical curiosities until the early 1980s, when applications of them began to emerge. During the past 10 years they have started to play a fundamental role in the study of geometric objects called ***fractals***. Fractals have revealed an order to natural phenomena that were previously dismissed as random and chaotic.

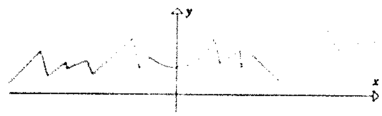


Figure 3.2.12

*BERNHARD BOLZANO (1781–1848). Bolzano, the son of an art dealer, was born in Prague, Bohemia (Czechoslovakia). He was educated at the University of Prague, and eventually won enough mathematical fame to be recommended for a mathematics chair there. However, Bolzano became an ordained Roman Catholic priest, and in 1805 he was appointed to a chair of Philosophy at the University of Prague. Bolzano was a man of great human compassion; he spoke out for educational reform, he voiced the right of individual conscience over government demands, and he lectured on the absurdity of war and militarism. His views so disenchanted Emperor Franz I of Austria that the emperor pressed the Archbishop of Prague to have Bolzano recant his statements. Bolzano refused and was then forced to retire in 1824 on a small pension. Bolzano's main contribution to mathematics was philosophical. His work helped convince mathematicians that sound mathematics must ultimately rest on rigorous proof rather than intuition. In addition to his work in mathematics, Bolzano investigated problems concerning space, force, and wave propagation.

**KARL WEIERSTRASS (1815–1897). Weierstrass, the son of a customs officer, was born in Ostenfelde, Germany. As a youth Weierstrass showed outstanding skills in languages and mathematics. However, at the urging of his dominant father, Weierstrass entered the law and commerce program at the University of Bonn. To the chagrin of his family, the rugged and congenial young man concentrated instead on fencing and beer drinking. Four years later he returned home without a degree. In 1839 Weierstrass entered the Academy of Münster to study for a career in secondary education, and he met and studied under an excellent mathematician named Christof Gudermann. Gudermann's ideas greatly influenced the work of Weierstrass. After receiving his teaching certificate, Weierstrass spent the next 15 years in secondary education teaching German, geography, and mathematics. In addition, he taught handwriting to small children. During this period much of Weierstrass's mathematical work was ignored because he was a secondary schoolteacher and not a college professor. Then, in 1854, he published a paper of major importance which created a sensation in the mathematics world and catapulted him to international fame overnight. He was immediately given an honorary Doctorate at the University of Königsberg and began a new career in college teaching at the University of Berlin in 1856. In 1859 the strain of his mathematical research caused a temporary nervous breakdown and led to spells of dizziness that plagued him for the rest of his life. Weierstrass was a brilliant teacher and his classes overflowed with multitudes of auditors. In spite of his fame, he never lost his early beer-drinking congeniality and was always in the company of students, both ordinary and brilliant. Weierstrass was acknowledged as the leading mathematical analyst in the world. He and his students opened the door to the modern school of mathematical analysis.

2.5.2 THEOREM. *For any polynomial*

$$p(x) = c_0 + c_1x + \cdots + c_nx^n$$

and any real number a,

$$\lim_{x\to a} p(x) = c_0 + c_1a + \cdots + c_na^n = p(a)$$

Proof.

$$\begin{aligned}\lim_{x\to a} p(x) &= \lim_{x\to a}(c_0 + c_1x + \cdots + c_nx^n)\\ &= \lim_{x\to a} c_0 + \lim_{x\to a} c_1x + \cdots + \lim_{x\to a} c_nx^n\\ &= \lim_{x\to a} c_0 + c_1 \lim_{x\to a} x + \cdots + c_n \lim_{x\to a} x^n\\ &= c_0 + c_1a + \cdots + c_na^n = p(a)\end{aligned}$$ ∎

Example 2 If we apply Theorem 2.5.2 to the limit pr... the intermediate steps and write immediately

$$\lim_{x\to 5}(x^2 - 4x + 3) = 5^2 - 4(5) + 3 = 8$$

LIMITS INVOLVING 1/x

The following limits are suggested by the graph of ... numerical calculations in Table 2.5.2.

