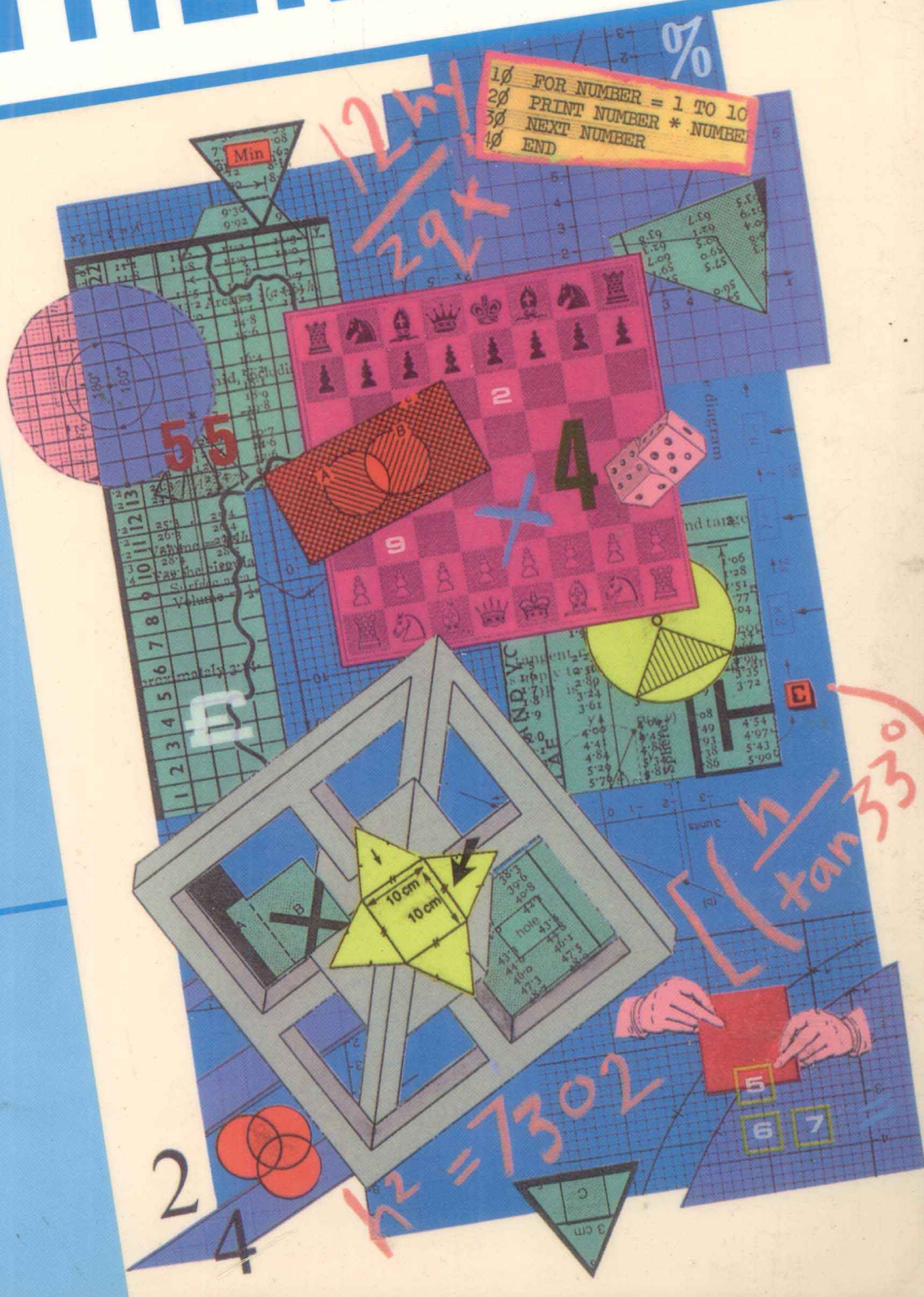
Letts STUDY GUIDE

AGE 11-14

KEY STAGE 3

MATHEMATICS



Ray Williams

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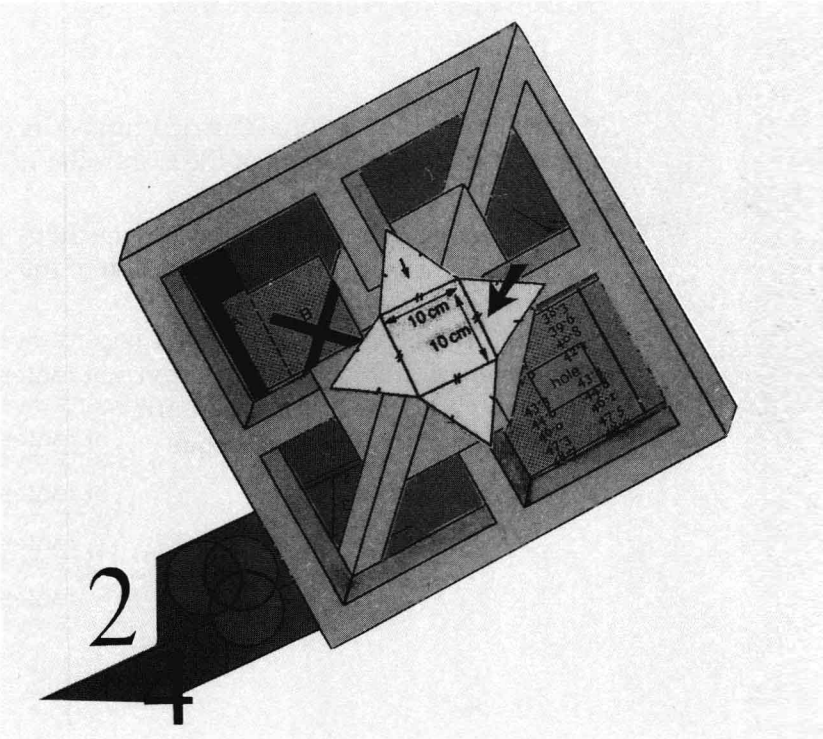
KEY STAGE 3

MATHEMATICS

Ray Williams

- A clear introduction to the new National Curriculum
- Topic by topic coverage, with lots of diagrams and illustrations
- Investigations designed to encourage active learning
- Frequent questions to test your knowledge
- Index and glossary of terms
- Sample National Test questions and answers



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Ray Williams

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Introduction

SUCCESSFUL STUDYING AT KEY STAGE 3

During Key Stage 3 of the National Curriculum, you will have to study the following subjects:

English, Mathematics, Science, Technology, a modern foreign language (usually French or German), Geography and History. If you go to school in Wales, you will also be required to learn Welsh.

This stage of your education is very important because it lays the foundation which you will need to embark upon your GCSE courses. The National Curriculum requires you and all 11–14 year olds to follow the same programmes of study, which define the knowledge and skills you will need to learn and develop during your course.

At school, your teachers will be monitoring your progress. At the end of Key Stage 3, your performance will be assessed and you will be given a National Curriculum level. Most students should reach Level 5 or Level 6, some may reach Levels 7 or 8, or perhaps even higher. In English, Mathematics and Science, you will have to take a National Test towards the end of your last year at Key Stage 3. The results of your tests, also marked in levels, will be set alongside your teachers' assessment of your work to give an overall picture of how you have done.

How this book will help you

This book is designed for you to use at home to support the work you are doing at school. Think of it as a companion or study guide to help you prepare for class work, homework, and for the important National Tests. Inside the book, you will find the level descriptions which will be used to assess your performance. We have included them in the book so that, as you near the end of Key Stage 3, you will be able to check how well you are doing.

Also included at the end of the book is a bank of practice questions. These are of the same style and standard as the questions you will face in your National Tests. Attempting these questions in the months leading up to your tests should help you to do as well as you can.

