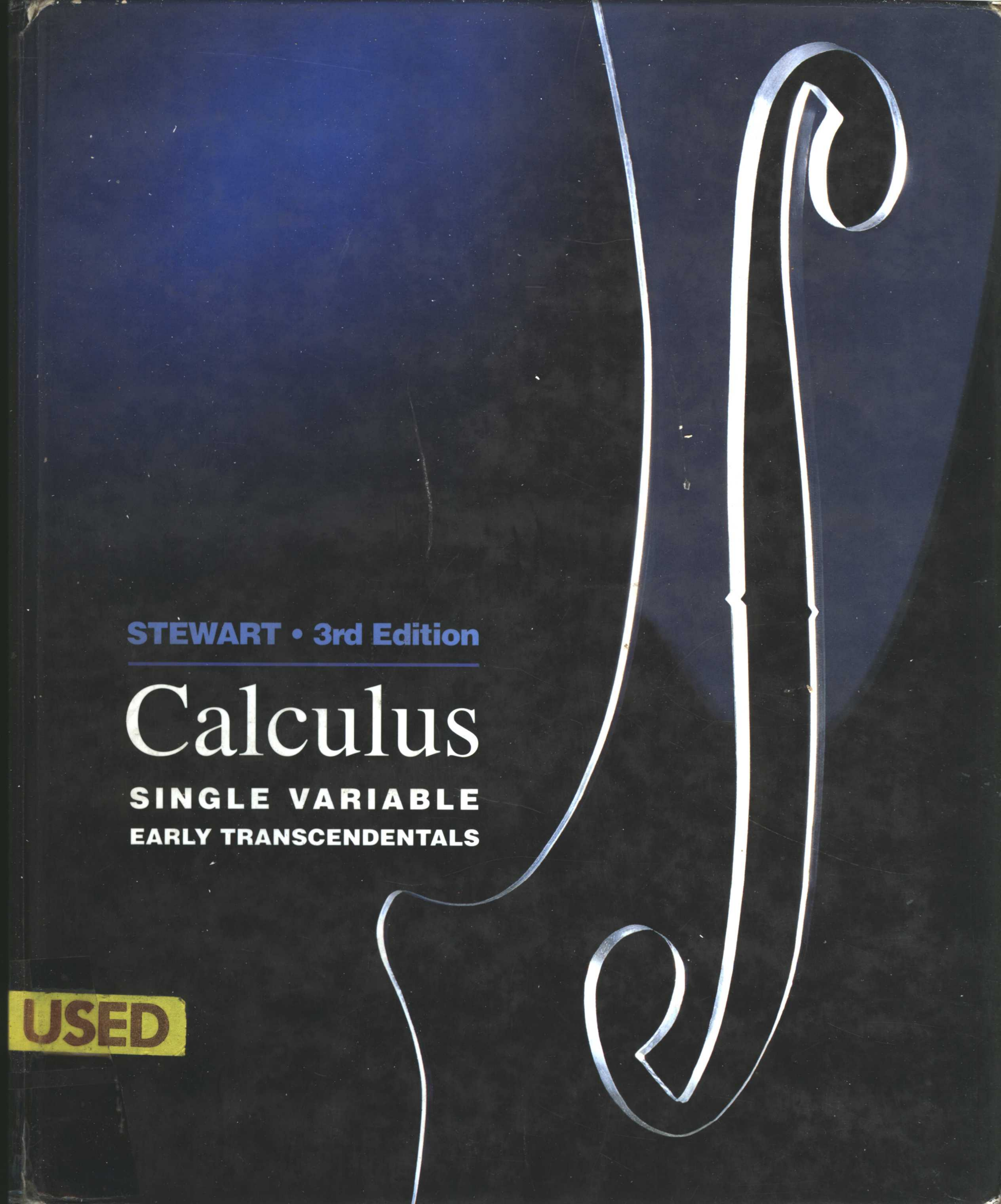

STEWART • 3rd Edition

Calculus

SINGLE VARIABLE

EARLY TRANSCENDENTALS

SINGLE VARIABLE CALCULUS
EARLY TRANSCENDENTALS

Third Edition

JAMES STEWART
McMaster University

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TO JEREMY HAYHURST

PREFACE

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.
>
> *George Polya*

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first two editions, I aim to convey to the student a sense of the utility of calculus and to develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding. Enough mathematical detail is presented so that the treatment is precise, but without allowing formalism to become obtrusive. The instructor can follow an appropriate course between intuition and rigor by choosing to include or exclude optional sections and proofs. Section 1.4, for example, on the precise definition of the limit is an optional section. Although a majority of theorems are proved in the text, some of the more difficult proofs are given in Appendix F.