$$\lim_{x\to 0^+}\frac{1}{x} = +\infty, \quad \lim_{x\to 0^-}\frac{1}{x} = -\infty, \quad \lim_{x\to +\infty}\frac{1}{x} = \ldots$$

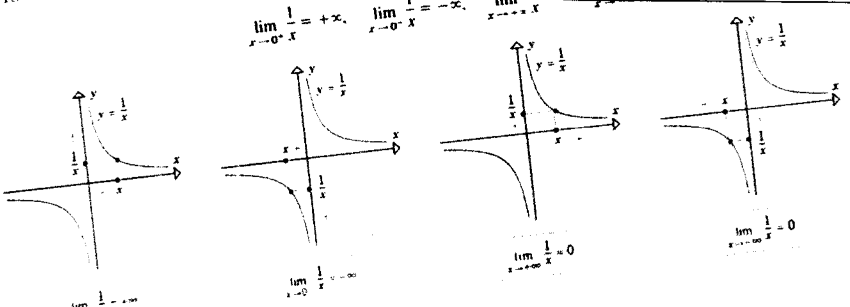


Figure 2.5.3

Table 2.5.2

	VALUES						CONCLUSION
x	1	10	100	1000	10,000	...	As $x \to +\infty$ the value of $1/x$ decreases toward zero.
$1/x$	1	.1	.01	.001	.0001	...	
x	−1	−10	−100	−1000	−10,000	...	As $x \to -\infty$ the value of $1/x$ increases toward zero.
$1/x$	−1	−.1	−.01	−.001	−.0001	...	
x	1	.1	.01	.001	.0001	...	As $x \to 0^+$ the value of $1/x$ increases without bound.
$1/x$	1	10	100	1000	10,000	...	
x	−1	−.1	−.01	−.001	−.0001	...	As $x \to 0^-$ the value of $1/x$ decreases without bound.
$1/x$	−1	−10	−100	−1000	−10,000	...	

Rule of Four

The term "rule of four" describes the presentations of ideas from symbolic, geometric, computational, and verbal viewpoints. Readers of earlier editions will recognize that this has always been an integral part of the Anton writing style. The style continues in the fifth edition.

Engineering Applications

The text now includes major applications of vector analysis to engineering and physics.

the value of div **F** will not vary much from its value div $\mathbf{F}(P_0)$ at the center, and we can reasonably approximate div **F** by the constant div $\mathbf{F}(P_0)$ on G. Thus, the Divergence Theorem implies that the flux $\Phi(G)$ of **F** across $\sigma(G)$ can be approximated as

$$\Phi(G) = \iint_{\sigma(G)} \mathbf{F}\cdot\mathbf{n}\, dS = \iiint_G \operatorname{div}\mathbf{F}\, dA \approx \operatorname{div}\mathbf{F}(P_0) \iiint \ldots$$

from which we obtain the following approximat…

$$\operatorname{div}\mathbf{F}(P_0) \approx \frac{\Phi(G)}{\operatorname{vol}(G)}$$

The expression on the right side of (8) is called … let the radius of the sphere approach zero [so t… plausible that the error in this approximation app… the point P_0 is given exactly by

$$\operatorname{div}\mathbf{F}(P_0) = \lim_{\operatorname{vol}(G)\to 0} \frac{\Phi(G)}{\operatorname{vol}(G)}$$

or equivalently,

$$\operatorname{div}\mathbf{F}(P_0) = \lim_{\operatorname{vol}(G)\to 0} \frac{1}{\operatorname{vol}(G)} \iint_{\sigma(G)} \mathbf{F}\cdot\mathbf{n}\, dS$$

□ INTERPRETATION OF DIVERGENCE IN FLUID FLOW

This limit, called the ***flux density of* F *at the point*** … of divergence. This results in a definition of diver… tion of a coordinate system, unlike the definition…

If P_0 is a point in an incompressible fluid at which… that $\Phi(G) > 0$ for a sufficiently small sphere G … volume of fluid going out through the surface o… happen if there is some point *inside* the sphere at… net outward flow through the surface would res… sphere, contradicting the incompressibility assum… would have to be a point *inside* the sphere at whic… inward flow through the surface would result in an… an incompressible fluid, points at which div $\mathbf{F}(P$… which div $\mathbf{F}(P_0) < 0$ are called ***sinks***. Fluid enters… sink. In an incompressible fluid without sources…

$$\operatorname{div}\mathbf{F}(P) = 0$$

□ GAUSS' LAW FOR INVERSE-SQUARE FIELDS

at every point P. In hydrodynamics this is called t… ***ible fluids*** and is sometimes taken as the defining …

Some of the major principles of physics are conseq… shall obtain by applying the Divergence Theorem… 18.1.2).