Reading this book, and doing the questions and activities, will help you get to grips with the most important elements of the National Curriculum in mathematics. Before you begin to read the book itself, take a few moments to read the following sections on 'Maths in the National Curriculum' and 'How to use this book'.

MATHS IN THE NATIONAL CURRICULUM

At Key Stage 3 in mathematics, you are expected to make progress in four Attainment Targets:

- Attainment Target 1: Using and Applying Mathematics
- Attainment Target 2: Number and Algebra
- Attainment Target 3: Shape, Space and Measures
- Attainment Target 4: Handling Data

In each Attainment Target, the mathematics is taught at various levels from Level 1 to Level 8. Exceptional performances from the most able students may, on occasion, exceed Level 8. By the end of Key Stage 3, pupils should expect to be working within Levels 3 to 7. This book is intended to help you in all the Attainment Targets at whatever Level you might be working.

HOW TO USE THIS BOOK

The National Curriculum states what you should be taught in schools but not how it should be taught. This means that schools will have different approaches and may not follow the same path that this book follows. However, by the end of the book you will have covered the mathematics contained in the National Curriculum for Key Stage 3.

The book is written in sections linked with the Attainment Targets:

Section 1 Using and Applying Mathematics
Section 2 Number
Section 3 Algebra
Section 4 Shape, Space and Measures
Section 5 Handling Data

Each section is sub-divided into chapters, each tackling a different topic or topics. One approach to using this book might be:

1. Select a chapter or topic.
2. Each topic usually begins with a 'Can you .....?' statement. These are the skills that are going to be tested in this topic.
3. Try the 'Now test yourself' questions relating to the topic, checking your answers as you go along.
4. Keep working through the different topics until the maths becomes too difficult. (The questions are graded, starting easy but getting gradually harder.)
5. Note the point where you find the work too difficult. This is your 'sticking point'.
6. Tackle another chapter/topic in the same way.
7. Return to your 'sticking points' at a later date to see if you can get any further.
8. Test yourself using the Practice National Test Questions and see what National Curriculum level you can reach.

You can use this book to reinforce the work that you are doing with your maths teacher, picking out the topics that you are doing in class.