The last several years have seen much discussion about change in the calculus curriculum and in methods of teaching the subject. I have followed these discussions with great interest and have conducted experiments in my own calculus classes and listened to suggestions from colleagues and reviewers. What follows is a summary of how I have responded to these influences in preparing the third edition. You will see that the *spirit* of reform pervades the book, but within the context of a traditional curriculum.

TECHNOLOGY

For the past five years I have experimented with calculus laboratories for my own students, first with graphing software for computers, then with graphing calculators, and finally with computer algebra systems. Those of us who have watched our students use these machines know how enlivening such experiences can be. We have seen from the expressions on their faces how these devices can engage our students' attention and make them active learners.

Despite my enthusiasm for technology, I think there are potential dangers for misusing it. When I first started using technology, I tended to use it too much, but then I started to see where it is appropriate and where it is not. Many topics in calculus can be explained with chalk and blackboard (and reinforced with pencil and paper exercises) more simply, more quickly, and more clearly than with technology. Other topics cry out for the use of machines. What is important is the *appropriate* use of technology, which can be characterized as involving the *interaction* between technology and calculus. In short, technology is not a panacea, but, when used appropriately, it can be a powerful stimulus to learning.

This textbook can be used either with or without technology and I use three special symbols to indicate clearly when a particular type of machine is required. The symbol means that an ordinary scientific calculator is needed for the calculations in an exercise. The icon indicates an example or exercise that requires the use of either a graphing calculator or a computer with graphing software. (Section 3 in Review and Preview discusses the use of these graphing devices and some of the pitfalls that can arise. Section 4.6 is a good example of what I mean by the interaction between technology and calculus.) The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, or Mathematica) are required. In all cases we assume that the student knows how to use the machine—we rarely give explicit commands.

Some of the exercises designated by or CAS are, in effect, calculus laboratories and require considerable time for their completion. Instructors should therefore consult the solutions manual to determine the complexity of a problem before assigning it. Some of those problems explore the shape of a family of curves depending on one or more parameters. (My students particularly enjoyed Exercise 40 on page 533. It was difficult to get them to leave the computer lab because they were having so much fun investigating the variety of fascinating shapes that these curves can have.) Other such projects involve technology in very different ways. See, for instance, pages 539 (Bézier curves), 588 (logistic sequences), and 482.

VISUALIZATION

One of the themes of the calculus reform movement is the Rule of Three: Topics should be presented numerically, graphically, and symbolically, wherever possible. I believe that, even in its first and second editions, my calculus text has had a stronger focus on numerical and graphical points of view than other traditional books. In the third edition I have taken this principle farther. See pages 114 and 591 for examples of how the Rule of Three comes into play. You will also see that I have included more work with tabular functions and more numerical estimates of sums of series.

I have added many examples and exercises that promote visual thinking. Given the graph of a function, I think it is important for a student to be able to sketch the graph of its derivative (page 116) and also to sketch the graph of an antiderivative (page 310) in a qualitative manner. See pages 158, 167, 185, 306, 377, 386, and 465 for other examples of exercises that test students' visual understanding.

In addition, I have added hundreds of new computer-generated figures to illustrate existing examples. These are not just pretty pictures—they constantly remind students of the geometric meaning behind the result of a calculation. I have also tried to provide more visual insight into formulas and their proofs (see, for instance, pages 126 and 242).

INCREASED EMPHASIS ON PROBLEM SOLVING

My educational philosophy was strongly influenced by attending the lectures of George Polya and Gabor Szego when I was a student at Stanford University. Both Polya and Szego consistently introduced a topic by relating it to something concrete or familiar. Wherever practical, I have introduced topics with an intuitive geometrical or physical description and attempted to tie mathematical concepts to the students' experience.

I found Polya's lectures on problem solving very inspirational and his books *How To Solve It, Mathematical Discovery,* and *Mathematics and Plausible Reasoning* have become the core text material for a mathematical problem-solving course that I instituted and teach at McMaster University. I have adapted these problem-solving strategies to the study of calculus both explicitly, by outlining strategies, and implicitly, by illustration and example.