18.7.2 GAUSS' LAW FOR INVERSE-SQUARE FIE…

$$\mathbf{F}(\mathbf{r}) = \frac{c}{\|\mathbf{r}\|^3}\mathbf{r}$$

is an inverse-square field in 3-space, and if … surrounds the origin and has outward orientatio…

$$\Phi = \iint_\sigma \mathbf{F}\cdot\mathbf{n}\, dS = 4\pi c$$

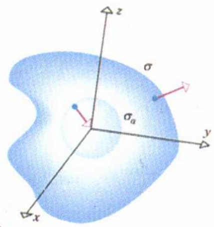


Figure 18.7.5

Recall from Formula (3) of Section 18.1 that **F** can be expressed …

$$\mathbf{F}(x,y,z) = \frac{c}{(x^2+y^2+z^2)^{3/2}}(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})$$

Since the components of **F** are not continuous at the origin, w… Divergence Theorem over the solid enclosed by σ. However, we … difficulty by constructing a sphere of radius a centered at the origin, … sufficiently small that the sphere lies entirely within the region enc… 18.7.5). We shall denote the surface of this sphere by σ_a. The solid G e… and σ is a three-dimensional *simply connected solid* in which σ is the … σ_a is the boundary of a cavity inside the solid. The components of **F** sati… of the Divergence Theorem on G.

Just as we were able to extend Green's Theorem to multiply connec… plane, so it is possible to extend the Divergence Theorem to multiply con… space, provided the surface integral in the theorem is taken over the enti… the outside boundary oriented outward (away from G) and the boundari… oriented inward (toward the cavities). Thus, if **F** is the inverse-square fie… σ_a is oriented inward, then the Divergence Theorem yields

$$\iiint_G \operatorname{div}\mathbf{F}\, dV = \iint_\sigma \mathbf{F}\cdot\mathbf{n}\, dS + \iint_{\sigma_a} \mathbf{F}\cdot\mathbf{n}\, dS$$

But we showed in Example 7 of Section 18.1 that div $\mathbf{F} = 0$, so (12) yiel…

$$\iint_\sigma \mathbf{F}\cdot\mathbf{n}\, dS = -\iint_{\sigma_a} \mathbf{F}\cdot\mathbf{n}\, dS$$

We can evaluate the surface integral over σ_a by expressing the integrand … components; however, it is easier to leave it in vector form. At each point on th… unit normal **n** points inward along a radius from the origin, and hence $\mathbf{n} = -\mathbf{r}$… (13) yields

$$\begin{aligned}\iint_\sigma \mathbf{F}\cdot\mathbf{n}\, dS &= -\iint_{\sigma_a} \frac{c}{\|\mathbf{r}\|^3}\mathbf{r}\cdot\left(-\frac{\mathbf{r}}{\|\mathbf{r}\|}\right) dS\\ &= \iint_{\sigma_a} \frac{c}{\|\mathbf{r}\|^4}(\mathbf{r}\cdot\mathbf{r})\, dS\\ &= \iint_{\sigma_a} \frac{c}{\|\mathbf{r}\|^2}\, dS\\ &= \frac{c}{a^2}\iint_{\sigma_a} dS \qquad \boxed{\|\mathbf{r}\| = a \text{ on } \sigma_a}\\ &= \frac{c}{a^2}(4\pi a^2) \qquad \boxed{\text{The integral is the surface area of the sphere.}}\\ &= 4\pi c\end{aligned}$$

which establishes (10).