You can also use it as a revision book, preparing yourself for your class tests and Key Stage 3 National Tests.

Finally, you can use the Practice National Test Questions as preparation for the real thing to give you an idea of what to expect.

Calculators

Questions designed to be done using a calculator will have this symbol in the margin

Questions designed to be done without using a calculator will have this in the margin

Where there is no symbol then you can assume that the use of a calculator is optional.

CHAPTER 1

Using and applying maths skills

This part of the National Curriculum will be tested in the classroom by your own teacher. It will take place at any time but you should be told in advance.

You could be asked to do several different activities over a period of time and your teacher will mark the work.

There are certain skills that you can practise that will enable you to tackle a task of a practical nature or an investigation. Skills like:

(a) testing theories
(b) making generalizations
(c) breaking down tasks into smaller ones
(d) developing a line of thought
(e) conducting experiments
(f) interpreting diagrams.

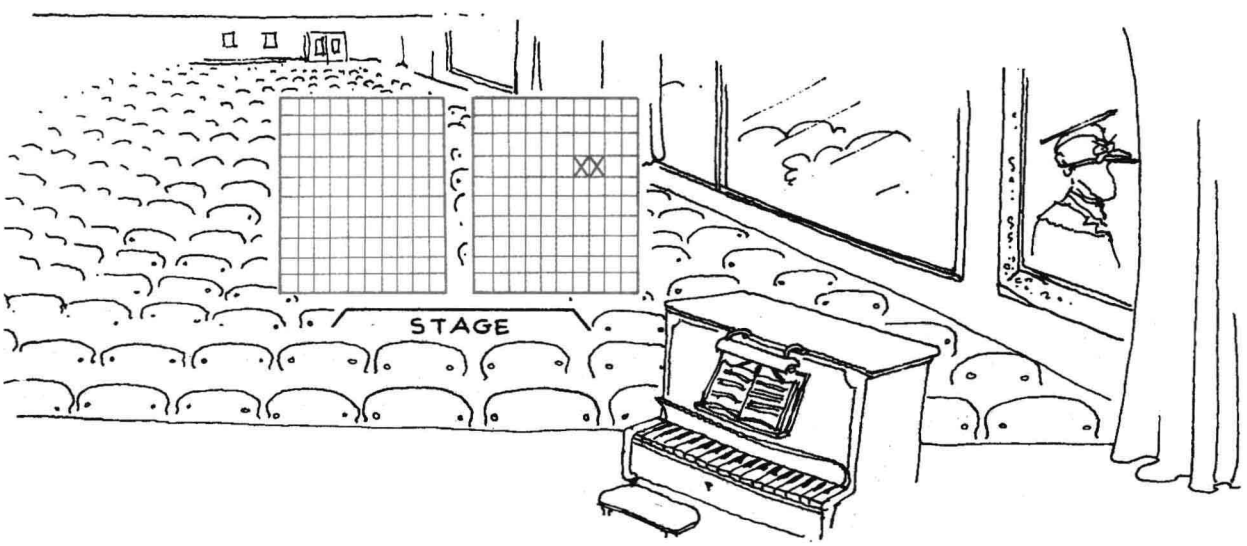
These next few activities try to give you practice in developing those skills.

PRACTICAL MATHS

Can you solve practical problems using mathematics?

Now test yourself

You are going to invent a seating plan for a school pantomime using a system of co-ordinates for numbering the tickets.

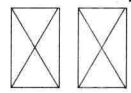


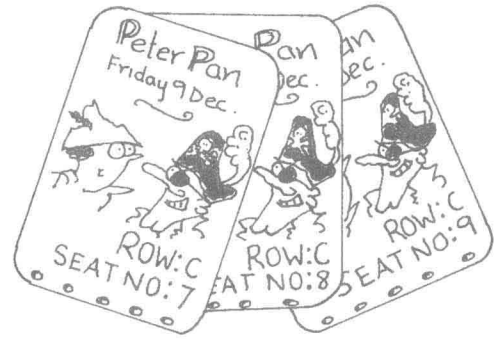
1 Copy the seating plan shown or design one of your own.