Students usually have difficulties in situations that involve no single well-defined procedure for obtaining the answer. I think nobody has improved very much on Polya's four-stage problem-solving strategy and, accordingly, I have included in this edition a version of Polya's strategy in Section 4 of Review and Preview, together with several examples and exercises involving precalculus material. I have also rewritten the solutions to certain examples in a more patient manner to make the problem-solving principles more apparent. (See, for instance, Example 1 on page 168.)

The classic calculus situations where problem-solving skills are especially important are related rates problems, maximum and minimum problems, integration, testing series, and solving differential problems. In these and other situations I have adapted Polya's strategies to the matter at hand. In particular, I have retained from prior editions the two separate special sections devoted to problem solving: 7.6 (Strategy for Integration) and 10.7 (Strategy for Testing Series).

In the second edition I included what I call *Problems Plus* after even-numbered chapters. These are problems that go beyond the usual exercises in one way or another and require a higher level of problem-solving ability. The very fact that they do not occur in the context of any particular chapter makes them a little more challenging. For instance, a problem that occurs after Chapter 10 need not have anything to do with Chapter 10. I particularly value problems in which a student has to combine methods from two or three different chapters. In this edition I have added examples to the Problems Plus sections, not as solutions to imitate (there are no problems like them), but rather as examples of how to tackle a challenging calculus problem. (See Example 1 on page 319.) I have also added a large number of good new problems, including some with a geometric flavor (see Problems 9, 10, 18, 27 after Chapter 2 and Problem 32 after Chapter 10). I have been testing these Problems Plus on my own students by putting them on assignments, tests, and exams. Because of their challenging nature I grade these problems in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant. My aim is to teach my students to be unafraid to tackle a problem the likes of which they have never seen before.

REAL WORLD APPLICATIONS

I have eliminated a few of the more arcane applications in the second edition and replaced them with substantial applied problems that I believe will capture the attention of students. See, for instance, Problem 10 on page 379 (investigating the shape of a can), Problem 7 on page 481 (positioning a shortstop to make the best relay to home plate) and Problem 9 (choosing a seat in a movie theater) and Problem 10 (explaining the formation and location of rainbows) on page 482. These are all extended problems that would make good projects. They happen to be located in the Applications Plus sections, which occur after odd-numbered chapters (starting with Chapter 3) and are a counterpart to the Problems Plus. (Again the idea is often to combine ideas and techniques from different parts of the book.) But there are many new applied problems in the ordinary sections of the book as well. (See, for instance, Exercise 54 on page 303 and Exercise 32 on page 186).

OTHER CHANGES

■ I have added historical and biographical margin notes, some of them fairly extensive, in order to enliven the course and to show students that mathematics was created by living, breathing human beings.
■ The review material on inequalities, absolute values, and coordinate geometry that used to appear in the first three sections of Review and Preview has been moved to Appendixes A, B, and C. The Review and Preview now starts with functions.
■ Section 2.9, on linear approximation, has been expanded to include an optional subsection on quadratic approximation. The error in both types of approximation is estimated using graphing devices.
■ Chapter 10 contains more changes than any other chapter. I have added material on numerical estimates of sums of series based on which test was used to prove convergence: the Integral Test (page 601), the Comparison Test (page 607), or the Ratio Test (page 620). The last half of the chapter, on power series, has been completely reorganized and rewritten: Taylor's Formula occurs earlier, error estimates now include those from graphing devices, and applications of Taylor polynomials to physics are emphasized.
■ Section 7.7, on the use of tables of integrals, has been expanded to include the use of computer algebra systems.
■ About 20% of the exercises are new. In most cases, a relatively standard exercise has been replaced by one that uses technology or stimulates visual thinking without technology. Some of the new exercises encourage the development of communication skills by explicitly requesting descriptions, conjectures, and explanations. Many of these exercises are suitable as extended projects.

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The preparation of this and previous editions has involved much time spent reading the reasoned (but sometimes contradictory) advice from a large number of astute reviewers. I greatly appreciate the time they spent to understand my motivation for the approach taken. I have learned something from each of them.

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JAMES STEWART

TO THE STUDENT

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper at hand to make a calculation or sketch a diagram.

Some students start by trying their homework problems and only read the text if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should study the definitions to see the exact meanings of the terms.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected step-by-step fashion with explanatory words and symbols—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix I. There are often several different forms in which to express an answer, so if your answer differs from mine, don't immediately assume you are wrong. There may be an algebraic or trigonometric identity that connects the answers. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you are right and rationalizing the denominator will show that the expressions are equivalent.