It follows from Example 3 of Section 18.1 with $q = 1$ that a single charged particle… charge Q located at the origin creates an inverse-square field

$$\mathbf{F}(\mathbf{r}) = \frac{Q}{4\pi\epsilon_0\|\mathbf{r}\|^3}\mathbf{r}$$

15.7 KEPLER'S LAWS OF PLANETARY MOTION

One of the great advances in the history of astronomy occurred in the early 1600s when Johannes Kepler deduced from empirical data that all planets in our solar system move in elliptical orbits with the sun at a focus. Subsequently, Isaac Newton showed mathematically that such planetary motion is the consequence of an inverse-square law of gravitational attraction. In this section we shall use the concepts developed in the preceding sections of this chapter to derive three basic laws of planetary motion, known as* ***Kepler's laws.***

□ KEPLER'S LAWS

In 1609 Johannes Kepler published a book known as *Astronomia Nova* (or sometimes as *Commentaries on the Motions of Mars*) in which he succeeded in distilling thousands of years of observational astronomy into three beautiful laws of planetary motion.

15.7.1 KEPLER'S LAWS.

- First law (***Law of Orbits***). Each planet moves in an elliptical orbit with the sun at a focus.
- Second law (***Law of Areas***). Equal areas are swept out in equal times by the line from the sun to a planet.
- Third law (***Law of Periods***). The square of a planet's period (the time it takes the planet to complete one orbit about the sun) is proportional to the cube of the length of the semimajor axis of its elliptical orbit.

□ CENTRAL FORCES

To derive Kepler's laws, we shall assume that the force exerted by the sun on a planet is always directed toward the sun's center. In general, a force that is always directed toward a fixed point is called a ***central force***…

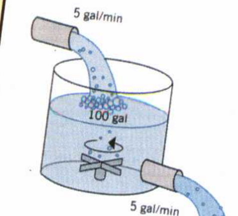


Figure 7.7.2

Example 7 (Mixing Problems) At time $t = 0$, a tank contains 4 lb of 100 gal of water. Suppose that brine containing 2 lb of salt per gallon of w enter the tank at a rate of 5 gal/min and that the mixed solution is draine the same rate (Figure 7.7.2). Find the amount of salt in the tank afte

Solution. Let $y(t)$ be the amount of salt (in pounds) at time t. We are $y(10)$, the amount of salt at time $t = 10$. We will begin by finding an the rate of change of the amount of salt in the tank at time t. Cle

$$\frac{dy}{dt} = \text{rate in} - \text{rate out}$$

where *rate in* is the rate at which salt enters the tank and *rate ou* leaves the tank. But

$$\text{rate in} = (2 \text{ lb/gal}) \cdot (5 \text{ gal/min}) = 10 \text{ lb/min}$$

At time t, the mixture contains $y(t)$ lb of salt in 100 gal of wat salt at time t is $y(t)/100$ lb/gal and

$$\text{rate out} = \left(\frac{y(t)}{100} \text{ lb/gal}\right) \cdot (5 \text{ gal/min}) = \frac{y(t)}{20} \text{ lb/min}$$

Therefore, (15) can be written as

$$\frac{dy}{dt} = 10 - \frac{y}{20}$$

or

$$\frac{dy}{dt} + \frac{y}{20} = 10$$

which is a first-order linear differential equation.

$$y(0) = 4$$

since the tank contains 4 lb of salt at time $t = 0$ Multiplying both sides of (16) by the integra

$$\mu = e^{\int (1/20)\,dt} = e^{t/20}$$

yields

$$\frac{d}{dt}(e^{t/20}y) = 10e^{t/20}$$

$$e^{t/20}y = \int 10e^{t/20}\,dt = 200e^{t/20} + C$$

$$y(t) = 200 + Ce^{-t/20}$$

From the initial condition, (17), it follo

$$4 = 200 + C \quad \text{or} \quad C = -196$$

so

$$y(t) = 200 - 196e^{-t/20}$$

Thus, at time $t = 10$ the amount of

$$y(10) = 200 - 196e^{-0.5} \approx 81$$

☐ EXPONENTIAL GROWTH

Many quantities increase or decre present. Some examples are hum the bloodstream, radioactivity, show how differential equati quantities.

Early Differential Equations Option

In keeping the trends of calculus reform, we have allowed for an early introduction of differential equations and their applications.