2 Label each row by a letter.

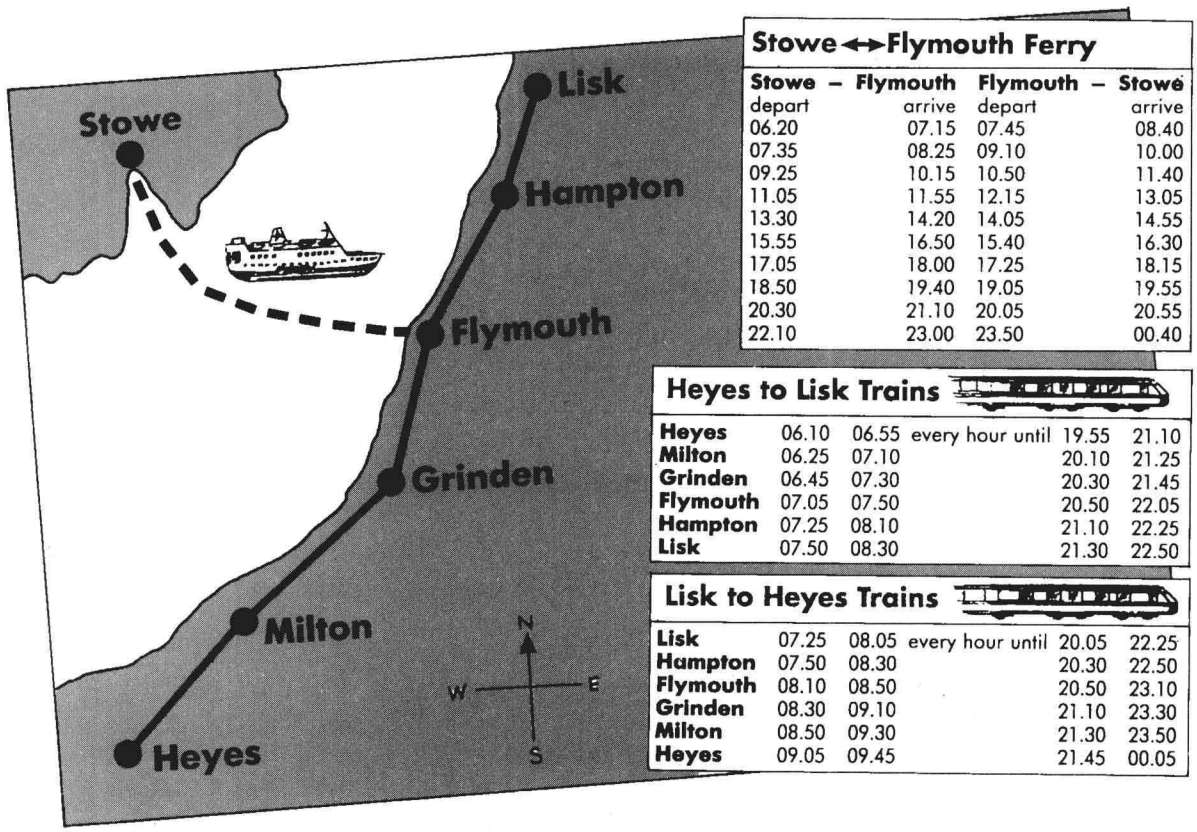
3 Give each seat in every row a number.

4 Mark on your seating plan where these three ticket-holders would sit.

5 How would you mark the tickets that represent the two seats marked by crosses? 



Now test yourself



Stowe ↔ Flymouth Ferry

Stowe – Flymouth		Flymouth – Stowe	
depart	arrive	depart	arrive
06.20	07.15	07.45	08.40
07.35	08.25	09.10	10.00
09.25	10.15	10.50	11.40
11.05	11.55	12.15	13.05
13.30	14.20	14.05	14.55
15.55	16.50	15.40	16.30
17.05	18.00	17.25	18.15
18.50	19.40	19.05	19.55
20.30	21.10	20.05	20.55
22.10	23.00	23.50	00.40

Heyes to Lisk Trains

Heyes	06.10	06.55	every hour until	19.55	21.10
Milton	06.25	07.10		20.10	21.25
Grinden	06.45	07.30		20.30	21.45
Flymouth	07.05	07.50		20.50	22.05
Hampton	07.25	08.10		21.10	22.25
Lisk	07.50	08.30		21.30	22.50

Lisk to Heyes Trains

Lisk	07.25	08.05	every hour until	20.05	22.25
Hampton	07.50	08.30		20.30	22.50
Flymouth	08.10	08.50		20.50	23.10
Grinden	08.30	09.10		21.10	23.30
Milton	08.50	09.30		21.30	23.50
Heyes	09.05	09.45		21.45	00.05

Sally lives at Milton and wishes to visit her grannie who lives at Stowe. Her journey involves a train ride and a ferry to get across the water. She needs to use different timetables to plan her journey.

6 Plan a day visit giving the times of the trains and ferries she could catch. Whatever journeys you decide she should take, work out how long she could stay at her grannie's and how long she would have to wait to make the ferry and train connections.

SEARCHING FOR A PATTERN

Can you solve investigations by searching for a pattern?

Now test yourself

Explore the last digits of the multiples of various numbers;

8, 16, 24, 32, 40, 48 ... Multiples of 8

Record and present the results.

7 Write down the last digits in order for multiples of 2 to 10.

8 What types of numbers are they (e.g. odd, even, square, triangular, etc.)?

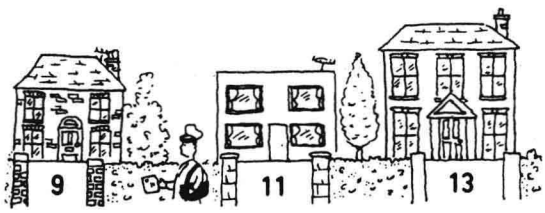
9 Can you spot any patterns?