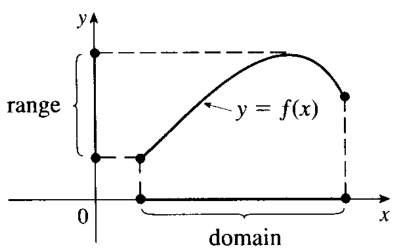
The symbol ▦ means that an ordinary scientific calculator is needed for the calculations in an exercise. The icon ▦ indicates an example or exercise that requires the use of either a graphing calculator or a computer with graphing software. (Section 3 in Review and Preview discusses the use of these graphing devices and some of the pitfalls that you may encounter.) The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, or Mathematica) are required. You will also encounter the symbol ⊘, which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Calculus is an exciting subject; I hope you find it both useful and interesting in its own right.

■ A NOTE ON LOGIC

In understanding the theorems it is important to know the meaning of certain logical terms and symbols. If P and Q are mathematical statements, then $P \Rightarrow Q$ is read as "P implies Q" and means the same as "If P is true, then Q is true." The *converse* of a theorem of the form $P \Rightarrow Q$ is the statement $Q \Rightarrow P$. (The converse of a theorem may or may not be true. For example, the converse of the statement "If it rains, then I take my umbrella" is "If I take my umbrella, then it rains.") The symbol $\Longleftrightarrow$ indicates that two statements are equivalent. Thus $P \Longleftrightarrow Q$ means that both $P \Rightarrow Q$ and $Q \Rightarrow P$. The phrase "if and only if" is also used in this situation. Thus "P is true if and only if Q is true" means the same as $P \Longleftrightarrow Q$. The *contrapositive* of a theorem $P \Rightarrow Q$ is the statement that $\sim Q \Rightarrow \sim P$, where $\sim P$ means *not* P. So the contrapositive says "If Q is false, then P is false." Unlike converses, the contrapositive of a theorem is always true.

CONTENTS



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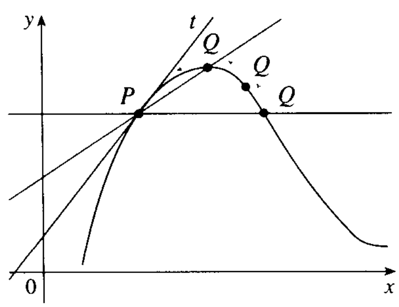

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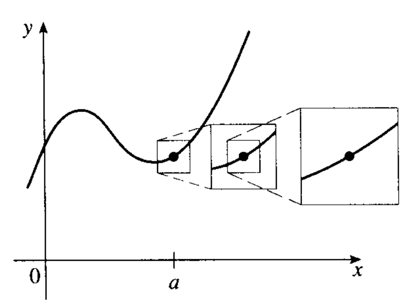

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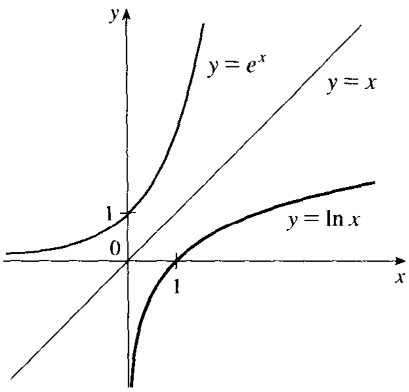
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1
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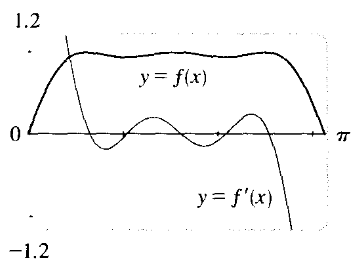
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y = f'(x)
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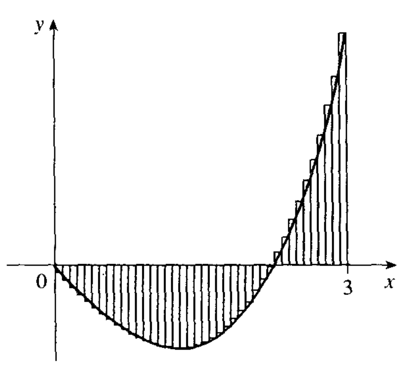
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3
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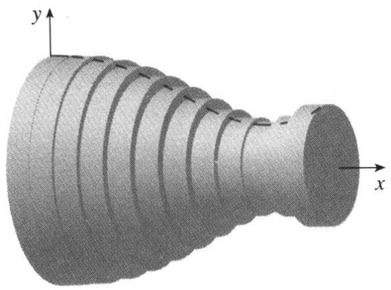
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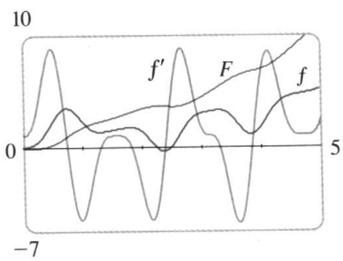
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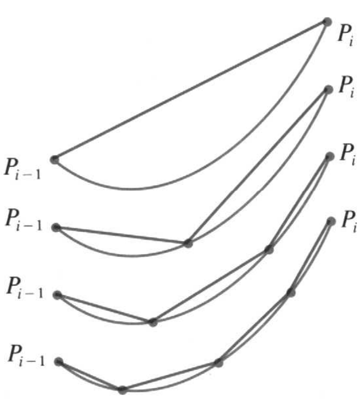
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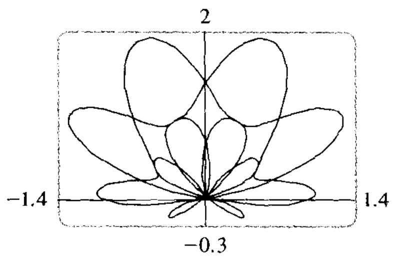