44. Use the definition of pH in Exercise 43 to find $[H^+]$ in a solution having a pH equal to
(a) 2.44 (b) 8.06.

45. The perceived loudness L of a sound in decibels (dB) is related to its intensity I in watts/square meter (W/m²) by the equation

$$L = 10 \log (I/I_0)$$

where $I_0 = 10^{-12}$ W/m². Damage to the average ear occurs at 90 dB or greater. Find the decibel level of each of the following sounds and state whether it will cause ear damage.

	SOUND	I
(a)	Jet aircraft (from 500 ft)	1.0×10^2 W/m²
(b)	Amplified rock music	1.0 W/m²
(c)	Garbage disposal	1.0×10^{-4} W/m²
(d)	TV (middle volume from 10 ft)	3.2×10^{-5} W/m²

In Exercises 46–48, use the definition of the decibel level of a sound (see Exercise 45).

46. If one sound is three times as intense as another, how much greater is its decibel level?

47. According to one source, the noise inside a moving automobile is about 70 dB, while an electric blender generates 93 dB. Find blender to tha

48. Suppose that th level of the origi echo, how many sound given that th as low as 10 dB?

49. On the **Richter scale**, related to the released tion

$$\log E = 4.4 + 1.5M$$

(a) Find the energy E of quake that registered M
(b) If the released energy of that of another, how muc on the Richter scale?

50. Suppose that the magnitudes of tw 1 on the Richter scale. Find the energy of the larger earthquake to earthquake. [*Note:* See Exercise 49

In Exercises 51 and 52, use Formula (5) o limit.

51. Find $\lim_{x \to 0} (1 - 2x)^{1/x}$. [*Hint:* Let $t = -2x$

52. Find $\lim_{x \to +\infty} (1 + 3/x)^x$. [*Hint:* Let $t = 3/x$.]

7.2 DERIVATIVES AND INTEGRALS OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

In this section we shall discuss derivatives and integrals involving logarithmic and exponential functions. Our work in this section will be informal in that it will be assumed that the functions considered are differentiable. The mathematical theory that justifies this will be considered later in this chapter.

☐ THE ROLE OF THE NATURAL LOGARITHM IN CALCULUS

In this section we shall be interested in logarithms and exponents from the viewpoint of functions. For $b > 0$ we shall call b^x the ***base b exponential function*** and $\log_b x$ the ***base b logarithm function***. In the case where $b = e$ we shall call e^x the ***natural exponential function*** and $\ln x = \log_e x$ the ***natural logarithm function***.

The natural logarithm function plays a special role in calculus that we can motivate by differentiating $f(x) = \log_b x$, where b is an arbitrary base. For purposes of this introductory discussion we shall assume that the function $\log_b x$ is differentiable, hence continuous for $x > 0$. This will be proved later in the chapter.

Table 7.1.1

x	2^x
3	8.000000
3.1	8.574188
3.14	8.815241
3.141	8.821353
3.1415	8.824411
3.14159	8.824962
3.141592	8.824974

Since we want 2^x to be a continuous function of x, and hence continuous at π, these rational powers of 2 would have to approach 2^π. This suggests that we might *define* 2^π as the limit* of these rational powers of 2. This idea is illustrated numerically in Table 7.1.1, which was generated with a calculator. From Table 7.1.1, the value of 2^π rounded to four decimal places is 8.8250.

In this informal section we shall accept without proof that the preceding limit procedure produces a definition of b^x for irrational x such that the following are true:

- b^x is a continuous function for all $b > 0$.
- b^x is a differentiable function for all $b > 0$.
- The standard properties of exponents such as $b^{u+v} = b^u b^v$ continue to hold.

The first two properties are consistent with the graphs shown in Figure 7.1.1*b*.

☐ REVIEW OF LOGARITHMS

In algebra a logarithm is defined as an exponent. More precisely, if $b > 0$ and $b \neq 1$, then for positive values of x one defines

$$\log_b x$$

(read, "the ***logarithm to the base b of x***") to be that power to which b must be raised to produce x. Thus,

$$\log_{10} 100 = 2$$

since 10 must be raised to the second power to produce 100. Similarly,

$$\log_2 8 = 3 \qquad \boxed{\text{since } 2^3 = 8}$$

Early Transcendental Option

The logarithm chapter has been rewritten to allow for an early transcendental option. The exposition is now structured so that the basic material can be moved forward for those who want an earlier treatment of logarithms and exponentials.