10 Are there any similarities between the last digits of the multiples of one number and the last digits of the multiples of any other number?

Now test yourself

Investigate the statement for various examples.

"If you add numbers of three consecutive houses you always get a multiple of 3"



11 Investigate for three odd house numbers.
(e.g. $5 + 7 + 9 = 21 = 7 \times 3$ and 21 is a multiple of 3).

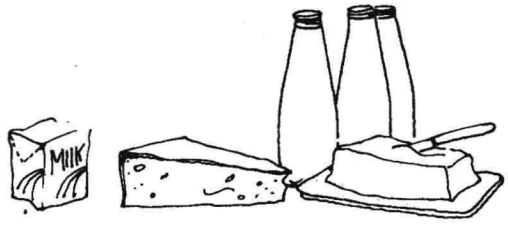
12 Test for more odd house numbers.

13 Test for more even house numbers.

14 Is the statement true for houses that are numbered consecutively (e.g. 1, 2 and 3 or 7, 8 and 9)?
Can you explain why in your own words?

Now test yourself

A mathematical milkman decided to multiply together the house numbers of adjacent houses. What did he find?



15 Work out several more. Don't forget numbers greater than a hundred!

16 What kind of numbers are your answers?

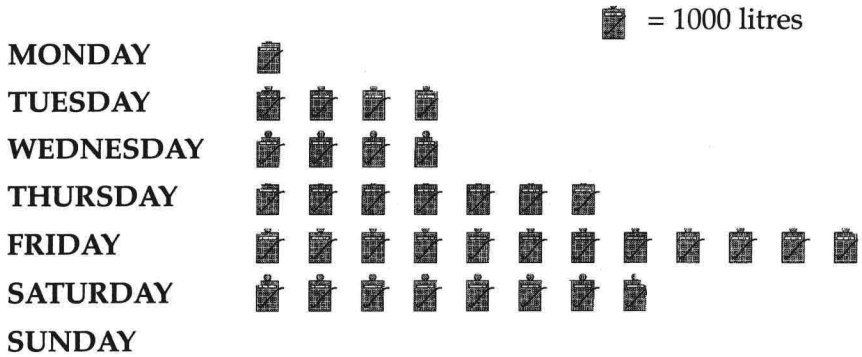
17 Are there any patterns?

18 Are there any numbers that cannot possibly be amongst your answers?
Hint: Start with easy numbers and look for patterns in your answers.

INTERPRETING CHARTS, TABLES AND DIAGRAMS

Can you interpret charts, tables and diagrams by writing sentences about the information they try to convey?

This pictogram shows the amount of petrol sold at a supermarket petrol station for 1 week.



What information does this pictogram tell us?

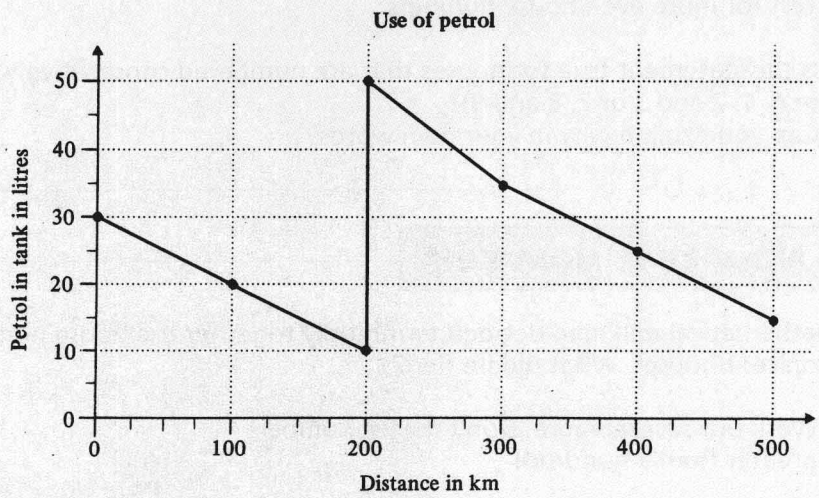
(a) It was closed on Sunday.
(b) Least petrol was sold on Monday. Could this be when the supermarket had a half-day closing?
(c) Most petrol was sold on Thursday, Friday and Saturday. Could this be when there were late-night openings?
(d) There were equal sales on Tuesday and Wednesday. Was the shop equally busy on those days?
(e) Does the petrol station reflect how busy the store was? I think so!
(f) The average daily sales over the six days was 6000 litres.

This ability to read diagrams is something worth practising.

Now test yourself

What can you write about these mathematical charts?

19 On a long journey by car, I recorded the number of litres of petrol in the tank every 100 km. The graph shows this information.