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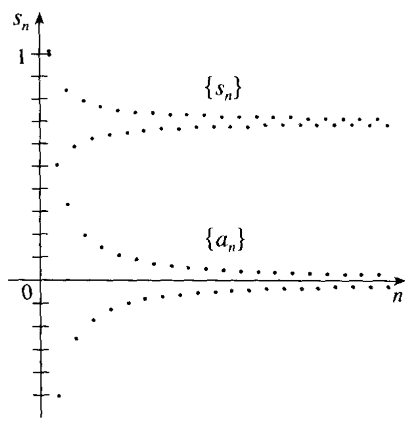

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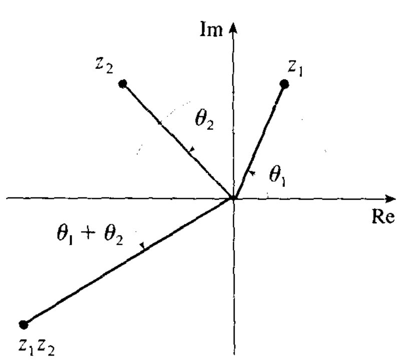

SINGLE VARIABLE CALCULUS
EARLY TRANSCENDENTALS

REVIEW AND PREVIEW

■ In most sciences one generation tears down what another has built and what one has established another undoes. In mathematics alone each generation builds a new story to the old structure.

HERMANN HANKEL

The fundamental objects that we deal with in calculus are functions. So we first review the basic ideas concerning functions, their graphs, and ways of combining them. We list the main types of functions that occur in calculus and its applications and we review the procedures for shifting, stretching, and reflecting their graphs. We then discuss the use of graphing calculators and graphing software for computers. This chapter also contains a discussion of the principles of problem solving that will be useful throughout the book. In the last section we give a preview of some of the principal ideas of calculus. Although it is not absolutely necessary to read this last section, it does provide an overview of the subject and a brief look at some of the reasons for studying calculus.

1 FUNCTIONS AND THEIR GRAPHS

The area A of a circle depends on the radius r of the circle. The rule that connects r and A is given by the equation $A = \pi r^2$. With each positive number r there is associated one value of A, and we say that A is a *function* of r.

The number N of bacteria in a culture depends on the time t. If the culture starts with 5000 bacteria and the population doubles every hour, then after t hours the number of bacteria will be $N = (5000)2^t$. This is the rule that connects t and N. For each value of t there is a corresponding value of N, and we say that N is a function of t.

The cost C of mailing a first-class letter depends on the weight w of the letter. Although there is no single neat formula that connects w and C, the post office has a rule for determining C when w is known.

Each of these examples describes a rule whereby, given a number (r, t, or w), another number (A, N, or C) is assigned. In each case we say that the second number is a function of the first number.

> A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

We usually consider functions for which the sets A and B are sets of real numbers. The set A is called the **domain** of the function. The number $f(x)$ is the **value of f at x** and is read "f of x." The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain, that is, $\{f(x) \mid x \in A\}$.