SUPPLEMENTS

GRAPHING CALCULATOR SUPPLEMENTS

The following supplement contains a collection of problems that are intended to be solved on a graphing calculator. The problems are not specific to a particular brand of calculator. Also provided is an overview of the types of calculators available and general instructions for calculator use.

- *Discovering Calculus with Graphing Calculators, Second Edition*
  ISBN: 0-471-00974-1

The following free supplement provides a brief overview of those aspects of graphing calculators that are relevant to the problems in this text. Topics include: choice of viewing window, roundoff error, techniques for finding roots, and common pitfalls associated with graphing calculators.

- *Graphing Calculator Survival Guide*
  ISBN: 0-471-13172-5

SYMBOLIC ALGEBRA SUPPLEMENTS

The following supplements are collections of problems for the student to solve. Each contains a brief set of instructions for using the software as well as an extensive set of problems utilizing the capabilities of the software. The problems range from very basic to those involving real-world applications.

- *Discovering Calculus with DERIVE™, Second Edition*
  Jerry Johnson, *University of Nevada–Reno*
  Benny Evans, *Oklahoma State University*
  ISBN: 0-471-00972-5

- *Discovering Calculus with MAPLE™*
  Kent Harris, *Western Illinois University*
  Robert J. Lopez, *Rose–Hulman Institute of Technology*
  ISBN: 0-471-55156-2

- *Discovering Calculus with MATHEMATICA™*
  Cecilia A. Knoll, Florida Institute of Technology
  Michael D. Shaw, Florida Institute of Technology
  Jerry Johnson, University of Nevada-Reno
  Benny Evans, Oklahoma State University
  ISBN: 0-471-00976-8

CD-ROM VERSION OF CALCULUS FOR IBM COMPATIBLE COMPUTERS

This supplement is an electronic version of Anton's *Calculus,* the *Student's Solutions Manual,* and the *Calculus Companion* on compact disk for use with IBM compatible computers equipped with a CD-ROM drive. All text material and illustrations are stored on disk with an interconnecting network of hyperlinks that allows the student to access related items that do not appear in proximity in the text. A complete keyword glossary and step-by-step discussions of key concepts are also included.

- *CD-ROM Version of Anton Calculus: An Electronic Study Environment*
  Developed by Smart Books. Inc.
  ISBN: 0-471-55803-6

CD-ROM MULTIMEDIA SUPPLEMENT FOR IBM COMPATIBLE COMPUTERS

This highly interactive multimedia CD provides opportunities for students to ask "what if" questions, change parameters, enter their own functions and see the effects of their mathematical decisions in real time. There are 24 multimedia modules accompanied by a laboratory workbook that covers key concepts and spans the entire calculus sequence.

■ *Calculus Connections: A Multimedia Adventure*
Douglas Quinney, University of Keele
Robert Harding, Cambridge University
IntelliPro, Inc.
ISBN: 0-471-13795-2

EARLY TRANSCENDENTAL SUPPLEMENT

This free supplement is designed for those who want an early treatment of exponentials and logarithms. In this short supplement the material in Section 7.2 is broken into smaller self-contained units for easy placement earlier in the text, and a guide for implementing the early transcendental option is provided.

■ *Early Transcendental Supplement to Accompany Anton Calculus 5/E*
ISBN: 0-471-13173-3

LINEAR ALGEBRA SUPPLEMENT

This free supplement is a brief introduction to those aspects of linear algebra that are of immediate concern to the calculus student. The emphasis is on methods rather than proof.

■ *Linear Algebra Supplement to Accompany Anton Calculus / 5E*
ISBN: 0-471-10677-1

STUDENT STUDY RESOURCES

The following supplement is a tutorial, review, and study aid for the student.

■ *The Calculus Companion to Accompany Anton Calculus / 5E*
William H. Barker and James E. Ward, *Bowdoin College*
ISBN: 0-471-10678-x

The following supplement contains detailed solutions to all odd-numbered exercises.

■ *Student's Solutions Manual to Accompany Anton Calculus / 5E*
Albert Herr, *Drexel University*
ISBN: 0-471-10589-9

RESOURCES FOR THE INSTRUCTOR

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