20 The sales in Fenton Music Store of CDs over a six-month period are represented in the following pictogram:

FENTON MUSIC STORE represents 20 CDs

OCTOBER	
NOVEMBER	
DECEMBER	
JANUARY	
FEBRUARY	
MARCH	

21 The figures below show the output of Grafton Fireplaces last year.

MONTH	JAN.	FEB.	MAR.	APR.	MAY
SALES	30	45	70	60	50

22 Last season my football team scored 100 goals. This pie chart shows when the goals were scored during the match.

TESTING BY EXAMPLE

Can you check for yourself whether your theory/rule/generalization is correct by trying out and testing it with a few examples?

Fiona was trying to answer the question:

'$5^2 - 4^2$: Is there a quick way to work out the difference between two squares?'

Fiona's work begins:

Problem:	You have $5^2 - 4^2$.	Do the sum and try to find a rule.
	$5^2 - 4^2$	
	$= 25 - 16$	5 add 4 = $\underline{9}$ which is the answer to the sum.
	$= \underline{9}$	
Try	$7^2 - 6^2$	$7 + 6 = \underline{13}$
	$= 49 - 36$	This rule works.
	$= \underline{13}$	

Rule: The difference between 2 squares is equal to the sum of the two numbers!

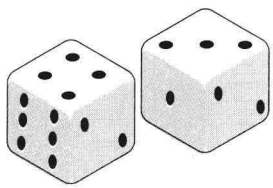
Fiona has made a generalization by stating a rule.
What she must now do is to test her rule by **trying a few more examples!**

Now test yourself

23 Continue with Fiona's work. **Hint:** Try $6^2 - 4^2$; then $6^2 - 3^2$; then $5^2 - 2^2$.
Is she right or wrong?
If she is wrong, can you find another rule?
Test this rule by trying some more examples.

TESTING A THEORY

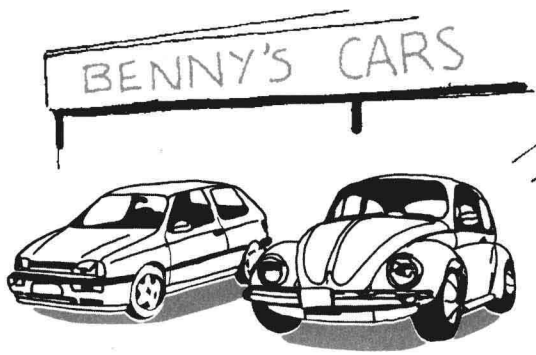
If you take a theory of your own, then can you use your own evidence to put together a good, clear explanation of what you have found out?



Suppose you have a theory that:

'When you roll 2 dice and add the scores together, you get more 7s than any other number!'

This can be tested by performing a simple **experiment**. Don't forget to roll the dice at least 100 times.



Another theory might be:

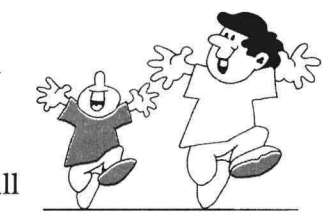
'Red is the most popular colour for a car.'

This can be tested by performing a simple **survey** of at least 500 cars.

Or how about:

'The taller you are, the wider your arm-span!'

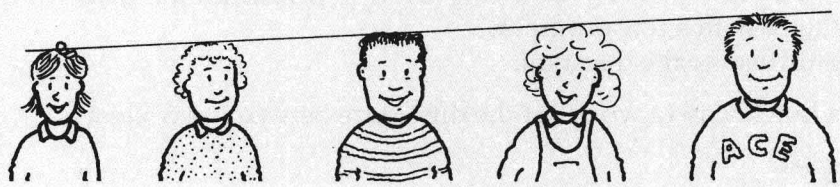
This can be tested by measuring people's heights and arm-spans and plotting a **scattergraph** to see if there is any positive correlation.



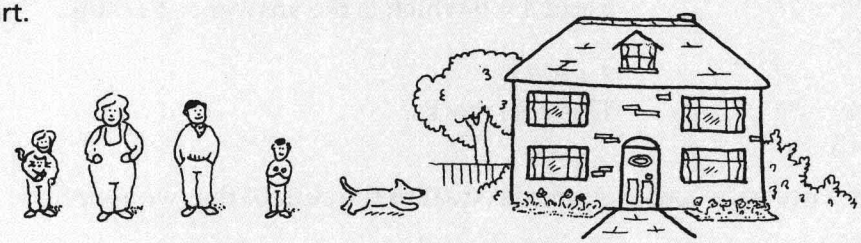
Always be on the lookout for evidence of mathematics in all walks of life that you can use in this way.

Now test yourself

24 Make a hypothesis about children's weight in relation to their height and test it by reference to a scattergraph.



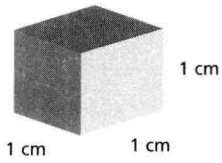
25 Make a hypothesis about the number of people living in the houses in your street and test it by reference to a frequency table and subsequent bar chart and pie chart.



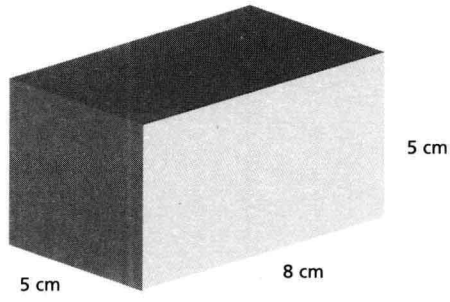
B BREAKING DOWN PROBLEMS

Can you solve quite complex problems by breaking them down into smaller, more manageable tasks?

Suppose you wanted to find out how many sugar cubes 1 cm by 1 cm by 1 cm would fit into a box measuring 5 cm by 5 cm by 8 cm.

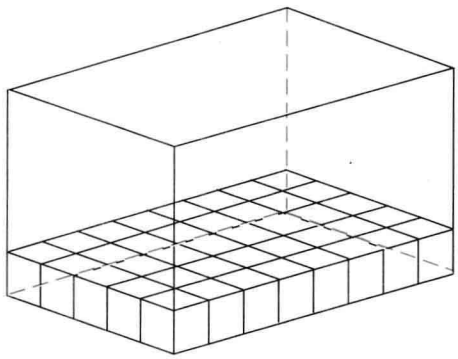


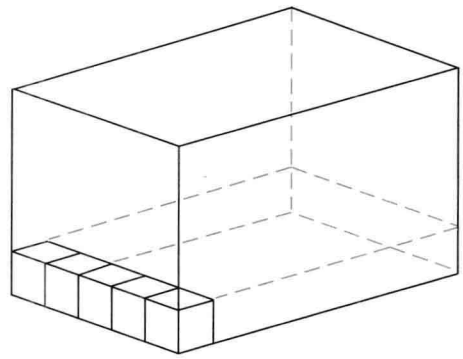
sugar cube



box

To solve the problem, consider the box to be made up of 5 layers of sugar cubes on top of each other.





This could be broken down even further as a layer could be considered as 8 rows of 5 sugar cubes.

By breaking the problem down like this, you can see that the total number of sugar cubes is:

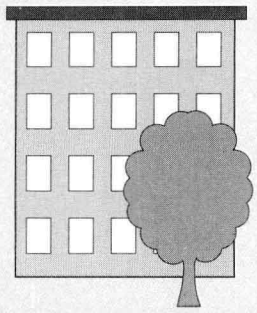
5 layers of 8 rows of 5 sugar cubes
= 5 × 8 × 5
= 200 sugar cubes.

Many problems can be tackled in this way!

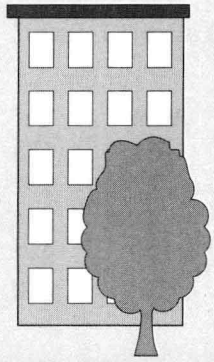
Now test yourself

26 You cannot see some of the windows in these buildings. How many windows are there?

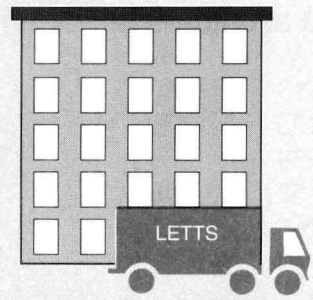
(a)



(b)

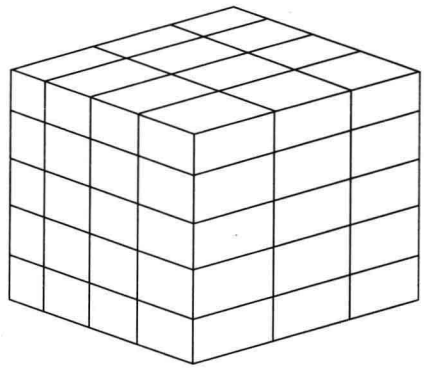


(c)

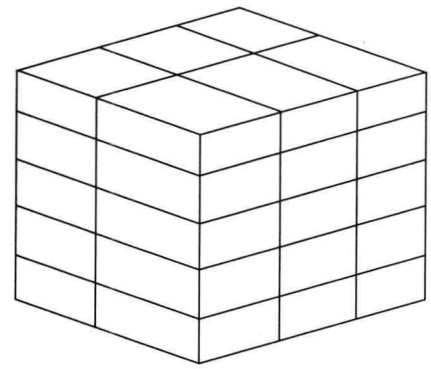


27 (a) How many 1 cm sugar cubes can be put into a box 20 cm long, 15 cm wide and 10 cm high?
(b) How many match boxes 2 cm by 5 cm by 8 cm can be put into a bigger box 40 cm long, 20 cm wide and 10 cm high?

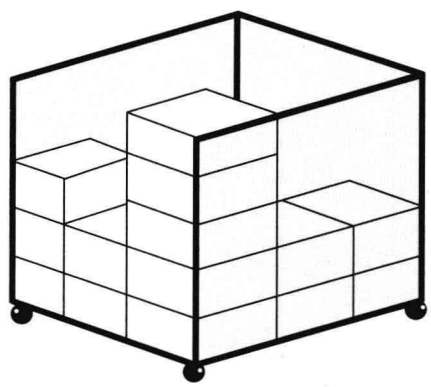
28 How many boxes are in these piles?
(a)



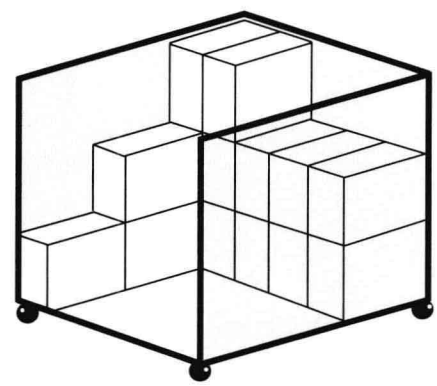
(b)



29 How many boxes will these trolleys hold?
(a)



(b)



ASKING QUESTIONS

If you are presented with a problem together with some information, can you ask yourself questions to arrive at a solution and even suggest related tasks?

Greenacres Nurseries plant two trees, both of which are 100 cm tall.

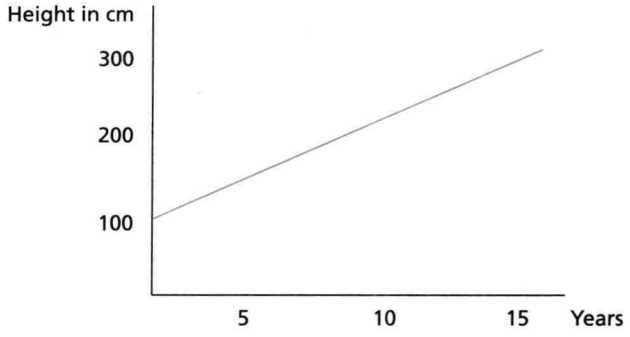
Tree A grows 13 cm each year.
Tree B grows by 10% each year.

Can you say which tree will be taller:
(a) after 2 years; (b) after 5 years; (c) after 10 years?

Tree height in centimetres

Year	A	B	Year	A	B
1	113	110	6	178	177.2
2	126	121	7	191	194.9
3	139	133.1	8	204	214.4
4	152	146.4	9	217	235.8
5	165	161	10	230	259.4

This table could lead to a graph:



Copy the graph axes. I have plotted the heights of tree A. On your axes, copy my graph and then plot the heights of tree B.

(a) After how many years are the trees the same size?
(b) How much taller is Tree B than Tree A after 10 years?
(c) Extend the graphs. Roughly when will Tree B be twice as tall as Tree A?

Now test yourself

Now use the solution to the Tree problem to help you solve these related problems.

30 The world's population in 1994 was 6.0 billion (i.e. 6 000 000 000) people. The world's population is increasing at 3% per year.
(a) In what year will the world's population be around 10 billion?
(b) If this increase was reduced to only 1%, in what year would the world's population be 10 billion?

31 If you invest your money in a bank or building society your money will grow.
(a) What will £100 be worth after 10 years if it grows at 10% per year?
(b) After how many years will you double your money with this annual increase?
(c) After how many years will you double your money with an annual increase of 5%?

THEORY AND EXPERIMENT

Can you see the difference between what is expected and what actually happens when performing a mathematical experiment?

In theory, the probability of an event happening can be worked out as a fraction. We say:

$$\text{Probability of an event} = \frac{\text{Total no. of favourable outcomes}}{\text{Total no. of possible outcomes}}$$

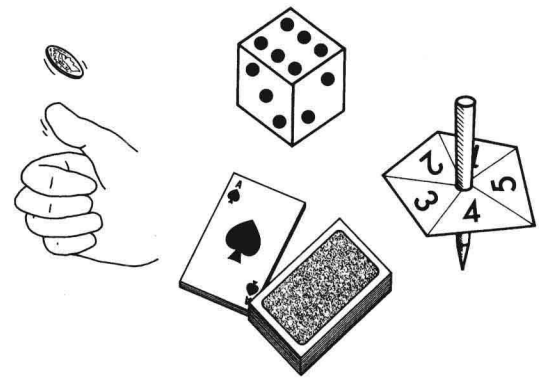
So:

(a) the probability of tossing a coin and it lands on 'heads' = $\frac{1}{2}$

(b) the probability of rolling a die and it lands on a '6' = $\frac{1}{6}$

(c) the probability of spinning a spinner and it falls on a '2' = $\frac{1}{5}$

(d) the probability of cutting a pack of cards and getting a 'spade' = $\frac{1}{4}$



Now test yourself

Remembering this, try to answer these questions.

32 How many heads would you expect if you tossed a coin 20 times?

33 How many 6s would you expect if you rolled a die 30 times?

34 How many scores of 2 would you expect if the spinner is spun 40 times?

35 How many spades would you expect if you cut a pack of playing cards 60 times?

Now perform each of the four experiments outlined above.
Are your results what you expected?
Don't worry, they are not always the same! That is what this exercise sets out to prove!